Physics. — *"Experimental proof of the existence of Ampère's molecular currents."* By Prof. A. EINSTEIN and Dr. W. J. DE HAAS. (Communicated by Prof. H. A. LORENTZ),

(Communicated in the meeting of April 23, 1915).

When it had been discovered by OERSTED that magnetic actions are exerted not only by permanent magnets, but also by electric currents, there seemed to be two entirely different ways in which a magnetic field can be produced. This conception, however, could hardly be considered as satisfactory and physicists soon tried to refer the two actions to one and the same cause. AMPÈRE succeeded in doing so by his celebrated hypothesis of currents circulating around the molecules without encountering any resistance.

The same assumption is made in the theory of electrons in the form e.g. in which it has been developed by H. A. LORENTZ, the only difference being that, like electric currents in general, the molecular currents are now regarded as a circulation of elementary charges or electrons.

It cannot be denied that these views call forth some objections. One of these is even more serious than it was in AMPÈRE's days; it is difficult to conceive a circulation of electricity free from all resistance and therefore continuing for ever. Indeed, according to MAXWELL's equations circulating electrons must lose their energy by radiation; the molecules of a magnetic body would therefore gradually lose their magnetic moment. Nothing of the kind having ever been observed, the hypothesis seems irreconcilable with a general validity of the fundamental laws of electromagnetism.

Again, the law of CURIE-LANGEVIN requires that the magnetic moment of a molecule shall be independent of the temperature, and shall still exist at the absolute zero. The energy of the revolving electrons would therefore be a true zero point energy. In the opinion of many physicists however, the existence of an energy of this kind is very improbable.

It appears by these remarks that after all as much may be said in favour of AMPÈRE's hypothesis as against it and that the question concerns important physical principles. We have therefore made the experiments here to be described. by which we have been able to show that the magnetic moment of an iron molecule is really due to a circulation of electrons.

The possibility of an experimental proof lies in the fact that every negative electron circulating in a closed path has a moment of momentum in a direction opposite to the vector that represents its magnetic moment, the ratio between the two moments having a definite value which is independent of the geometric dimensions and of the time of circulation. The magnetic molecule behaves as a gyroscope whose axis coincides wich the direction of the magnetisation. Every change of magnetic state involves an alteration of the orientation of the gyroscopes and of the moment of momentum of the magnetic elements. In virtue of the law of conservation of moment of momentum the change of "magnetic" moment of momentum must be compensated by an equal and opposite one in the moment of momentum of ponderable matter. The magnetisation of a body must therefore give rise to a couple, which makes the body rotate. [1])

§ 1. *Magnetic moment and moment of momentum of the molecule.*

The magnetic moment of a current of intensity i flowing along a circle of area F is given by the formula

$$\mathfrak{m} = iF,$$

or if the current consists in an electron circulating n times per second by

$$\mathfrak{m} = neF \quad \ldots \ldots \ldots \quad (1)$$

It may be represented by a vector perpendicular to the plane of the circle, the positive direction of this vector corresponding in the well-known way to the positive direction of the current.

The moment of momentum is

$$\mathfrak{M} = 2mnF, \quad \ldots \ldots \ldots \quad (2)$$

if we let coincide its positive direction with that of the magnetic moment.

Hence:

$$\mathfrak{M} = \frac{2m}{e}\,\mathfrak{m} \quad \ldots \ldots \ldots \quad (3)$$

For a body in which a certain number of electrons are circulating, this becomes

$$\Sigma\mathfrak{M} = \frac{2m}{e}\,\Sigma\mathfrak{m},$$

or if we denote the magnetisation $\Sigma\mathfrak{m}$ by I

[1]) This paper had gone to press when we learned that O. W. RICHARDSON (Phys. Rev. Vol. 26, 1908 p. 248) had sought already for the effect in question, without however obtaining a positive result.

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$$\Sigma \mathfrak{M} = \frac{2m}{e} I. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

§ 2. *Consequence of the existence of a magnetic moment of momentum.*

Any change of the moment of momentum $\Sigma\mathfrak{M}$ of a magnetized body gives rise to a couple Θ determined by the vector equation

$$\Theta = -\Sigma \frac{d\mathfrak{M}}{dt} = 1{,}13.10^{-7} \frac{dI}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where the numerical coefficient has been deduced from the known value of $\frac{e}{m}$ for negative electrons.

It has been our aim to verify the relation expressed by (5). We shall show in the first place that the calculated effect is not too small to be observed. Let the body be an iron cylinder with radius R, which can rotate about its vertical axis. We shall deduce from (5) the angular velocity ω the cylinder acquires by the reversal of a longitudinal magnetisation, which we suppose to have the saturation value I_s. Denoting by Q the moment of inertia of the cylinder, and writing λ for the above coefficient $1{,}13 \,.\, 10^{-7}$, we find

$$Q\omega = \int \Theta dt = 2\lambda I_s \,.$$

Now, if the saturation value of the magnetisation per cm³ is 1000, which is not a high estimate, we have $I_s = \frac{M}{7{,}8} \,.\, 1000$. The moment of inertia is $Q = \frac{1}{2} MR^2$, and we find for $R = 0{,}1$ cm

$$\omega = 0{,}6 \,.\, 10^{-2},$$

an angular velocity that can easily be observed.

§ 3. *Description of the method.*

At first sight it seems that equation (5) may be tested in the following way. A soft iron cylinder C is suspended by a thin wire D coinciding with the axis of the cylinder prolonged, the period of the torsional oscillations being a few seconds. Let the cylinder C be surrounded by a coil K whose axis coincides with that of C. Then, on reversing a current in K, a rotation of C ought to be observed. In reality, however, this simple method cannot be thought of. As the field of the coil will not be uniform the cylinder would probably show highly irregular motions completely masking the effect that is sought for.

Better results are obtained if the effect is magnified by resonance. For this purpose an alternating current having the same or nearly the same frequency as the oscillations of C about the wire D is made to flow through the coil.

For the oscillations of C about the vertical axis under the influence of the couple θ we have the equation

$$\theta = Q\ddot{\alpha} + \Theta\alpha + P\dot{\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

in which the angle α, the deviation from the position of equilibrium is reckoned positive in the same direction as the current in the windings. Q is the moment of inertia, Θ the torsion constant of the wire and P a small coefficient of friction. Instead of Θ and P we shall introduce two new constants

$$\omega_0 = \sqrt{\frac{\Theta}{Q}}, \quad \varkappa = \frac{P}{2Q} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

the first of which is 2π times the free frequency, as it would be in the absence of friction, whereas $\varkappa$ is the constant of damping. Indeed the free oscillations (the equation for which is deduced from (6) by putting $\theta = 0$) are given by

$$\alpha = Ce^{-\varkappa t} \cos\left(\sqrt{\omega_0^2 - \varkappa^2}\; t + p\right).$$

The differential equation (6) is easily solved if we develop θ as a function of t in a FOURIER series. Now according to (5) θ has the same phase as $\frac{dI}{dt}$. Hence, if the magnetisation were proportional to the current we could directly represent θ as a harmonic function whose phase would be $\frac{1}{4}\pi$ in advance of that of the current i in the coil. The proportionality will, however, hold for small intensities only. If the amplitude of i is made to increase so that the magnetisation approaches saturation, the magnetisation curve takes a different form. Finally, for very large amplitudes of i, the magnetisation will suddenly pass from one saturation value into the opposite one, simultaneously (except for a small difference of phase) with the change of direction of the current. For this limiting case the calculation will now be made.

The couple acting on the cylinder may be represented by fig. 1, in which the sinusoid refers to the current i[1]).

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1) The curve with the sharp peaks represents the value of $\frac{dI}{dt}$, to which the couple θ is proportional. It was obtained in the following way. The iron cylinder, which had its right position along the axis of the coil K, was surrounded by a narrow glass tube covered with windings and immediately beside this tube a similar one, equal to it and covered in the same way, was placed. The windings of the two

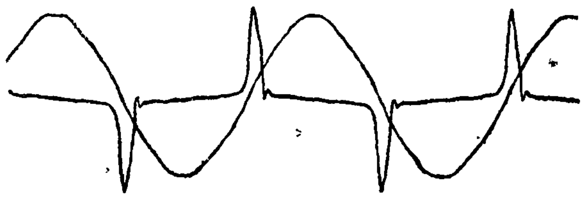

Fig. 1.

Each sharp peak corresponds to a reversal of the magnetisation and we have for each of them

$$\int \theta \, dt = \pm 2\lambda I_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Let the origin $t = 0$ coincide with a point in Fig. 1, where the current passes from the negative to the positive direction. Then we may write

$$i = A \sin \omega t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and θ may be developed in a series

$$\theta = \sum_{n=1}^{n=\infty} B \cos n \, \omega \, t \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Of this series the first term only need be considered here, as the effect corresponding to it is the only one that is multiplied by resonance, so that the other terms have no sensible influence on the motion of the cylinder. Now, multiplying (10) by $\cos \omega t$ and integrating over a full period $T = \frac{2\pi}{\omega}$ we find

$$\int_{-\frac{\pi}{2\omega}}^{+\frac{3\pi}{2\omega}} \theta \cos \omega t \, dt = \frac{\pi}{\omega} B_1.$$

On the left hand side θ is different from 0 only in the very small intervals at $t = 0$ and $t = \frac{\pi}{\omega}$. For the first of these we may put

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tubes were connected in such a way, that a current passing through them flowed round the tubes in opposite directions.

Under these circumstances, the current induced in the windings is exactly proportional to $\frac{dI}{dt}$, the demagnetizing action of the poles of the iron bar being eliminated, as well as the induction due to the field of the coil K. The graph for the induced current, and therefore for $\frac{dI}{dt}$ or θ was obtained by means of an oscillograph of SIEMENS and HALSKE. The alternations of the current i, represented by the sinusoid, were registered in the same way.

$\cos \omega t = 1$ and for the second $\cos \omega t = -1$ so that we find, using (8)

$$B_1 = \frac{4\lambda\omega}{\pi} I_s . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Instead of (6) we now get the equation

$$B_1 \cos \omega t = Q\ddot{\alpha} + \Theta\alpha + P\dot{\alpha}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

the periodic solution of which is

$$\alpha = \frac{B_1}{u} \cos (\omega t - v), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

if the constants u and v are determined by

$$\left.\begin{aligned} u \cos v &= (\omega_0{}^2 - \omega^2)\, Q \\ u \sin v &= 2\,\varkappa\,\omega\, Q \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Here the quantity u, to which we shall give the positive sign, determines the amplitude whereas the phase of the oscillations is given by the angle v. For the amplitude, which we shall denote by $|\alpha|$, we find

$$|\alpha| = \frac{B_1}{u} = \frac{4\lambda\, I_s}{\pi Q \sqrt{\dfrac{(\omega_0{}^2 - \omega^2)^2}{\omega^2} + 4\varkappa^2}} \quad . \quad . \quad . \quad . \quad (15)$$

For $\omega = \omega_0$ it becomes a maximum $|\alpha|_m$, viz.

$$|\alpha|_m = \frac{2\lambda\, I_s}{\pi\, Q\, \varkappa} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

As to the phase, we first remark that according to (14) $v = \frac{\pi}{2}$ for $\omega = \omega_0$. If the frequency of the alternating current is higher than that of the cylinder, we have $v > \frac{\pi}{2}$ and in the opposite case $v < \frac{\pi}{2}$. When ω is made to differ more and more from ω_0, the phase v approaches the value π in the first case and 0 in the second. If the constant of damping $\varkappa$ is small we may say that these limiting values will be reached at rather small distances from ω_0 already. In our experiments this was really the case and we may therefore say, excepting only values of ω in the immediate neighbourhood of ω_0 that $v = \pi$ for $\omega > \omega_0$ and $v = 0$ for $\omega < \omega_0$. Taking into account what has been said about the positive direction one will easily see that, if the current i and the deviation α had the same phase, the cylinder would at every moment be deviated in the direction the current in the coil has just then. In reality the phase of the oscillations of the cylinder is behind that of the current by an amount $v - \frac{\pi}{2}$; this follows from (9) and (13). Remembering further that in the deduction of (11) it has been assumed that the circulating electrons are negative and that if they were positive ones, the sign of B_1 and the phase of the effect would be reversed we are led to the following conclusion:

Negative electrons.

$\omega > \omega_0$. The phase of the oscillations of the cylinder is a quarter of a period behind that of the current.
$\omega < \omega_0$. It is a quarter of a period in advance.
$\omega = \omega_0$. The vibration has the same phase as the current.

Positive electrons.

$\omega > \omega_0$. The phase of the oscillations of the cylinder is a quarter of a period in advance of that of the current.
$\omega < \omega_0$. It is a quarter of a period behind that of the current.
$\omega = \omega_0$. The vibration of the cylinder and the current have opposite phases.

It is important to notice that there is a quarter of a period difference of phase between the active couple $B_1 \cos \omega t$ and the current $i = A \sin \omega t$ and likewise between the active couple and the alternating magnetisation. This is always so, independently of the relative values of ω and ω_0 and of the sign of the circulating electrons.

§ 4. *Short description of the apparatus.*

The alternating field which has been mentioned several times already was excited by two coils placed with their axes along the same vertical line and with a distance of about 1 cm between them. They were mounted on a brass foot to which three foot screws could give different inclinations. The coils were connected in series and gave a field of about 50 Gauss. The iron cylinder was suspended along their axis. This cylinder, 1.7 mm thick and in the first experiments 7 cm long, was carefully turned of soft iron. Centrally in its top there was bored a narrow hole of diameter 0.3 mm in which a fitting glass wire was sealed. At its middle the cylinder wore a very light mirror made from a silvered microscope covering glass. The light of a single wire lamp was thrown on the mirror through the space between the two coils. The reflected rays formed an image on a scale placed at a distance of 45 cm When the cylinder was set vibrating this image was broadened into a band, the width of which determined the double deviation.

In order to obtain resonance, it must of course be possible to regulate the length of the glass wire. For this purpose we used a clamping arrangement by which the glass wire could be tightly held at different points of its length.

The clamp and the suspending wire with the cylinder could rotate together about a vertical axis in a fixed column. The effective current was read on a precision instrument. Finally, the whole apparatus was surrounded by an arrangement by which the terrestrial magnetic field could be compensated. We shall revert to it further on.

§ 5. *The experiments.*

Let us now examine the principal disturbing causes.

1. At the ends of the cylinder alternating poles are induced. Acting on these the horizontal component of the terrestrial field can give rise to a couple alternating with the same frequency as the current and tending to rotate the cylinder about a horizontal axis. (Effect I).

Rotations of this kind have not, however, been observed by us.

2. According to the views of WEISS the ferromagnetic crystals are lying irregularly in all directions. It may therefore happen that some of them are directed in such a way that their magnetism is not reversed by the alternating field. In this case there will be a permanent horizontal component of the magnetisation, which, acted upon by the alternating horizontal component of the magnetic field in the coil, will give rise to an alternating couple around the vertical axis with the same frequency and phase as the alternating field (Effect II).

3. The axis about which the cylinder rotates will not coincide accurately with its magnetic axis.

A permanent horizontal magnetic force such as that of terrestrial magnetism, will therefore produce torsional oscillations of the cylinder. The couple which excites these oscillations has the same phase as the magnetisation and (in the case of strong currents) as the alternating current itself.

4. It is easily seen that the FOUCAULT currents which are induced

in the cylinder cannot have any influence in our experiment, their sole effect being a slight retardation of the magnetic reversals. So far as we can see, the above effects are the only ones that have the same frequency as the current in the coil and are therefore magnified by resonance. When now the coil was connected to the main alternating current conductors the image on the scale remained perfectly at rest so long as the length of the suspending wire was not such as to make the frequency of a free vibration of the cylinder coincide very nearly with that of the alternating field. The resonance appeared and disappeared again by a change of length of the wire by 1 mm, the whole length being 8 cm.

In order to find the length required for resonance and to make sure that the suspended apparatus did not vibrate in one of its higher modes, we used the following method by which we could also determine the moment of inertia of the cylinder.

At the lower end of the iron cylinder we sealed a short copper cross bar whose moment of inertia was 10,7.

For the moment of inertia of the cylinder calculation had given 0,0045.

It follows from this that the period of oscillation of the cylinder becomes $\sqrt{\frac{10,7}{0,0045}} = 48,8$ times greater by adding the small cross bar. If therefore we chose the length of the wire so as to have a frequency 1 [1]) with the cross-bar, the frequency without it would be about 48,8. This is nearly equal to the frequency of the alternating current.

We were sure by this that the suspended system would vibrate in its fundamental mode. In order to determine the moment of inertia more accurately however, the cylinder was now placed within the coil and the length of the wire was increased until the resonance was at its maximum. Then the frequency of the free vibrations might be supposed to be equal to that of the alternating current which was found to be 46,2. After this the arrangement was removed from the coil and the cross bar fixed to it. We then found the frequency 1,14. From these numbers we deduce

$$Q = 10,7 \left(\frac{1,14}{46,2}\right)^2 = 0,0065.$$

After these preparations it was found that Effect II, i. e. the oscillation caused by permanent poles in the cylinder, was of no

[1]) By frequency we always mean the number of complete oscillations in a second.

importance. The double deviation remained unchanged when the position of the axis of the coil with respect to a vertical line was changed by means of the foot screws, a change which gave rise to horizontal alternating fields.

Effect III, however, which was caused by the action which stationary magnetic fields can exert on the alternating poles on account of their excentric position could easily be observed. The double deviation changed immediately when a permanent magnet was brought near the coil. The influence of the terrestrial magnetism was also apparent. When it was not compensated we got, in the case of resonance, a broadening of the image on the scale up to 3 cm for a scale distance of 45 cm. In all further experiments the terrestrial field has therefore been compensated, the measurements required for this being made with an earth inductor and a ballistic galvanometer. The horizontal and vertical components of the terrestrial field were compensated separately by means of hoops of about 1 m. diameter on which copper wire was wound. The current was taken from storage cells, and precision Ampèremeters of SIEMENS and HALSKE served for continually controlling its strength.

Whether the compensation was obtained could be tested by turning the upper end of the suspending wire. The amplitude of the oscillations changed by this so long as the terrestrial magnetism was still acting on the iron magnetized by the alternating current. After compensation however this azimuthal sensibility of the effect had disappeared. After all there remained a well marked double deviation of 4,5 mm.

We now had to make sure that this was really the effect we sought for. For this purpose we first availed ourselves of the circumstance that the acting couple must differ a quarter of a period in phase from the current and the magnetisation. We brought a permanent magnet near the coil, thereby calling forth effect III and adding to the couple $B_1 \cos \omega t$, with which we are concerned, a new one, which has the same or the opposite phase as the magnetisation and therefore differs a quarter of a period in phase from $B_1 \cos \omega t$. Whatever be the sign of this additional couple, the amplitude of the resulting one must become larger than B_1. We found indeed that the broadening of the image always increased when we brought a magnet near the coil.

Further the theory requires that the magnitude of the effect depends on the intensity of the alternating field in the same way as the magnetisation itself. This was likewise confirmed by experiment.

Finally we shall compare the observed magnitude of the effect with the theoretical one. If we take 1200 for the magnetisation reached by the iron, we get (the volume of the cylinder being 0,16 cm^3) $I_s = 192$. By direct observation of the oscillations in the alternating field we found

$$K = 0,533.$$

As

$$Q = 0,0065,$$

it follows from (16) that

$$|\alpha| = 0,0036.$$

For a scale distance of 45 cm this gives for the double deviation $4|\alpha| \,.\, 45 = 0,65$; as has been said already, we have found 0,45 by our experiments.

As to this difference we must observe that the theoretical value is an upper limit, as the magnetism does not change its sign instantaneously.

On account of the demagnetising influence of the free poles the field in the coil must be rather strong if on its reversal the magnetisation is to take immediately a constant value in the new direction.

§ 6. *Determination of the phase.*

We have seen that the active couple differs a quarter of a period in phase from the alternating magnetisation. Further it follows from § 3 that by comparing the phase of the effect (P_1) with that of the alternating current (P_2) we shall be able to decide, whether the electrons circulating round the iron molecules are really *negative* ones. We have tried to effect this by proceeding in the following way.

The single wire lamp used for the scale reading was connected with the main alternating current conductors in parallel with the coil that contained the iron cylinder. If then we brought a permanent magnet near the lamp, the incandescent wire was set into motion by alternating electromagnetic forces, so that, besides the oscillations due to the vibrations of the mirror, the image also performed those that were caused by the motion of the wire.

By observing whether the addition of this last vibration increased or decreased the amplitude of the image, we could compare the phase P_1 with that of the new vibrations. Now this latter is determined by the phase of the glowing wire and this in its turn depends on the phase of the current in it, whereas the difference between this phase and P_2 is determined by the self-induction of the coil. It would therefore be possible to compare the phases P_1 and P_2.

Unfortunately, when our experiments had been brought to a conclusion and one of us had left Berlin it came out that a mistake had been made in the application of the method, so that we must consider as a failure this part of our investigation. The negative sign of the circulating electrons is however made very probable by the agreement between the magnitude of the observed effect and the value we have deduced for it from that of the ratio $\frac{e}{m}$ for negative electrons.

§ 7. *More accurate measurements.*

The measurements thus far described furnished a satisfactory confirmation of the theory, but were much lacking in precision. The field in the coil was too weak practically to cause the sudden reversals of the magnetisation assumed in the theory. Further the coefficient of damping $\varkappa$ could not be determined with any accuracy. Even the question may arise whether the influence of the damping is represented rightly by the term $P\dot{\alpha}$ in equation (6).

For these reasons we have somewhat modified our apparatus. In order to quicken the reversals of the magnetisation we used instead of the former short coil one of 62 cm length (about 100 windings to a cm) the amplitude of whose field, for an effective strength of 1,45 Ampère was 260 Gauss in its central part and therefore 130 Gauss at the ends. In order to diminish the demagnetizing influence of the poles we further used a cylinder of 16 cm length and 0,17 cm diameter. The mirror was now suspended by a thin walled tube that was sealed to the lower end of the iron cylinder. It just projected beneath the lower end of the coil. In order to avoid a determination of the coefficient of damping and assumptions about the law of damping a series of experiments were made in which, for a definite length of the wire, the amplitude $|\alpha|$ was determined for different frequencies of the alternating current, so that a "resonance curve" could be drawn.

The alternating current was furnished by a generator placed in the cellar of the building and moved by the current of a battery of storage cells. The apparatus in the working room comprised a variable resistance connected in parallel to the windings of the field magnets. By varying this resistance we could change within certain limits the exciting current in the motor and therefore the number of its revolutions and the frequency of the induced alternating current. The current which passed through the variable resistance was controlled by an ampèremeter. When all other things were kept constant the frequency of the alternating current was a function of the strength of the current in the variable resistance. Besides we used a resonance frequency meter of HARTMANN and BRAUN, with which we could accurately determine definite frequencies (45 ; 45,5; 46 up to 55). The intermediate frequencies were interpolated by means of the ampèremeter. The amplitude of the vibrations of the cylinder was measured in the same way as in the former experiments. However, in order to increase the precision we now took a scale distance of 145 cm.

In fig. 2 the results have been plotted graphically. The numbers

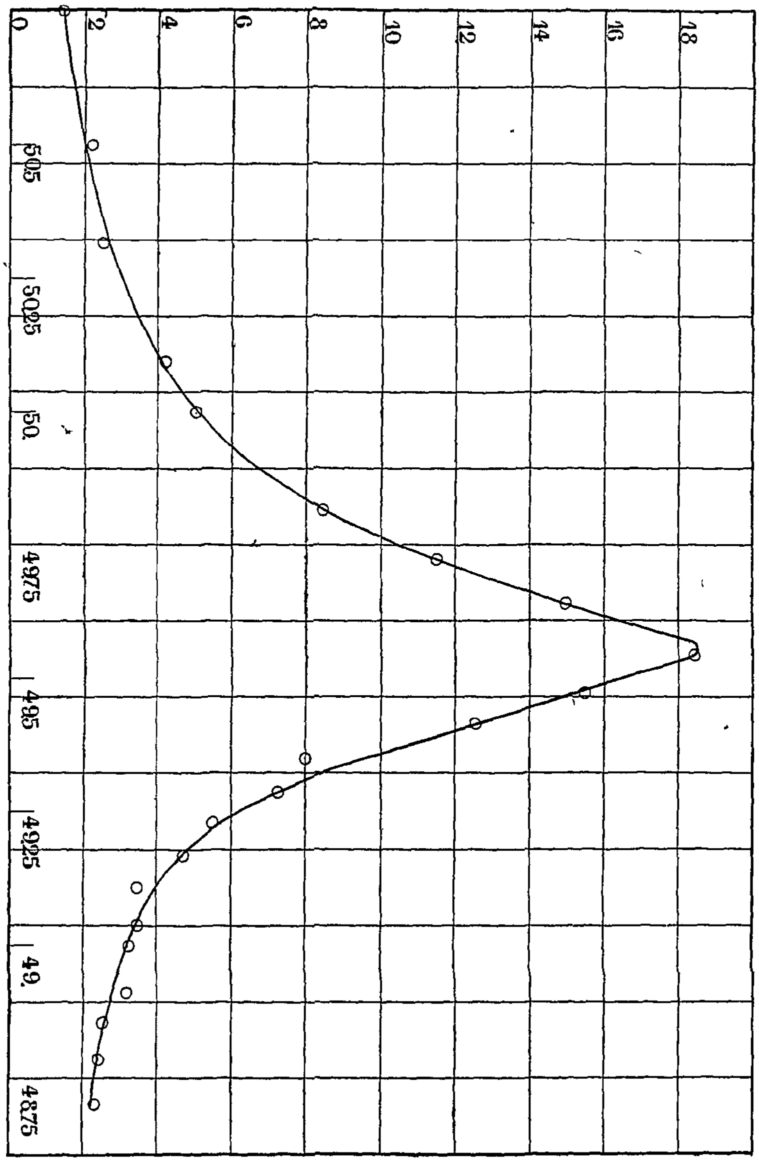

Fig. 2.

on the horizontal [1]) axis give the frequencies of the alternating current, those on the vertical axis 10 times the double deviation in centimeters.

For the calculation we each time used two points at the same height combined with the ordinate of the highest point of the curve. If for shortness' sake we put

$$\frac{4\lambda I_s}{\pi Q}=\mu,$$

it follows from (15) that

$$\frac{\mu}{|\alpha|}=\sqrt{\frac{(\omega^2-\omega_0^2)^2}{\omega^2}+4\varkappa^2}.$$

Now, if $\omega_1\,(>\omega_0)$ and $\omega_2\,(<\omega_0)$ are the two values of ω corresponding to the same amplitude $|\alpha|$ we have the equations

$$\frac{\mu}{|\alpha|}=\sqrt{\frac{(\omega_1^2-\omega_0^2)^2}{\omega_1^2}+4\varkappa^2}\ \text{ and }\ \frac{\mu}{|\alpha|}=\sqrt{\frac{(\omega_0^2-\omega_2^2)^2}{\omega_2^2}+4\varkappa^2}.$$

By elimination of ω_0 and $\varkappa$ from these and from

$$\frac{\mu}{|\alpha|_m}=2\varkappa$$

we find

$$\frac{\mu^2}{|\alpha|^2}-\frac{\mu^2}{|\alpha|^2_m}=(\omega_1-\omega_2)^2.$$

Let v be the difference in frequency of the two chosen points, so that $\omega_1-\omega_2=4\pi v$ and let us put

$$\frac{|\alpha|}{|\alpha|_m}=b.$$

Then we find, after introducing the value of μ

$$\lambda=\pi^2\frac{Q}{I_s}\,|\alpha|_m\,.\,v\sqrt{\frac{b^2}{1-b^2}}. \quad \ldots\ldots \quad (17)$$

When the resonance curve has been drawn, (17) gives a value of λ for each ordinate $|\alpha|$. If this value or what amounts to the same $v\sqrt{\frac{b^2}{1-b^2}}$ is constant, this proves that the influence of the damping can really be represented by a linear term in the equation of motion.

The following table contains the values of v and b, taken from the diagram and those of $v\sqrt{\frac{b^2}{1-b^2}}$ we have deduced from them.

[1]) If the figure is brought into the right position by a rotation of 90°.

Ordinates	v	b	$\sqrt{\frac{b^2}{1-b^2}}$	$v\sqrt{\frac{b^2}{1-b^2}}$
15	0,0911	0,812	1,32	0,120
12	0,152	0,649	0,853	0,130
9	0,221	0,488	0,560	0,124
7	0,293	0,380	0,413	0,121
5	0,403	0,271	0,280	0,114
4	0,489	0,217	0,222	0,108
3	0,618	0,163	0,165	0,0957

The last column shows that for the greater deviations, not less than 7 mm, the curve agrees satisfactorily with theory, $v\sqrt{\frac{b^2}{1-b^2}}$ being sufficiently constant. If we pass on to smaller ordinates this quantity seems to decrease very rapidly. It must be remarked however that the small ordinates cannot be measured with sufficient precision. We shall therefore use the first four ordinates only. The mean of the numbers deduced from them is

$$v\sqrt{\frac{b^2}{1-b^2}} = 0{,}124.$$

Further it follows from the curve that

$$|\alpha|_m = \frac{1{,}85}{145{,}4} = 0{,}320 \,.\, 10^{-2}.$$

The moment of inertia of the vibrating system was determined by measuring the change of frequency produced by the addition of a small moment of inertia, which is accurately known.

We found [1]) for it

$$Q = 0{,}0126$$

If now we take 1300 for the magnetization (calculated from the hysteresis curve of the material and the constants of the coil) we find for the magnetic moment of the cylinder

$$I_s = 470.$$

With these numbers equation (17) leads to the value

[1]) It may be mentioned here that, assuming a pure cylindrical form, we calculated for the moment of inertia of the cylinder without the glass tube and the little mirror $Q = 0{,}0102$.

$$\lambda = 1{,}1 \,.\, 10^{-7},$$

which agrees very well with the theoretical one 1,13. 10^{-7}.

We must observe, however, that we cannot assign to our measurements a greater precision than of $10^0/_0$.

It seems to us that within these limits the theoretical conclusions have been fairly confirmed by our observations..

The experiments have been carried out in the "Physikalisch-Technische Reichsanstalt". We want to express our thanks for the apparatus kindly placed at our disposition.

Physics. — *"On a possible influence of the* FRESNEL-*coefficient on solar phenomena"*. By Prof. P. ZEEMAN.

(Communicated in the meeting of September 25, 1915).

We shall prove here, that the presence of the term $-\frac{\lambda}{\mu}\frac{d\mu}{d\lambda}$ of LORENTZ in the expression for the FRESNEL coefficient (cf. also my paper Vol. **18**, p. 398 of these Proceedings) may give rise to a change in the propagation of lightwaves if in a moving, refracting medium a change of velocity occurs. I suppose the medium to have everywhere the same density and to be flowing with a velocity v parallel to the axis of X in a system of coordinates that is at rest with respect to the observer. In the direction of the Z axis a velocity gradient exists in such a way, that the velocity decreases with the distance to the X axis and becomes zero at the distance $z = \Delta$. If now the incident lightbeam (with a plane wave front) is parallel to the axis of X, the parts of the wave fronts which are near this axis will be more carried with the medium than those at a greater distance. The wave front will thus be rotated.

If the velocity decreases linearly in the direction of the Z axis the wavefront will remain plane. In a time t the angle of rotation, (supposed to be small) will be $\alpha = \frac{\varepsilon . v . t.}{\Delta}$, where ε is the FRESNEL coefficient and where v and Δ have the above mentioned meaning. More in general we may consider an element of the wave front and then write $\frac{dv}{dz}$ for $\frac{v}{\Delta}$. Moreover t may be expressed as a function of the velocity of light and the path through which the rays have travelled, so that we find

$$\alpha = \frac{\varepsilon l}{c/\mu}\frac{dv}{dz} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

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