Modelling of crack formation and growth using FEM for selected structural materials at static loading

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Abstract: The purpose of this paper is to show the results of a study focused on the occurrence of damage heterogeneous materials, especially on the issue of modelling crack formation and propagation. In the beginning the attention is paid to the direct application of the finite element method to different types of materials in order to find critical parameters determining behaviour of materials at damage process. The applications of damage mechanics and possible approaches to model the origin of a crack propagation through modifications in FEM systems are presented and some practical applications are tested. Main effort is devoted to cement fibre composites and the search for new methods for their more accurate modelling, especially close to the field stress concentrator, respectively ahead of the crack tip. Modified XFEM method has been used as a suitable tool for numerical modelling.

Key-Words: Computational damage; crack growth modelling; finite element method.

Received: March 23, 2023. Revised: August 24, 2023. Accepted: September 25, 2023. Published: November 1, 2023.

1 Introduction

The question of ensuring the safety of structural components and predicting their service life is increasingly associated with the development of new devices and components. In the case of constructions, it may be directly dependent on the occurrence of defects that may arise during the production stage or through their lifetime. One of the concepts used in construction and safety assessment is a set of theories and methods known in fracture mechanics. This scientific field, combining continuum mechanics with material engineering, describes the behaviour of defects in structures. It is a complex defect-stress-material relationship. To understand the relationships end extend lifetime, it is necessary to modernize construction practices and also use new numerical methods.

The aim of fracture mechanics is to describe or predict the behaviour of bodies containing defects. In many cases, cracks can lead to total failure of the structure due to fracture. There are two basic approaches for deriving the conditions in the moment of initiation of unstable crack propagation. The first one uses the weakest link theory, the second model considers the accumulation of damage during loading. Failure of structural materials is understood as a continuous process in which the stages of plastic deformation, nucleation and initiation of cracks is intermingled. The final stage in the development of failure of bodies, which is the subject of investigation of fracture mechanics, is the propagation of cracks (unstable or stable). The goal of the presented works was how to find out the mutual relations between physical regularities and the physical laws themselves the essence of the breaking process on the one hand and the theory of continuum mechanics on the other.

The promotion of non-traditional materials, constructions and technologies in the modern construction industry also requires new approaches to the investigation of their physical properties, where proven ones cannot be relied upon. Simulation of the behaviour of material samples, structural elements and buildings as complex structures becomes necessary. After these motivational comments in *Section* 1 some notes to fracture mechanics and to model problems are presented in *Section* 2. The overview of classical approaches and crack growth modelling in *Section* 3, will be followed by some details of damage mechanics and cohesion approach in *Section* 4, with special attention to the modified finite element method in *Section* 5. The principal investigation as to building materials by *Section* 6 is supplied by references to potential applications in *Section* 7 and concluding remarks with future research priorities in *Section* 8.

2 Some notes about fracture mechanics

Fracture mechanics deals with the problem of bodies with cracks. The term fracture refers to the division of a body into two or more parts. For the purpose of fracture mechanics, we define a crack as a violation of the cohesion of bodies along a surface bounded by a curve that is either closed or it ends on the surface of the body.

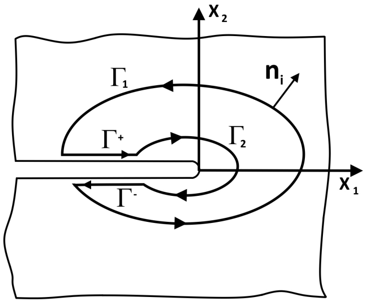

Figure 1: Contour integral definition.

With a simplified view of the mentioned problem, it is crucial to define the stability criteria, which establish the conditions under which an existing or emerging crack will propagate and thus enable the calculation of the corresponding critical stress or the critical length of the crack.

The concept of stability criterion can be extended to the case of general stress concentrators. In such a case, we are looking for conditions under which crack will start to propagate from the considered concentrator. In general, any fracture-mechanical quantity can be used for the formulation of stability criteria, and the criteria can be formulated on the basis of energy, [1], [2], using the deformation energy density, crack driving force, G, J integral, or based on stresses and strains at the crack tip (concept of stress intensity factor, crack opening). For linear fracture mechanics, the more commonly used stability criterion is the K_{IC} criterion (the G_{IC} criterion can be used equivalently). For non-linear fracture shear it is then the J_{IC} criterion, [3], or the crack opening displacement (CTOD) condition, with a number of criteria for combined stress, [4], [5], [6].

The described procedure works quite reliably if we limit ourselves to the first singular term when describing the stress around the crack front, so we are talking about one-parameter fracture mechanics. It is precisely in the case of multiaxial stresses, when we switch to multi-parameter fracture mechanics and work with concepts as are T-stress and Q-parameter, [7]. The general form of the equation in case of static loading for the overall decomposition in the classical (differential) approach, is based on the decomposition of the overall deformation, represented by the strain tensor ϵ, as a square matrix of order 3, into four additional components, namely the elastic, plastic, creep and thermal, [8], denoted as ϵ^e, ϵ^p, ϵ^c and ϵ^θ here. All these quantities will considered as variable in the time t, with values prescribed at $t = 0$ are prescribed (which can be marked as Cauchy initial conditions).

Namely in the isotropic and homogeneous case, taking $i, j \in \{1, 2, 3\}$, we can consider

$$\epsilon^e_{ij} = ((1+\nu)/E)\,\sigma_{ij} - (\nu/E)\,\sigma_*\delta_{ij}\ ;$$

here we need the Kronecker symbol $\delta_{ij} = 1 - \operatorname{sgn}|i-j|$, $\sigma_* = (\sigma_{11} + \sigma_{22} + \sigma_{33})/3$ refers to the principal stress and E, ν is the couple of material parameters: the Young modulus and the Poisson ratio. The contribution of ϵ^p can be expressed using the curve of real stress versus real deformation during plastic loading (nonlinear in general), as an additional material characteristic; The increment of plastic deformation during creep, is evaluated using the finite element method, using the initial deformation approach, typically. The contribution of ϵ^θ comes (if no more detailed thermodynamic analysis is available) from temperature difference, applying the thermal expansion factor. Consequently, very important is evaluation of ϵ^c from

$$\dot\epsilon^c_{ij} = ((3\dot\epsilon^c_{ef})/(2\sigma_{ef}))\,\sigma'_{ij}$$

where the lower index $_{ef}$ means certain scalar effective value, σ' being the deviatoric part of σ, i. e. $\sigma'_{ij} = \sigma_{ij} - \sigma_*\delta_{ij}$, with $i, j \in \{1, 2, 3\}$ again.

Contour integrals play an important role in linear and nonlinear fracture mechanics, see Fig. 1. Well known is the J-integral, [3], [8]. An analogous integral for bodies subjected to creep deformation is called C-integral, [9], [10], [11], [12]. To understand the implementation of individual types of curve integrals, the manual for the WARP3D software system is recommended, [13].

3 Crack growth modelling using classical FEM approaches

In the course of the 1960s, theories were gradually created that were able to describe the behaviour of bodies with cracks by considering plastic zones of larger scale, and thus get closer to more realistic conditions arising during loading of bodies with a crack. Currently, there are several procedures that can be used to solve the problem of simulating crack propagation using FEM. Among the first and oldest are the modelling of stable crack growth using the node release method, where starting criterion is necessary to define. Modelling of crack propagation by the node release method is possible only in the case of a 2D problem, see Fig. 2, where effective plastic deformation (ϵ_{ef}) is displayed.

The first criterion is based on knowledge of the crack length relationship as a function of time, which is assigned to the time period required to solve the modelled problem. The second type of criterion uses the stress definition for crack propagation. The third type of criterion uses the critical value of crack opening, which is the deformation characteristic, as a boundary condition for crack propagation. The principle is to separate the deformable surface of the body from the analytically defined non-deformable surface. During the release of the nodes, the stresses between

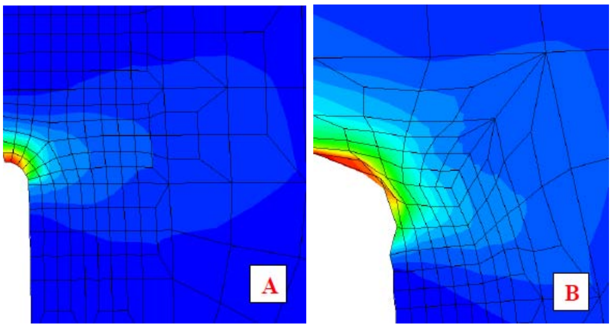

Figure 2: Node release approach for 2D.

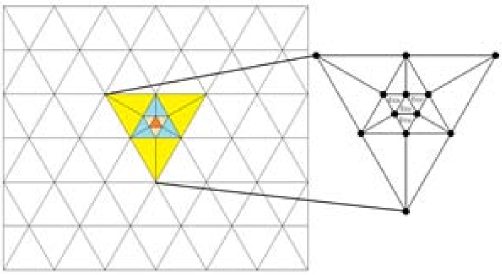

Figure 3: Remeshing technique.

the two contact elements are gradually reduced to zero. Other important methods are based on constant remeshing, [14], see Fig. 3.

The second option is the method of "disappearing" elements, within its framework are also included the latest approaches using cohesive elements. In fact, they are actually a generalized contact. Both methods mentioned above will be discussed in more detail in the following session.

4 Damage mechanics and cohesion approach

The prediction of crack propagation through interface elements based on fracture mechanics has been selected for crack growth modelling. Damage is being implemented in constitutive models and accumulation of damage is processed, [15]. As an example of the use of damage mechanics for ductile failure, the Gurson-Tvergaard-Needleman model can be pointed out, [16], [17]. Based on the evaluation of the experiments performed, it is recommended introducing two (or three) optional parameters q_1, q_2, q_3, usually $q_1 = 1.5$, $q_2 = 1$, $q_3 = q_1^2$. Analysis of the behaviour of these parameters was performed, [18]. Here, σ_{YS} is the yield stress of the matrix material, σ_m is the principal (hydrostatic) stress and f is the volume fraction of voids. The difference between the original model and the modified one is shown in Fig. 4, the schematic situation in front of the crack front in Fig. 5. The energy balance of such so-called complete model, [19], implemented as a user procedure in Abaqus software, [20], reads equation for plastic potential describing plastic flow in porous materials

$$\sum_{i,j=1}^{3} \frac{2S_{ij}S_{ij}}{3\sigma_{YS}^2} + 2q_1 f \cosh \frac{3q_2^2\sigma_m}{2\sigma_{YS}} - q_3 f^2 = 1 \,. \quad (1)$$

For practical calculations, f (volume fraction of cavities) in (1) is usually replaced by certain effective volume fraction f^*, introduced as

$$f^* = f_c - ((f_u^* - f_c)/(f_F - f_c))(f - f_c)\,;$$

here f_c is the critical volume at which voids (cavities) coalesce, f_F is the volume of voids at ultimate damage and $f_u^* = 1/q_1$. This effective value f^* applies to condition when f_c is less than f.

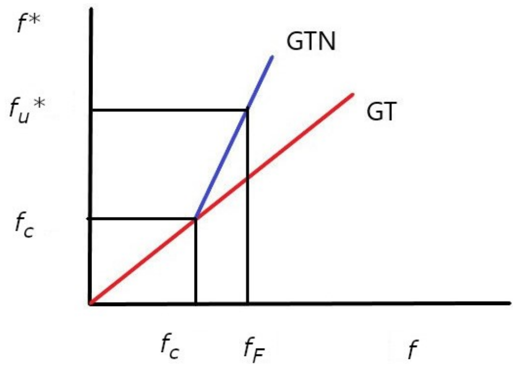

Figure 4: Definition of the Gurson-Tvergaard (GT) and Gurson-Tvergaard-Needleman (GTN) damage model.

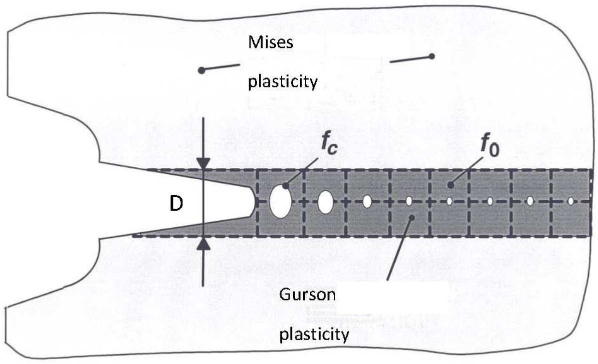

Figure 5: Schematic illustration of damage models in front of the crack.

Another type of elements for modelling crack propagation are cohesive elements. These elements are originally evolved from contact elements and they are based on the idea of material separation with the creation of new surfaces, [21]. Practically, it is a certain phenomenological description that characterizes the behaviour of the material using the so-called traction separation law, thanks to which we can then predict local violations; schematic shape of cohesive element for the simplified 2-dimensional geometry is presented by Fig. 6.

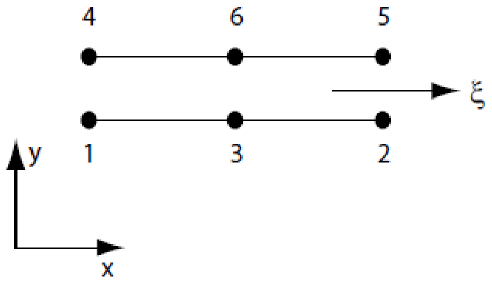

Figure 6: Schematic shape of cohesive element for 2D in the local coordinates.

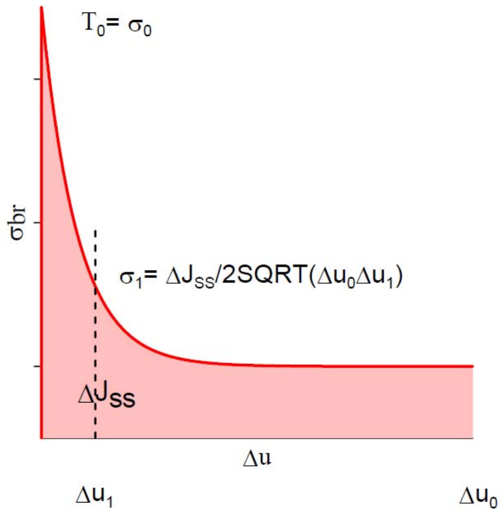

Figure 7: Traction separation law for fibre composites optimized for the numerical stability of the iterative algorithm.

There are several options for implementing the cohesion law (traction separation law) into a commercial FEM system. The presented work is based on long-term experience with the commercial system Abaqus, which makes it relatively easy to write custom user procedures for some special types of damage, new types of elements, or user control of some system options. Just the possibility of writing a user procedure UEL (User's Element) became the basis for the creation and implementation of the procedure for traction separation law, [22], [23], [24]. The original version is shown in Fig. 7, this traction separation law was further modified. The following Fig. 8 shows comparison of these two methods for a specific material. As crucial in this real case, it is necessary to point out the exact modelling of the dependence of actual stress versus actual deformation.

A numerical implementation of both the material model for ductile GTN material including the construction of a special element into the Abaqus software system for the cohesive model shown in Fig. 7 was documented, [25]. The relevant equations and the derived procedure for the realization of special elements are presented here. At the same time, a numerical analysis is performed and some aspects of practical calculations are pointed out. An example for application of the cohesive approach for ceramics with long fibres is described, [26], for the viscoelastic model, [27].

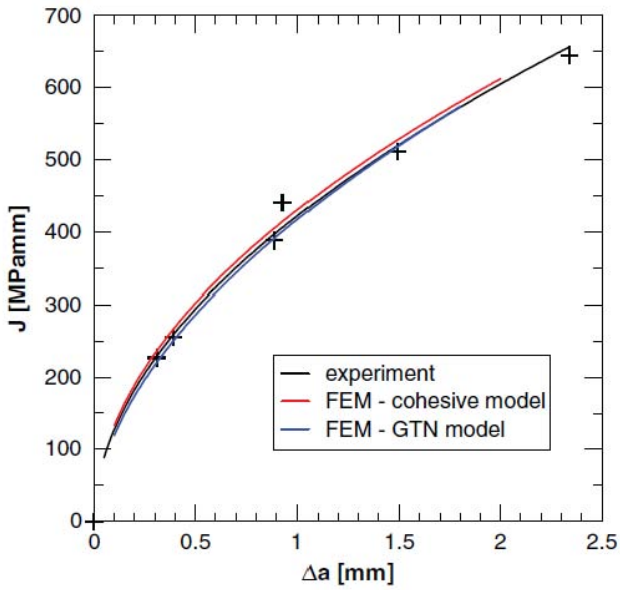

Figure 8: Determination of the J - R curve, showing the applied integral J versus crack extension, for both methods, material forged steel.

5 Crack growth via XFEM modelling

There is no doubt that the FEM is widely used mainly in the area of solving differential equations, however, as already indicated in the article, the FEM mesh may not always be ideal for modelling crack propagation. And this is one of the most significant interest in solid mechanics problems. First models were based on the weak (strain) discontinuity that could pass through finite element mesh using variational principle, [28]. Other authors and investigators considered strong (displacement) discontinuity by modifying the principle of virtual work statement (which is also the case for models with the traction separation law), [29], [30], [31], including stability and convergence of such problems, [32], and improvement of the precision of numerical procedure, [33], [34], [35].

In the strong discontinuity approach, the displacement consist of regular and enhanced components, where the enhanced components yield via jump across discontinuity surface, [30]. There is a modification of the basic equation, well-known from the standard finite element method (FEM), for the relationship between displacements $u^e(x)$ on particular points x of the e-th element (3-dimensional vectors of functions in general) and displacements at selected element nodes u^i, utilizing some standard shape functions $N_i(x)$, i. e.

$$u^e(x) = \sum_{i \in E_A} N_i(x) u^i \,; \tag{2}$$

here E_A denotes the set of nodes corresponding to the standard e-th element.

For the extended finite element method (XFEM), some more terms must be added to slightly modified displacements inside element end displacements at element nodes of (2). The second term realizes the technique of penetration into the element and the movement of the crack, the third criterion the failure (decohesion) and the possible direction of movement according to the preferred criteria. So the previous equation for the case of crack growth for 2-dimensional modelling (not limited to one element) is modified into the form

$$u^h(x) = \sum_{i \in E_A} N_i(x) u^i \tag{3}$$
$$+ \sum_{j \in C_B} N_j(x) H_j(x) + \sum_{k \in C_C} N_k(x) \sum_{m=1}^{4} \Phi_k^m(x) c_k^m$$

including certain specialized shape functions $N_i(x)$, $N_j(x)$ and $N_k(x)$ for intrinsic version of XFEM, same shape function for extrinsic version, where C_A, C_B, C_C are the sets of points corresponding to Fig. 9 and $H(x)$ is the Heaviside function. The first term in (3) corresponds to the standard method of finite elements, the second realizes the formation of a crack and the third appropriate criterion of its formation, while $\Phi_k^m(x)$ characterizes the local situation in front of the crack, m takes on values from 1 to 4, which corresponds to the basic possible directions of crack propagation in 2D.

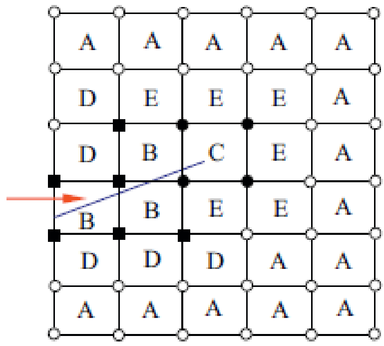

Figure 9: Illustration of enrichment function; domains A, B, C, D, E under the influence of crack growth.

Fig. 9 also characterizes the crack movement. Area A is affected by a crack, areas D and E are at risk of a crack creation and the situation here is already affected by the crack. In area B the crack moves and the second term of the (3) applies, in area C there is a crack tip and the third term of the equation (3) applies.

For practical calculations, the time discretization method and the extended finite element method can

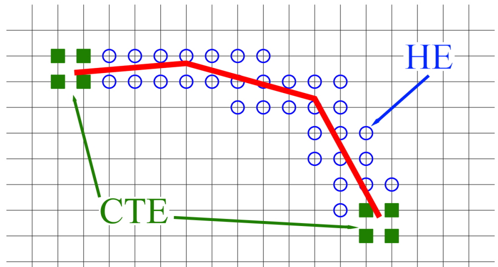

Figure 10: Crack propagation scheme for XFEM, HE stands for Heaviside Enrichment, CTE for Crack Tip Enrichment.

be used (chart can be seen in Fig. 10,) working with adaptive enrichment of the set of basis functions near singularities. This method (including a number of its modifications with their own names and designations) it already has quite a rich history with the remarkable progress in recent years, [36], [37], [39], [40], [41], [42], [43].

The FEM software Abaqus$^{\text{TM}}$ utilizes just (3) where the third term represents a simple criterion based on the minimum deformation energy, [5], [44], [45]. The mentioned equation, however, represents only one class of XFEM-based methods, the so-called extrinsic formulation. In this case, the number of degrees of freedom (DOF) increases, but the standard shape function from (2) does not change.

For the second group of XFEM methods, we use the label intrinsic method. In this case, there is no change in the number of degrees of freedom, but in the shape functions, which is numerically much more demanding and procedures based on the method of least squares are used. It should be noted that the third term of (3), implementing the criterion of initialization and crack growth, does not change. The second term of the equation will therefore change significantly and may look like

$$u^h(x) = \sum_{j \in C_N} \tilde{N}_j(x) \tilde{a} \,;$$

here C_N is complete set of nodes, whereas $\tilde{a}$ refers to a set of cautiously designed parameters, whose relation to u_i has to be evaluated from certain non-trivial auxiliary problem, e. g. using the least squares technique, [37], [38]. This entry into the history of XFEM and its division into extrinsic and intrinsic methods, [46], is only informative and serves to understand the damage modelling of fibre composites, in particular in the following considerations; cf. concurrent approaches, [47], [48], [49], too.

6 Damage modelling of structral fibre composites

Fibre cement composites belong to the class of perspective concretes with higher mechanical resistance against the formation of cracks. This allows for a finer and more economical construction; therefore, a new perspective on the creation of building structures or replacement is necessary steel constructions. These structures subjected to loads can result in stresses in the body exceeding the strength of the material, and thus lead to gradual failure. Such failures are often initiated by surface or near-surface cracks, which reduces the strength of the material.

A separate serious problem is the setting of material parameters at the macroscopic level, supported by appropriate experiments, if there is at least some information about the structure of the material, e.g. about random or intentionally preferred fibre directions; problems of this kind with an emphasis on non-destructive or minimally invasive ones test methods (especially x-ray, tomographic and electromagnetic, working with stationary magnetic or harmonic electromagnetic field) is covered, [50], as well as temperature-dependent problems, [51], [52]. A more general procedure for randomly oriented fibres was developed, [53], [54]; an example of a real distribution in reinforced concrete, [55], is in Fig. 11.

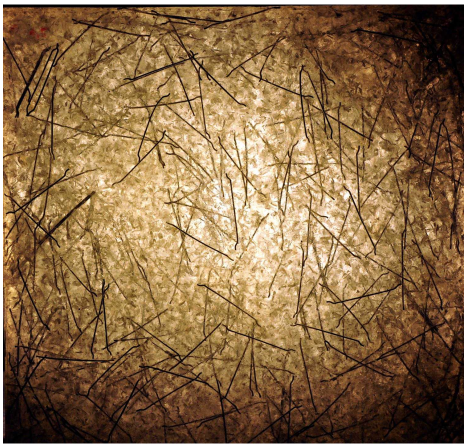

Figure 11: Radiographic image of fibre distribution in wire composite.

The formation of microcracks, [50], can be taken into account by introducing a failure factor based on the approach modifying the stress field and working with the non-local Eringen model, [56], a schematic diagram describing the stress calculation ahead the crack tip is shown in Fig. 12. About the interfaces between the matrix and the fibres, but also inside the matrix, possibly also the fibres, depending on the gradually activated macrocracks, it is usually assumed that they can be described by a cohesion model according to [57], [58], or, [59].

The XFEM application is able to suppress the disadvantages in the simulation of cohesive crack propagation; however, it must handle the absence of a sharp singularity at the crack tip with a more complex derivation of the required stresses from displacements. A complete computational model should generally include the initiation and propagation of cracks, their bridging by fibres, loss of cohesion between fibres and matrix, their mutual sliding by friction and destruction of fibres; special functions are required, e.g., for stress singularities in case opening and closing of cracks. The scheme of crack propagation through the original FEM elements is represented in Fig. 10. A unified scale-spanning approach covering elastic and plastic behaviour along with fracture a with other defects leads to the concept of structured deformation, [60].

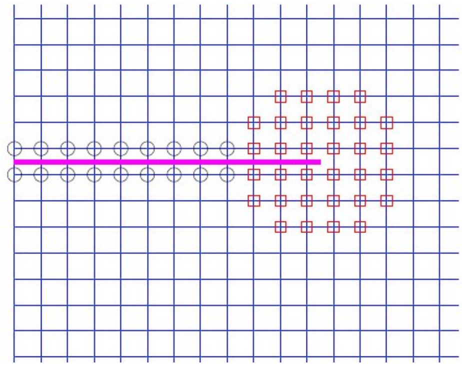

Figure 12: Calculating the stress in front of the crack tip: a non-local approach.

By considering models based on micromechanics, the macro-constitutive equations of unidirectional or randomly distributed fibres of reinforced materials, taking into account the possibility of forming and spreading cracks in the matrix, as well as to the separation and breakage of fibres. The computational model is finally used in numerical simulations in order to outline its reliability in evaluating both the fibre-matrix interaction phenomenon, so the ability to predict fracture failure of fibre composites. Mechanical behaviour of fibre-reinforced brittle matrix composites, with an emphasis on cementitious composites, will be investigated based on both discontinuous approach and modified approaches based on FEM.

XFEM is mesh-independent as to internal boundary such as material interfaces and cracks. These internal boundaries usually cause weak or strong field discontinuities variables that will be taken into account in XFEM by incorporating enrichment functions into the standard FEM approximation.

7 Application of the XFEM approach

For simplicity, let us start with the small-strain static formulation of the problem of deformation of linear elastic body, as the first step for the implementation of XFEM-based computational approach to the analysis of both micro- and macrocracking in quasi-brittle composites, with obvious applications to cementitious composites with various types of reinforcement. Such weak formulation is usually based on the conservations principles from classical thermomechanics, namely of mass, momentum and energy, supplied by appropriate constitutive equations, whose material parameters come from experimental identification procedures. In our case the conservation of linear momentum on a deformable body Ω in the 3-dimensional Euclidean space, supplied by some Cartesian coordinate system $x = (x_1, x_2, x_3)$, reads

$$((\varepsilon(w), \sigma)) = (w, f) + \langle w, g\rangle \qquad (4)$$

for all virtual displacements $w = (w_1, w_2, w_3)$, related to an initial configuration, whereas (for any fixed x) σ is a symmetric matrix with 3×3 elements, f denotes prescribed body forces on Ω, g represents prescribed surface forces on certain part Γ of the boundary $\partial\Omega$ of Ω, whereas $\Theta = \partial\Omega \setminus \Gamma$ corresponds to the supported part of $\partial\Omega$; $\varepsilon(w)$ then means a strain tensor, introduced as $\varepsilon_{ij}(w) = (\partial w_i/\partial x_j + \partial w_j/\partial x_i)/2$ with $i, j \in \{1, 2, 3\}$. All virtual displacements can be restricted to those with zero values on Θ; the same is expected from (still unknown) real displacements u. In (4) the simplified notation of integrals

$$((\varepsilon(w), \sigma)) = \sum_{i,j=1}^{3} \int_\Omega \varepsilon_{ij}(w)\, \sigma_{ij}\, \mathrm{d}x\,,$$
$$(w, f) = \sum_{i,j=1}^{3} \int_\Omega w_i f_i\, \mathrm{d}x\,,$$
$$\langle w, g\rangle = \sum_{i,j=1}^{3} \int_\Gamma w_i g_i\, \mathrm{d}s(x)$$

is used; moreover from the Hooke law we need to evaluate $\sigma = C\varepsilon(u)$ using an order 4 symmetric tensor with 21 different material characterisics in general, which can be reduced to 2 characteristics E, μ again (cf. *Section* 2). Thus, we obtain a weak formulation of an elliptic mixed (Dirichlet and Neumann) boundary value problem for a partial differential equation of elliptic type, whose solvability can rely on the classical Lax-Milgram theorem in special Sobolev spaces. Its numerical analysis can be preformed using the standard FEM techniques effectively: for the unknown values of u in discrete points x we come to the sparse system of linear algebraic equations.

However, such interpretation of (4) covers the pure elastic static case only, without implementation of development of any fracture. As certain remedy, [61], [62], we can rewrite (4) into its quasi-static form, introducing, due to the Kelvin parallel viscoelastic model, $\sigma = C(\varepsilon(u) + \beta\varepsilon(v))$, understanding v as the displacement rate, i. e. the time derivative of u, utilizing the homogeneous Cauchy initial condition (all zero values of u in the initial zero time), α being a structural damping factor, as an additional material characteristic. Consequently, (4) can be understood as a weak formulation of a parabolic mixed boundary value problem, with an unknown displacement $u(t)$ developed in time $t \geq 0$, whose solvability can rely on the method of discretization in time and on the properties of special Rothe sequences (namely piecewise simple abstract functions and linear Lagrange splines composed of such functions) in special Bochner-Sobolev spaces. Their practical construction is easy: taking u^s for any $t = sh$ approximately, h being certain time step, $s \in \{1, 2, \ldots\}$, and v^s similarly, too, we are able to rewrite (4) as

$$((\varepsilon(w), \alpha C\varepsilon(v^s))) \qquad (5)$$
$$+\, ((\varepsilon(w), C\varepsilon(u^s))) = (w, f) + \langle w, g\rangle$$

for $hv^s = u^s - u^{s-1}$, starting from the zero-valued u^0 everywhere on Ω. Therefore we have to analyse, step-by-step, linear elliptic problems; the standard FEM techniques are available again, with the similar results as above.

Unfortunately, this is the end of simple linear computations. Both i) the formation of microscopic fractured zones and ii) the initiation, opening and closing of macroscopic cracks needs certain modifications of (5), disturbing its linearity substantially. The case i) can be handled by careful introducing of certain non-local regularizing damage factor $\mathfrak{D}$ with values between 0 and 1, working with some stress invariants and with the Eringen model typically, respecting namely the specific material behaviour under tension and compression. This results in some stiffness loss, expressed by the replacement of C by $(1 - \mathfrak{D}(t))\, C$. The case ii), at least for the a priori known potential positions of cracks, forces an additional right-hand-side term of (5): similarly to $\langle w, g\rangle$ we need $\langle w, \gamma(Du)$ on Λ instead of Γ, with Λ representing all internal interfaces, where Du are differences in displacements on Λ and $\gamma(.)$ must be seen as a non-trivial new material characteristics, including the cohesive properties of Λ. Denoting still by the upper index ^{s} our approximations for $t = sh$, for any such time we can implement the iterative procedure to

$$((\varepsilon(w), (1 - \mathfrak{D}^s_\times)\, \alpha C\varepsilon(v^s))) \qquad (6)$$
$$+\, ((\varepsilon(w), (1 - \mathfrak{D}^s_\times)\, C\varepsilon(u^s))) = (w, f) + \langle w, g\rangle$$
$$+\langle w, \gamma(Du^s_\times)\rangle_\Lambda$$

where (if no better information is available) u^{s-1} can be taken as an initial guess of $u^s_\times$, then replaced by u^s, obtained from (6), for the next iterative step, etc.

Whereas the main difficulty of i) is the complicated design and evaluation of $\mathfrak{D}(.)$, for the standard FEM applications ii) suffers for the unpleasant duty of repeated remeshing, to cover potential creation of still new parts of Λ, which can be very expensive. This highlights the priority of XFEM (and similar methods), thanks to the advanced choice of nodal functions. Nevertheless, the basic form of (6) stays unchanged.

Let us also mention the need of further improvements for numerous engineering applications, exceeding the scope of this article, namely: a) the implementation of finite strains, b) the replacement of empiric linearized constitutive relations as Hooke, Kelvin, etc., ones, by proper relations coming from thermodynamic considerations, c) the passage from a quasi-static to a dynamic formulation, adding the inertia forces, mass damping and a more general dissipative mechanism in general. Although a), b), c) are not avoided by recent software packages, the related mathematical existence and convergence theory contains still open questions because no system of propositions, theorems, etc., comparable with that for linear and quasilinear problems, is available; consequently one must rely on experimental validation of computational results only.

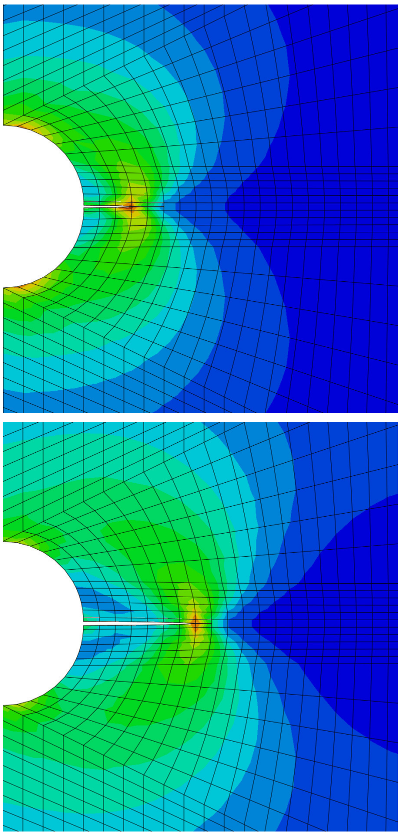

Figure 13: The application of Mazars exponential model.

A sample with a cement matrix and steel fibres was selected for computational modelling. Numerical results show the surface propagation of cracks in the damaged body depending on the location fibre and material properties. The reinforcing effect of the fibres plays a significant role in the direction of crack propagation. A sample with a cement matrix and steel fibres was selected for computational modelling. The attention is paid in particular to Eringen's model for generating the multiplicative damage factor, related quasi-static analysis, the existence of weak solution of the corresponding boundary value and initial value problem with a parabolic partial system differential equations.

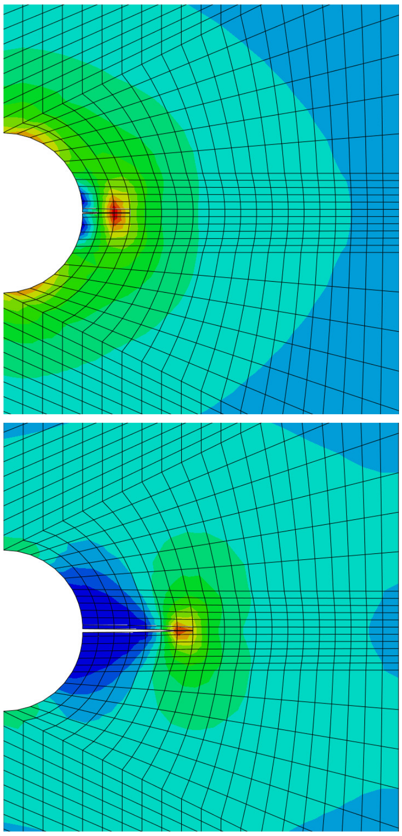

Figure 14: The application of non-local approaches.

The proposed procedure thus combines the possibilities of several approaches for modelling crack propagation in fibre composites. The XFEM method is primary, the stress in front of the crack front is recalculated according to the non-local approach, in the entire body according to the exponential or power law of violation, [63], [64], [65], [66], which implies some degree of averaging (especially as to stress) ahead of the crack front. The following Fig. 13 and Fig. 14 present some results, due to the well-tried so-called Mazars model, [67], [68], [69], [70].

Mazars damage model is based on a strain formulation and generally is used for the physically nonlinear analysis of concrete structures. The main objective is modification of damage model, in which both tension and compression damage evolution laws are regularized using a classical fracture energy methodology. Its hypothesis is founded on the base of an elastic damage isotropic behaviour. This model assumes the following premises: i) damage evolves/occurs only due to positive strains in the principal directions, which indirectly promotes "smeared crack grow"; ii) only one scalar damage variable is defined, this is due to the damage model being isotropic; iii) represents the material as totally damaged, and it is limited in the interval; iv) no permanent strains are admitted, and the unloading path is linear, therefore, no hysteresis loops.

8 Conclusions

The practical application of this work is to show some techniques suitable for the crack growth modelling in case of heterogeneous materials, in order to find critical parameters determining the behaviour of materials and damage. The basic step that conditions the realistic modelling of the behaviour of the material is finding or using a suitable criterion that determines the formation of a new crack and then the subsequent growth of resulting crack. It stands to reason that in searching for a suitable criterion (often based on fracture mechanics), the first approximation a model with an a priori crack or stress concentrator is used.

Special attention is concentrated on modelling building fibre composites, especially on the use of a modified FEM and on the presentation of some techniques that can improve XFEM. However, the problem of local stress determination for these composites is more general for the entire group of fibre composites regardless to understanding of the size effect. The field of application of XFEM is still in the forefront of interest of the mentioned group of authors, especially the solution of crack propagation in fibre composites.

The combination of the Mazars model, which is very popular and is implemented in numerous systems for FEM, with the refinement and determination of the stress distribution in front of the crack tip is a promising solution for structural composites modelling. Generally speaking, the numerical stability of crack propagation modelling in the case of some stress averaging in front of the crack tip is much better than in the case of sharp concentrators, especially their effect on XFEM.

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Contribution of individual authors

Vladislav Kozák elaborated both for the physical formulations and the computational simulations. Jiří Vala was responsible for the mathematical analysis.

Conflicts of interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

Sources of funding for research presented in a scientific article or scientific article itself

This research has been supported from the project of specific university research at Brno University of Technology No. FAST-S-22-7867.

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