Supplementary material for: Multiple Mediation Analysis for Interval-Valued Data

Contents

1 Simulation Study for the $m = 2$ mediation case

This section contains the tabular results of the simulation study for the $m = 2$ case (see Section 6 of the manuscript).

Simulation design. As described in the simulation design for the $m = 1$ case.

Data Generation. As described for the previous case except as follows. The mediator variables $\mathbf{M^c}$ and $\mathbf{M^r}$ as well as the dependent variables $\mathbf{y^c}$ and $\mathbf{y^r}$ are obtained by applying the interval model depicted in Figure 2-B of the manuscript with the following parameters: $\mathbf{A^c} = \text{diag}(4.8, 2.1)$, $\mathbf{A^r} = \text{diag}(3.1, -8.6)$, $\text{vec}(\mathbf{\Xi}) = (2.7, -0.98, 4.1, 2.3)^T$, $\mathbf{\Pi} = \text{diag}(2.04, 1.1)$, $\alpha^{\mathbf{c}} = 3.0$, $\alpha^{\mathbf{r}} = -5.3$, $\beta = (2.3, 1.9)^T$, $\gamma^{\mathbf{c}} = (1.9, 0.9)$, $\gamma^{\mathbf{r}} = (2.1, -4.3)$, and $\delta = -3.25$. Note that, diag(.) is the operator that transforms a vector into a diagonal matrix whereas vec(.) transforms a $kn \times 1$ vector into a $n \times k$ matrix.

Outcome measures. As described in the simulation design for the $m = 1$ case.

Results. The results are provided in the Table 1

n, ϵ	IMedA-ALS		SEM-ML		SEM-WLS		2SMA	
	amse	PA	amse	PA	amse	PA	amse	PA
$n = 50$								
ϵ_1	0.41	96.11	0.64	80.37	37.42	58.93	0.64	80.42
ϵ_2	0.43	94.54	0.67	78.46	50.23	59.97	0.66	78.88
ϵ_3	0.46	93.00	0.71	77.18	60.93	52.87	0.70	77.68
ϵ_4	0.48	91.99	0.71	77.22	46.97	53.90	0.69	78.10
$n = 250$								
ϵ_1	0.40	96.52	0.46	93.46	37.52	67.88	0.46	93.53
ϵ_2	0.40	96.27	0.48	92.88	48.71	68.02	0.46	93.26
ϵ_3	0.41	96.05	0.49	92.03	56.76	66.46	0.47	92.68
ϵ_4	0.42	95.78	0.50	91.72	83.22	66.84	0.47	92.79
$n = 500$								
ϵ_1	0.40	96.52	0.43	94.99	41.23	72.24	0.43	95.07
ϵ_2	0.40	96.39	0.45	94.47	34.91	72.66	0.43	94.86
ϵ_3	0.40	96.37	0.46	93.98	39.25	69.86	0.44	94.64
ϵ_4	0.40	96.23	0.47	93.62	51.15	71.05	0.44	94.70
$n = 1000$								
ϵ_1	0.40	96.52	0.42	95.84	32.84	77.52	0.42	95.91
ϵ_2	0.40	96.50	0.43	95.38	8.45	77.66	0.42	95.77
ϵ_3	0.40	96.46	0.45	94.92	18.36	74.91	0.42	95.59
ϵ_4	0.40	96.40	0.46	94.60	49.70	77.24	0.42	95.68

Table 1. Second Monte Carlo study: Percentage of agreement (PA) index and average root mean square errors (AMSE) for the array of parameters of the multiple mediation model ($m = 2$)

2 2SMA algorithm and Lavaan R-code

The two-steps mediation analysis (2SMA) is based on a set of OLS regressions that are hierarchically estimated in order to guarantee the identification of all the IMedA's parameters. In particular, given the IMedA model:

$$\mathrm{S}_1 : \begin{cases} \mathbf{M^c} = \mathbf{1}\mathbf{A}^c + \mathbf{X}\boldsymbol{\Xi} + \mathbf{E}^c \\ \mathbf{M^r} = \mathbf{1}\mathbf{A}^r + (\mathbf{1}\mathbf{A}^c + \mathbf{X}\boldsymbol{\Xi})\boldsymbol{\Pi} + \mathbf{E^r} \end{cases} \quad \mathcal{S}_2 : \begin{cases} \mathbf{y^c} = \mathbf{1}\alpha^c + \mathbf{X}\boldsymbol{\beta} + \mathbf{M^c}\boldsymbol{\gamma^c} + \mathbf{M^r}\boldsymbol{\gamma^r} + \boldsymbol{\epsilon}^c \\ \mathbf{y^r} = \mathbf{1}\alpha^r + (\mathbf{1}\alpha^c + \mathbf{X}\boldsymbol{\beta} + \mathbf{M^c}\boldsymbol{\gamma^c} + \mathbf{M^r}\boldsymbol{\gamma^r})\delta + \boldsymbol{\epsilon}^r \end{cases} \tag{1}$$

the parameters are estimated in two main steps, one for the system $\mathcal{S}_1$ and another one for $\mathcal{S}_2$, as follows:

I Step: *estimate* $\widehat{\boldsymbol{\Xi}} = (\mathbf{X}^T\mathbf{X})^{-1}\ \mathbf{X}^T\mathbf{M^c}$

$\widehat{\mathbf{A}}^c = \overline{\mathbf{M}}^{\mathbf{c}} - \overline{\mathbf{X}}\widehat{\boldsymbol{\Xi}}$

compute $\mathbf{M}^{\mathbf{c}^*} = \mathbf{1}\widehat{\mathbf{A}}^c + \mathbf{X}\widehat{\boldsymbol{\Xi}}$

estimate $\widehat{\boldsymbol{\Pi}} = (\mathbf{M}^{\mathbf{c}^*T}\mathbf{M}^{\mathbf{c}^*})^{-1}\ \mathbf{M}^{\mathbf{c}^*T}\mathbf{M^r}$

$\widehat{\mathbf{A}}^r = \overline{\mathbf{M}}^{\mathbf{r}} - \overline{\mathbf{M}}^{\mathbf{c}^*}\widehat{\boldsymbol{\Pi}}$

compute $\mathbf{M}^{\mathbf{r}^*} = \mathbf{1}\widehat{\mathbf{A}}^r + \mathbf{M}^{\mathbf{c}^*}\widehat{\boldsymbol{\Pi}}$

II Step: *estimate* $\widehat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{X})^{-1}\ \mathbf{X}^T\mathbf{y^c}$

$\widehat{\gamma}^{\mathbf{c}} = (\mathbf{M}^{\mathbf{c}T}\mathbf{M^c})^{-1}\ \mathbf{M}^{\mathbf{c}T}\mathbf{y^c}$

$\widehat{\gamma}^{\boldsymbol{r}} = (\mathbf{M}^{\mathbf{r}T}\mathbf{M^r})^{-1}\ \mathbf{M}^{\mathbf{r}T}\mathbf{y^c}$

$\widehat{\alpha}^c = \overline{\mathbf{y}}^{\mathbf{c}} - \overline{\mathbf{X}}\widehat{\boldsymbol{\beta}} - \overline{\mathbf{M}}^{\mathbf{c}}\widehat{\gamma}^{\mathbf{c}} - \overline{\mathbf{M}}^{\mathbf{r}}\widehat{\gamma}^{\boldsymbol{r}}$

compute $\mathbf{y}^{\mathbf{c}^*} = \mathbf{1}\widehat{\alpha}^c + \mathbf{X}\widehat{\boldsymbol{\beta}} + \mathbf{M^c}\widehat{\gamma}^{\mathbf{c}} + \mathbf{M^r}\widehat{\gamma}^{\boldsymbol{r}}$

estimate $\widehat{\delta} = (\mathbf{y}^{\mathbf{c}^*T}\mathbf{y}^{\mathbf{c}^*})^{-1}\ \mathbf{y}^{\mathbf{c}^*T}\mathbf{y^r}$

$\widehat{\alpha}^r = \overline{\mathbf{y}}^{\mathbf{r}} - \overline{\mathbf{y}}^{\mathbf{c}^*}\widehat{\delta}$

compute $\mathbf{y}^{\mathbf{r}^*} = \mathbf{1}\widehat{\alpha}^r + \mathbf{y}^{\mathbf{c}^*}\widehat{\delta}$

Note that unlike IMedA-ALS, 2SMA does not involve an alternating gradient-descendent approach in minimizing the loss function associated to the regression model. On the contrary, it adopts several gradient-descent procedures that separately minimize the loss function. In this way, the estimation of a given subset of parameters does not affect the estimation of another subset. As a consequence, each subset of parameters satisfies the convergence of the algorithm toward a proper local/global stationary point whereas a global convergence is not allowed in this context. Therefore, in some circumstances, this estimation procedure may possibly yield biased results as the parameters are independently estimated.

The following R-syntax has been used to estimate mediation paths with SEM-ML and SEM-WLS approaches.

```
### How to define model in Lavaan
# model <- "mc ~ 1 + xc + xl
#           ml ~ 1 + mc
#           yc ~ 1 + xc + xl + mc + ml
#           yl ~ 1 + yc"
# I equation: MC
# II equation: ML
# III equation: yc
# IV equation: yl
###

library("lavaan")
# Note: xc, xl, yc, yl, MC1, ML1 are column-vectors of real data generated by Matlab
# and passed through RCMD BATCH connection.
Data <- data.frame(xc,xl,yc,yl,MC1,ML1)
names(Data) <- c("xc","xl","yc","yl","MC1","ML1")
model <- " MC1 ~ 1 + xc + xl
    ML1 ~ 1 + MC1
    yc ~ 1 + xc + xl + MC1 + ML1
    yl ~ 1 + yc"

data.estimator = "ML" #or "WLS"
fit <- sem(model, data = Data, estimator=data.estimator,likelihood="normal",
    std.ov=FALSE,fixed.x=TRUE,orthogonal=TRUE, control=list(iter.max=500))
summary(fit)
convergence <- lavInspect(fit, what = "converged")
print(convergence)
est.pars <- parameterEstimates(fit)
print(est.pars)
```

3 Scenario analysis for IMedA-ALS

In this section we describe the results of a short scenario analysis carried out to evaluate the flexibility of the IMedA model in capturing all the possible linear relations among the observed variables. In particular, we test the ability of the proposed model to recover the true structure of the observed data over an extended set of possible scenarios. The IMedA model is defined as:

$$\mathrm{S}_1: \begin{cases} \mathbf{M^c} = \mathbf{1}\mathbf{A}^c + \mathbf{X}\boldsymbol{\Xi} + \mathbf{E}^c \\ \mathbf{M^r} = \mathbf{1}\mathbf{A}^r + (\mathbf{1}\mathbf{A}^c + \mathbf{X}\boldsymbol{\Xi})\boldsymbol{\Pi} + \mathbf{E^r} \end{cases} \quad \mathcal{S}_2: \begin{cases} \mathbf{y^c} = \mathbf{1}\alpha^c + \mathbf{X}\boldsymbol{\beta} + \mathbf{M^c}\boldsymbol{\gamma^c} + \mathbf{M^r}\boldsymbol{\gamma}^r + \boldsymbol{\epsilon}^c \\ \mathbf{y^r} = \mathbf{1}\alpha^r + (\mathbf{1}\alpha^c + \mathbf{X}\boldsymbol{\beta} + \mathbf{M^c}\boldsymbol{\gamma^c} + \mathbf{M^r}\boldsymbol{\gamma^r})\delta + \boldsymbol{\epsilon}^r \end{cases} \tag{2}$$

that can be considered as a constrained version of the more general model:

$$\overline{\mathcal{S}}_1: \begin{cases} \mathbf{M^c} = \mathbf{1}\mathbf{A}^c + \mathbf{X}\boldsymbol{\Xi} + \mathbf{E}^c \\ \mathbf{M^r} = \mathbf{1}\mathbf{A}^r + \mathbf{X}\boldsymbol{\Xi}_2 + \mathbf{E^r} \end{cases} \quad \overline{\mathcal{S}}_2: \begin{cases} \mathbf{y^c} = \mathbf{1}\alpha^c + \mathbf{X}\boldsymbol{\beta} + \mathbf{M^c}\boldsymbol{\gamma^c} + \mathbf{M^r}\boldsymbol{\gamma}^r + \boldsymbol{\epsilon}^c \\ \mathbf{y^r} = \mathbf{1}\alpha^r + \mathbf{X}\boldsymbol{\beta_2} + \mathbf{M^c}\boldsymbol{\gamma_2^c} + \mathbf{M^r}\boldsymbol{\gamma_2^r} + \boldsymbol{\epsilon}^r \end{cases} \tag{3}$$

In particular, the systems $\mathcal{S}_1$ and $\mathcal{S}_2$ are more parsimonious than $\overline{\mathcal{S}}_1$ and $\overline{\mathcal{S}}_2$ as they require 12m parameters against 16m, respectively. In order to proceed with the scenario analysis, we decide to analyse the characteristics of the systems $\mathcal{S}_2$ and $\overline{\mathcal{S}}_2$, as those ones related to the other two systems can be easily obtained just by generalizing the ensuing results (note that $\mathcal{S}_1$ and $\overline{\mathcal{S}}_1$ are formally the same of $\mathcal{S}_2$ and $\overline{\mathcal{S}}_2$). For the sake of simplicity, $\mathcal{S}_2$ and $\overline{\mathcal{S}}_2$ can be re-written in terms of their *regression cores*, as follows:

$$\mathrm{S}_2^*: \begin{cases} \mathbf{y^c} = \mathbf{X}\boldsymbol{\beta} \\ \mathbf{y^r} = \mathbf{X}\boldsymbol{\beta}\delta \end{cases} \quad \overline{\mathcal{S}}_2^*: \begin{cases} \mathbf{y^c} = \mathbf{X}\boldsymbol{\beta} \\ \mathbf{y^r} = \mathbf{X}\boldsymbol{\beta_2} \end{cases} \tag{4}$$

that consist of a set of two equations modeling centers and ranges by the matrix of interval-valued independent variables. Note that, because $\mathcal{S}_2^* \subset \mathcal{S}_2$ and $\overline{\mathcal{S}}_2^* \subset \overline{\mathcal{S}}_2$, we can study the more simple regression systems as proxies for the more complex ones. By looking at their structures we can immediately notice how $\overline{\mathcal{S}}_2^*$ is able to model all the relations among the observed variables $\mathbf{y^c}$, $\mathbf{y^r}$, $\mathbf{x^c}$, and $\mathbf{x^r}$ (remember that $\mathbf{X} = (\mathbf{x^c}, \mathbf{x^r})$) and, therefore, can be considered the *golden rule* in terms of flexibility: its four parameters are capable to capture all the possible linear relationships among the observed variables. As a consequence, we can assume $\overline{\mathcal{S}}_2^*$ as the "reference point" for any comparison of $\overline{\mathcal{S}}_1^*$.

Simulation design. With regards to the interval-valued variables involved in the models' representations, we can define the following covariance matrix:

$$\begin{array}{c} \\ \mathbf{x^c} \\ \mathbf{x^r} \\ \mathbf{y^c} \\ \mathbf{y^r} \end{array} \begin{array}{c} \begin{array}{cccc} \mathbf{x^c} & \mathbf{x^r} & \mathbf{y^c} & \mathbf{y^r} \end{array} \\ \begin{pmatrix} 1 & & & \\ 0 & 1 & & \\ a & b & 1 & \\ c & d & e & 1 \end{pmatrix} \end{array}$$

that represent all the possible linear relationships among them. Because the covariance is a linear operator, the five parameters a, b, c, d, e can take just three subsets of real values (i.e., $\mathbb{R}_0^+$, $\mathbb{R}_0^-$, $\{0\}$). As a consequence, all the possible linear relations among the variables are $3^5 = 243$. However, not all scenarios can be tested in our context: indeed, because of the linear constraints realized by the regression systems, the admissible scenarios simply reduce to 81 (e.g., constant models are here meaningless). Finally, we selected a set of meaningful scenarios through which we test $\overline{\mathcal{S}}_1^*$ and $\overline{\mathcal{S}}_2^*$ (see Tables 2-3).

	$\{a,b\} > 0$		$\{a,b\} < 0$		$\{a > 0, b < 0\}$		$\{a < 0, b > 0\}$
1	$\{c,d\} > 0$	5	$\{c,d\} > 0$	9	$\{c,d\} > 0$	13	$\{c,d\} > 0$
2	$\{c,d\} < 0$	6	$\{c,d\} < 0$	10	$\{c,d\} < 0$	14	$\{c,d\} < 0$
3	$\{c > 0, d < 0\}$	7	$\{c > 0, d < 0\}$	11	$\{c > 0, d < 0\}$	15	$\{c > 0, d < 0\}$
4	$\{c < 0, d > 0\}$	8	$\{c < 0, d > 0\}$	12	$\{c < 0, d > 0\}$	16	$\{c < 0, d > 0\}$

Table 2. Scenario analysis: admissible scenarios (panel I)

	$\{a = 0, b > 0\}$		$\{a = 0, b < 0\}$		$\{a > 0, b = 0\}$		$\{a < 0, b = 0\}$
17	$\{c = 0, d > 0\}$	19	$\{c = 0, d > 0\}$	21	$\{c > 0, d = 0\}$	23	$\{c > 0, d = 0\}$
18	$\{c = 0, d < 0\}$	20	$\{c = 0, d < 0\}$	22	$\{c < 0, d = 0\}$	24	$\{c < 0, d = 0\}$

Table 3. Scenario analysis: admissible scenarios (panel II)

Note that Tables 2-3 reports the covariance parameters that refer to the twenty-four relationships between exogenous ($\mathbf{x^c}$, $\mathbf{x^r}$) and endogenous ($\mathbf{y^c}$, $\mathbf{y^r}$) variables. The covariance parameter e of the relationship between $\mathbf{y^c}$ and $\mathbf{y^r}$ can be considered by simply replicating panels I and II for three times, obtaining so eighty-one scenarios. At this point, the scenario analysis is carried out by generating new datasets for each scenario according to a pre-fixed covariance matrix, by applying the regression systems $\overline{\mathcal{S}}_1^*$ and $\overline{\mathcal{S}}_2^*$, and by evaluating the final results in terms of reconstruction of the original data structures.

Data generation. We created 81 semi-positive defined covariance matrices by guaranteeing that the covariance parameters are in the desiderated natural ranges: for values in $\mathbb{R}_0$ the parameters a, b, c, d, e take as large values as possible whereas for values in $\{0\}$ the parameters take values in the interval $[0-\epsilon, 0+\epsilon]$ with ϵ being a small positive quantity closed to zero. Each covariance matrix defines a specific scenario. Next, for each covariance matrix, 1000 $n \times 4$ datasets are drawn from a normal multivariate distribution by constraining the sample covariance matrix to be as close as possible to the fixed covariance matrix.[1] For the sake of simplicity, we used $n = 50$. In order to guarantee the positiveness of ranges, the second and fourth columns of the generated data matrix are linearly rescaled to lie in $\mathbb{R}^+$. Finally, on each new dataset we ran the two regression systems $\overline{\mathcal{S}}_1^*$ and $\overline{\mathcal{S}}_2^*$ and saved the obtained results (i.e., model's parameters and predicted values). Overall, the simulation procedure generated 1000 x 81 = 81000 new datasets as well as an equivalent number of parameters and reconstructed datasets.

Outcome measures. Each simulation was evaluated considering the ability of the models $\overline{\mathcal{S}}_1^*$ and $\overline{\mathcal{S}}_2^*$ in recovering the drawn data sample. Particularly, we computed *means*, *variances*, *RMSE*, and *AoR* on the sample data $\mathbf{y_q^c}$, $\mathbf{y_q^r}$, $\widetilde{\mathbf{Y}}_\mathbf{q} = [\mathbf{y_q^c} - \mathbf{y_q^r}, \mathbf{y_q^c} + \mathbf{y_q^r}]$ and the corresponding reconstructed data $\mathbf{y_q^{c^*}}$, $\mathbf{y_q^{r^*}}$, $\widetilde{\mathbf{Y}}_\mathbf{q} = [\mathbf{y_q^{c^*}} - \mathbf{y_q^{r^*}}, \mathbf{y_q^{c^*}} + \mathbf{y_q^{r^*}}]$ (with $q = 1...1000$). Formally, the root mean squares error (RMSE) is computed as:

$$\text{RMSE} = \sqrt{n^{-1} \cdot \|\mathbf{y} - \mathbf{y}^*\|^2}$$

where $\mathbf{y} \in \{\mathbf{y^c}, \mathbf{y^r}, \widetilde{\mathbf{Y}}\}$ whereas $\mathbf{y}^* \in \{\mathbf{y^{c^*}}, \mathbf{y^{r^*}}, \widetilde{\mathbf{Y}^*}\}$. Note that RMSE gives information about the amount of error of reconstructed data in resembling the sample data. By contrast, the amount of reconstruction (AoR) index and gives information about the *amount of reconstruction* performed by the two regressions systems. The index is defined as follows:

$$\text{AoR} = \frac{\min\{\|\mathbf{y}^+\|, \|\mathbf{y}\ \|\}}{\max\{\|\mathbf{y}^+\|, \|\mathbf{y}\ \|\}} \qquad \text{with} \quad \mathbf{y}^+ = \frac{\mathbf{y}^{*T}\mathbf{y}}{\|\mathbf{y}\|} \cdot \frac{\mathbf{y}}{\|\mathbf{y}\|}$$

where $\mathbf{y}$ and $\mathbf{y}^*$ are defined as above. Note that AoR takes values in $[0, 1]$ with 0 indicating that no reconstruction occurred whereas 1 means that complete reconstruction occurred. In this context, variances gives information about the rigidity/flexibility of the $\overline{\mathcal{S}}_1^*$ and $\overline{\mathcal{S}}_2^*$ systems in order to reconstruct the sample data. In particular, when $\text{var}(\mathbf{y}^*_{\overline{\mathcal{S}}_1^*}) < \text{var}(\mathbf{y}^*_{\overline{\mathcal{S}}_2^*})$ we state the system $\overline{\mathcal{S}}_1^*$ less flexible than $\overline{\mathcal{S}}_2^*$ in recovering $\mathbf{y}^*$.

Results. Tables 4-10 show the obtained results. In particular, Tables 4, 6, 8 report the AoR and RMSE measures whereas Tables 5, 7, 9 show the mean values and variances. Table 10 shows the regression parameters for the equation $\mathbf{y^r}$ computed for both the $\overline{\mathcal{S}}_1^*$ and $\overline{\mathcal{S}}_2^*$ systems. Overall, the results show how $\overline{\mathcal{S}}_1^*$ shows the same performances of $\overline{\mathcal{S}}_2^*$ in almost all scenarios. In particular, $\overline{\mathcal{S}}_1^*$ reconstructs well the data interval-valued structures $\widetilde{\mathbf{Y}}$ like $\overline{\mathcal{S}}_1^*$ does. Moreover, the variances computed on the reconstructed data nicely shows how $\overline{\mathcal{S}}_1^*$ is flexible enough to show equivalent

[1]Several dissimilarities measures can be used in comparing two covariance matrices $\mathbf{A}$ and $\mathbf{B}$. In our context, we resort to use the following measure: $\mathcal{D}(\mathbf{A}, \mathbf{B}) = 1 - [\text{Tr}(\mathbf{A} \cdot \mathbf{B}) \cdot (\|\mathbf{A}\|_f \cdot \|\mathbf{B}\|_f)^{-1}$ with f being the Frobenius norm. Note that $\mathcal{D}(\mathbf{A}, \mathbf{B}) \in [0, 1]$. Among others, such matrix distance yields more strong results in terms of stability and robustness. For further details, see: Herdin, M., Czink, N., zcelik, H., Bonek, E. (2005, June). Correlation matrix distance, a meaningful measure for evaluation of non-stationary MIMO channels. In *Vehicular Technology Conference, 2005. VTC 2005-Spring. 2005 IEEE 61st* (Vol. 1, pp. 136-140). IEEE.

performances of $\overline{\mathcal{S}}_2^*$. This is partially guaranteed by the fact that δ tends to be closed - in absolute value - to the mean value between β_2^c and β_r^2. By contrast, considering the interval-valued components $\mathbf{y^c}$ and $\mathbf{y^r}$ the system $\overline{\mathcal{S}}_2^*$ outperforms $\overline{\mathcal{S}}_1^*$ in terms of AoR, RMSE, and variance just in 24 cases on 81 (note that in Tables such cases are in gray). In particular, in these cases the mean amount of error of recovering (i.e., $1 - AoR$) is almost equal to 5% whereas the error of estimation (RMSE) increases considerably. As a consequence, $\overline{\mathcal{S}}_1^*$ becomes progressively less flexible as also stated by the rapid decrease of the variances. Formally, this is due to the behavior of δ that tends to approximate the mean value between β_2^c and β_r^2. Indeed, in the aforementioned cases δ tends to be closed to zero and consequently the component $\mathbf{y^r}$ proportionally tends to be under-recovered. Nevertheless, as a consequence of the general least squares properties, in such a case $\mathbf{y^{r^*}}$ tends to assume the mean values of $\mathbf{y^r}$. This is the reason why $\overline{\mathcal{S}}_1^*$ is still comparable with $\overline{\mathcal{S}}_2^*$ in recovering $\widetilde{\mathbf{Y}}$.

scenario	$\text{AoR}_{\overline{\mathcal{S}}_1^*}$			$\text{AoR}_{\overline{\mathcal{S}}_2^*}$			$\text{RMSE}_{\overline{\mathcal{S}}_1^*}$			$\text{RMSE}_{\overline{\mathcal{S}}_2^*}$		
	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$
1	1.000	0.996	1.000	1.000	1.000	1.000	0.060	0.276	0.282	0.003	0.000	0.003
2	0.815	0.929	0.825	0.815	0.931	0.825	6.015	1.104	6.117	6.015	1.085	6.113
3	0.943	0.916	0.941	0.943	0.985	0.947	3.234	1.201	3.453	3.232	0.515	3.273
4	0.932	0.913	0.930	0.932	0.983	0.936	3.545	1.234	3.756	3.544	0.542	3.586
5	0.816	0.927	0.825	0.816	0.928	0.826	6.015	1.123	6.121	6.015	1.119	6.120
6	1.000	0.993	0.999	1.000	0.993	0.999	0.135	0.344	0.370	0.135	0.341	0.367
7	0.936	0.911	0.934	0.936	0.983	0.940	3.460	1.242	3.679	3.458	0.538	3.501
8	0.940	0.917	0.939	0.941	0.984	0.944	3.321	1.201	3.534	3.319	0.518	3.360
9	0.957	0.911	0.953	0.957	0.970	0.958	2.851	1.245	3.114	2.849	0.729	2.942
10	0.975	0.915	0.970	0.975	0.978	0.975	2.177	1.214	2.496	2.175	0.623	2.264
11	0.985	0.992	0.985	0.985	0.994	0.986	1.662	0.363	1.702	1.661	0.312	1.691
12	0.795	0.930	0.807	0.795	0.931	0.807	6.427	1.094	6.520	6.426	1.088	6.519
13	0.968	0.914	0.964	0.968	0.975	0.969	2.442	1.216	2.732	2.441	0.654	2.528
14	0.964	0.910	0.960	0.964	0.973	0.965	2.588	1.247	2.876	2.587	0.681	2.676
15	0.802	0.931	0.813	0.802	0.931	0.813	6.329	1.096	6.424	6.329	1.095	6.424
16	1.000	1.000	1.000	1.000	1.000	1.000	0.092	0.003	0.092	0.092	0.000	0.092
17	0.944	0.961	0.945	0.944	0.961	0.945	3.185	0.823	3.291	3.185	0.823	3.291
18	0.831	0.923	0.840	0.832	0.924	0.840	5.525	1.150	5.646	5.525	1.149	5.645
19	0.838	0.921	0.846	0.838	0.922	0.846	5.407	1.167	5.533	5.407	1.166	5.533
20	0.971	0.992	0.973	0.971	0.992	0.973	2.302	0.375	2.333	2.299	0.373	2.329
21	0.998	0.992	0.998	0.998	0.992	0.998	0.542	0.369	0.657	0.541	0.362	0.653
22	0.833	0.921	0.841	0.833	0.921	0.841	5.514	1.164	5.637	5.514	1.163	5.637
23	0.835	0.921	0.843	0.835	0.922	0.843	5.489	1.162	5.612	5.489	1.162	5.612
24	0.966	0.999	0.969	0.966	0.999	0.969	2.472	0.126	2.476	2.472	0.124	2.475

Table 4. Scenario analysis ($d > 0$): AoR and RMSE for both the $\overline{\mathcal{S}}_1^*$ and $\overline{\mathcal{S}}_2^*$ systems. Note that in gray are represented the scenarios where $\overline{\mathcal{S}}_1^*$ largely differs from $\overline{\mathcal{S}}_2^*$.

scenario	$\text{MVs}_{\overline{\mathcal{S}}_1^*}$			$\text{MVs}_{\overline{\mathcal{S}}_2^*}$			$\text{Vs}_{\overline{\mathcal{S}}_1^*}$			$\text{Vs}_{\overline{\mathcal{S}}_2^*}$		
	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$
1	12.012	3.995	12.012	12.011	3.996	12.005	40.131	1.724	553.594	40.153	1.802	551.338
2	12.021	3.991	12.021	12.020	3.991	12.049	17.528	0.522	73.050	17.541	0.563	72.654
3	11.988	3.984	11.988	11.988	3.984	11.983	33.273	0.314	83.630	33.292	1.528	77.257
4	11.928	4.023	11.928	11.927	4.023	11.877	31.936	0.253	64.825	31.963	1.518	59.740
5	12.033	4.001	12.033	12.033	4.000	12.002	18.207	0.515	74.786	18.216	0.522	74.729
6	11.950	3.994	11.950	11.951	3.994	11.986	39.913	1.678	536.203	39.931	1.679	536.221
7	12.053	4.008	12.053	12.054	4.008	12.033	32.220	0.221	56.994	32.246	1.514	52.446
8	12.010	4.007	12.010	12.010	4.007	12.061	33.040	0.333	88.181	33.058	1.542	81.475
9	12.007	4.020	12.007	12.007	4.020	11.949	38.911	0.236	73.496	38.933	1.284	69.359
10	11.987	4.004	11.987	11.988	4.004	12.011	41.789	0.300	100.673	41.819	1.418	95.278
11	11.948	3.995	11.948	11.948	3.995	11.940	38.308	1.671	512.425	38.328	1.706	511.459
12	12.022	3.980	12.022	12.023	3.980	12.082	18.474	0.585	86.175	18.481	0.599	86.079
13	12.009	3.999	12.009	12.008	3.999	11.973	40.912	0.294	96.581	40.943	1.377	91.442
14	11.964	3.995	11.964	11.964	3.995	11.912	40.426	0.216	70.080	40.449	1.340	66.001
15	12.026	4.026	12.026	12.026	4.026	12.036	20.607	0.560	92.062	20.615	0.562	92.082
16	11.986	4.005	11.986	11.986	4.006	12.007	39.910	1.812	578.486	39.926	1.811	578.543
17	11.951	3.986	11.951	11.950	3.987	11.913	29.935	1.115	267.672	29.963	1.114	267.706
18	12.000	4.002	12.000	12.001	4.003	12.014	8.825	0.460	32.404	8.829	0.465	32.639
19	11.960	4.010	11.960	11.960	4.010	12.003	10.114	0.411	33.128	10.115	0.414	33.274
20	12.017	4.004	12.017	12.018	4.005	11.994	35.302	1.661	470.070	35.361	1.661	470.556
21	11.927	4.015	11.927	11.927	4.015	11.940	40.100	1.677	537.489	40.105	1.682	537.405
22	12.002	3.980	12.002	12.002	3.980	11.983	9.095	0.418	30.173	9.096	0.420	30.315
23	12.048	3.995	12.048	12.049	3.995	12.139	9.323	0.430	31.956	9.320	0.435	32.189
24	12.014	3.995	12.014	12.013	3.995	12.009	33.593	1.810	486.395	33.598	1.811	486.423

Table 5. Scenario analysis ($d > 0$): means (MVs) and variances (Vs) for both the $\overline{\mathcal{S}}_1^*$ and $\overline{\mathcal{S}}_2^*$ systems. Note that in gray are represented the scenarios where $\overline{\mathcal{S}}_1^*$ largely differs from $\overline{\mathcal{S}}_2^*$.

scenario	$\text{AoR}_{\overline{\mathcal{S}}_1^*}$			$\text{AoR}_{\overline{\mathcal{S}}_2^*}$			$\text{RMSE}_{\overline{\mathcal{S}}_1^*}$			$\text{RMSE}_{\overline{\mathcal{S}}_2^*}$		
	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$
1	0.811	0.930	0.821	0.811	0.931	0.821	6.091	1.102	6.191	6.091	1.098	6.190
2	0.985	0.982	0.984	0.984	0.982	0.984	1.677	0.554	1.768	1.677	0.551	1.767
3	0.950	0.924	0.948	0.950	0.985	0.953	3.031	1.147	3.243	3.028	0.507	3.071
4	0.939	0.913	0.937	0.939	0.983	0.943	3.369	1.224	3.587	3.367	0.534	3.410
5	0.993	0.999	0.994	0.993	1.000	0.994	1.099	0.130	1.107	1.099	0.061	1.100
6	0.805	0.930	0.816	0.805	0.930	0.816	6.158	1.100	6.256	6.158	1.098	6.256
7	0.936	0.923	0.935	0.936	0.986	0.940	3.439	1.162	3.632	3.436	0.498	3.473
8	0.942	0.910	0.940	0.942	0.982	0.946	3.284	1.243	3.514	3.282	0.553	3.329
9	0.970	0.911	0.965	0.970	0.978	0.971	2.374	1.235	2.679	2.372	0.620	2.453
10	0.964	0.914	0.960	0.964	0.974	0.965	2.592	1.223	2.869	2.590	0.669	2.677
11	0.799	0.932	0.811	0.799	0.932	0.811	6.343	1.082	6.436	6.343	1.081	6.436
12	0.999	0.991	0.998	0.999	0.991	0.998	0.491	0.403	0.637	0.491	0.402	0.636
13	0.954	0.910	0.950	0.954	0.972	0.956	2.932	1.247	3.189	2.931	0.690	3.012
14	0.967	0.914	0.963	0.967	0.975	0.968	2.478	1.222	2.766	2.477	0.659	2.564
15	0.998	0.992	0.998	0.998	0.992	0.998	0.524	0.376	0.647	0.524	0.376	0.647
16	0.790	0.933	0.802	0.790	0.933	0.802	6.481	1.072	6.570	6.481	1.071	6.570
17	0.834	0.921	0.842	0.834	0.921	0.842	5.477	1.169	5.602	5.477	1.168	5.602
18	0.977	0.997	0.979	0.977	0.997	0.979	2.034	0.231	2.047	2.034	0.231	2.047
19	0.994	0.998	0.994	0.994	0.998	0.994	1.044	0.208	1.065	1.044	0.207	1.064
20	0.834	0.922	0.842	0.834	0.922	0.842	5.507	1.165	5.631	5.507	1.164	5.630
21	0.833	0.923	0.842	0.833	0.923	0.842	5.486	1.162	5.609	5.486	1.161	5.609
22	0.949	0.975	0.952	0.949	0.975	0.952	3.024	0.661	3.096	3.024	0.661	3.096
23	0.997	0.988	0.996	0.997	0.988	0.996	0.742	0.454	0.871	0.742	0.454	0.871
24	0.833	0.922	0.841	0.833	0.922	0.841	5.510	1.165	5.634	5.510	1.164	5.633

Table 6. Scenario analysis ($d < 0$): AoR and RMSE for both the $\overline{\mathcal{S}}_1^*$ and $\overline{\mathcal{S}}_2^*$ systems. Note that in gray are represented the scenarios where $\overline{\mathcal{S}}_1^*$ largely differs from $\overline{\mathcal{S}}_2^*$.

scenario	$MVs_{\overline{\mathcal{S}}_1^*}$			$MVs_{\overline{\mathcal{S}}_2^*}$			$Vs_{\overline{\mathcal{S}}_1^*}$			$Vs_{\overline{\mathcal{S}}_2^*}$		
	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$
1	12.007	4.005	12.007	12.006	4.005	11.989	17.045	0.574	78.191	17.051	0.584	78.170
2	12.023	3.997	12.023	12.023	3.997	12.066	36.981	1.492	441.587	36.993	1.496	441.522
3	11.997	4.013	11.997	11.996	4.013	11.966	32.627	0.474	124.030	32.664	1.564	115.746
4	11.988	3.994	11.988	11.987	3.994	11.968	33.297	0.251	66.870	33.313	1.500	61.610
5	11.974	4.012	11.974	11.975	4.012	11.960	37.524	1.818	546.971	37.536	1.832	546.664
6	11.967	3.992	11.967	11.967	3.992	11.955	15.608	0.584	72.830	15.613	0.588	72.869
7	11.978	4.017	11.978	11.979	4.016	11.975	31.621	0.415	104.981	31.642	1.550	97.162
8	12.024	3.977	12.024	12.025	3.977	12.058	34.053	0.241	65.595	34.080	1.519	60.679
9	11.992	3.979	11.992	11.992	3.979	11.974	41.289	0.257	85.000	41.318	1.431	80.146
10	11.977	3.998	11.977	11.977	3.998	12.001	40.186	0.284	91.685	40.208	1.363	86.545
11	11.924	3.994	11.924	11.924	3.994	11.967	20.806	0.606	100.474	20.814	0.608	100.477
12	11.958	3.994	11.958	11.958	3.994	11.982	39.582	1.638	519.188	39.595	1.639	519.196
13	11.925	3.995	11.925	11.924	3.995	11.920	38.523	0.205	63.665	38.545	1.318	59.821
14	11.965	4.000	11.965	11.964	4.000	11.926	41.267	0.293	97.056	41.298	1.384	91.887
15	12.049	4.005	12.049	12.049	4.005	12.080	39.962	1.660	530.546	39.976	1.659	530.595
16	11.970	3.982	11.970	11.969	3.982	11.956	17.158	0.641	87.645	17.164	0.644	87.720
17	11.954	3.988	11.954	11.954	3.987	11.936	9.523	0.394	29.796	9.522	0.397	29.956
18	12.069	4.015	12.069	12.068	4.015	12.064	35.691	1.784	509.196	35.718	1.783	509.272
19	12.014	4.001	12.014	12.014	4.001	12.022	38.571	1.773	547.686	38.603	1.772	547.818
20	12.049	4.012	12.049	12.049	4.012	12.016	8.954	0.418	29.709	8.959	0.419	29.798
21	11.982	4.015	11.982	11.982	4.015	12.016	9.038	0.401	28.924	9.035	0.406	29.136
22	11.954	4.012	11.954	11.954	4.012	11.933	31.126	1.367	341.207	31.131	1.367	341.232
23	12.035	4.000	12.035	12.034	4.000	12.039	39.758	1.594	507.060	39.764	1.594	507.082
24	12.033	4.006	12.033	12.033	4.005	12.037	8.943	0.425	30.195	8.942	0.428	30.392

Table 7. Scenario analysis ($d < 0$): means (MVs) and variances (Vs) for both the $\overline{\mathcal{S}}_1^*$ and $\overline{\mathcal{S}}_2^*$ systems. Note that in gray are represented the scenarios where $\overline{\mathcal{S}}_1^*$ largely differs from $\overline{\mathcal{S}}_2^*$.

scenario	$\text{AoR}_{\overline{\mathcal{S}}_1^*}$			$\text{AoR}_{\overline{\mathcal{S}}_2^*}$			$\text{RMSE}_{\overline{\mathcal{S}}_1^*}$			$\text{RMSE}_{\overline{\mathcal{S}}_2^*}$		
	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$
1	0.892	0.950	0.898	0.892	0.951	0.898	4.420	0.923	4.517	4.420	0.922	4.517
2	0.904	0.950	0.909	0.904	0.950	0.909	4.160	0.929	4.264	4.160	0.928	4.264
3	1.000	0.896	0.991	1.000	1.000	1.000	0.015	1.335	1.335	0.005	0.001	0.005
4	1.000	0.898	0.991	1.000	1.000	1.000	0.025	1.324	1.324	0.024	0.002	0.024
5	0.909	0.945	0.912	0.909	0.946	0.912	4.111	0.977	4.227	4.111	0.961	4.223
6	0.905	0.954	0.909	0.905	0.954	0.910	4.148	0.893	4.244	4.148	0.890	4.243
7	1.000	0.899	0.991	1.000	1.000	1.000	0.020	1.318	1.318	0.019	0.000	0.019
8	0.999	0.897	0.991	1.000	1.000	1.000	0.270	1.333	1.363	0.270	0.038	0.274
9	1.000	0.898	0.991	1.000	1.000	1.000	0.245	1.327	1.353	0.245	0.032	0.248
10	1.000	0.898	0.991	1.000	1.000	1.000	0.249	1.331	1.357	0.248	0.031	0.251
11	0.901	0.943	0.905	0.901	0.943	0.905	4.237	0.994	4.354	4.237	0.993	4.353
12	0.894	0.947	0.899	0.894	0.947	0.899	4.381	0.957	4.485	4.381	0.956	4.485
13	1.000	0.899	0.991	1.000	1.000	1.000	0.240	1.327	1.352	0.240	0.029	0.243
14	1.000	0.898	0.991	1.000	1.000	1.000	0.259	1.325	1.353	0.258	0.033	0.261
15	0.893	0.949	0.899	0.893	0.952	0.899	4.384	0.935	4.483	4.383	0.907	4.477
16	0.906	0.946	0.910	0.906	0.947	0.910	4.125	0.962	4.237	4.125	0.954	4.235
17	0.902	0.952	0.907	0.902	0.952	0.907	4.217	0.906	4.315	4.217	0.905	4.314
18	0.904	0.955	0.908	0.904	0.955	0.908	4.183	0.879	4.276	4.183	0.878	4.275
19	0.900	0.955	0.905	0.900	0.955	0.905	4.246	0.882	4.338	4.246	0.882	4.338
20	0.901	0.953	0.906	0.901	0.953	0.906	4.220	0.904	4.318	4.220	0.903	4.317
21	0.902	0.955	0.907	0.902	0.955	0.907	4.212	0.886	4.306	4.212	0.886	4.306
22	0.907	0.952	0.911	0.907	0.952	0.911	4.113	0.907	4.213	4.113	0.906	4.213
23	0.900	0.950	0.905	0.900	0.950	0.905	4.238	0.927	4.339	4.238	0.927	4.339
24	0.906	0.953	0.910	0.906	0.953	0.910	4.134	0.908	4.234	4.134	0.907	4.234

Table 8. Scenario analysis ($d = 0$): AoR and RMSE for both the $\overline{\mathcal{S}}_1^*$ and $\overline{\mathcal{S}}_2^*$ systems. Note that in gray are represented the scenarios where $\overline{\mathcal{S}}_1^*$ largely differs from $\overline{\mathcal{S}}_2^*$.

scenario	$\text{MVs}_{\overline{\mathcal{S}}_1^*}$			$\text{MVs}_{\overline{\mathcal{S}}_2^*}$			$\text{Vs}_{\overline{\mathcal{S}}_1^*}$			$\text{Vs}_{\overline{\mathcal{S}}_2^*}$		
	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$	$\mathbf{y^c}$	$\mathbf{y^r}$	$\widetilde{\mathbf{Y}}$
1	11.997	3.994	11.997	11.996	3.994	12.029	20.278	0.954	154.814	20.287	0.956	154.824
2	11.992	3.999	11.992	11.991	3.999	11.987	22.020	0.936	164.815	22.030	0.937	164.827
3	12.039	3.986	12.039	12.038	3.986	11.987	43.652	0.004	1.293	43.667	1.842	1.159
4	12.026	3.993	12.026	12.025	3.993	12.013	43.128	0.002	0.608	43.143	1.808	0.571
5	11.954	3.988	11.954	11.955	3.988	11.872	27.971	0.827	184.925	27.988	0.860	184.437
6	11.984	4.020	11.984	11.984	4.020	11.963	22.248	0.977	173.645	22.256	0.982	173.567
7	11.990	3.984	11.990	11.991	3.983	11.959	43.534	0.002	0.544	43.549	1.794	0.512
8	11.989	3.994	11.989	11.990	3.994	11.947	42.710	0.000	0.019	42.725	1.829	0.015
9	11.988	3.994	11.988	11.988	3.994	11.972	48.403	0.004	1.600	48.423	1.820	1.461
10	11.987	4.000	11.987	11.988	4.000	11.985	48.175	0.001	0.504	48.195	1.825	0.456
11	11.996	3.992	11.996	11.996	3.992	12.035	21.495	0.774	132.756	21.504	0.776	132.743
12	11.993	4.011	11.993	11.993	4.011	11.983	20.559	0.881	144.396	20.570	0.883	144.384
13	12.017	4.010	12.017	12.016	4.010	11.997	46.908	0.002	0.766	46.929	1.816	0.718
14	12.021	3.998	12.021	12.021	3.998	12.006	48.028	0.002	0.939	48.049	1.810	0.854
15	11.976	3.991	11.976	11.975	3.991	11.969	20.078	0.918	146.880	20.090	0.971	146.082
16	11.990	3.988	11.990	11.989	3.988	11.995	22.493	0.851	152.539	22.502	0.867	152.374
17	12.002	3.980	12.002	12.002	3.980	12.004	21.776	0.965	167.816	21.795	0.965	167.798
18	12.002	4.003	12.002	12.001	4.003	12.015	22.079	1.007	177.314	22.110	1.007	177.374
19	11.959	3.978	11.959	11.960	3.978	11.958	21.623	1.003	173.139	21.640	1.003	173.130
20	11.954	3.995	11.954	11.954	3.995	11.953	21.797	0.972	169.081	21.816	0.972	169.072
21	12.003	4.004	12.003	12.003	4.004	11.943	21.802	0.994	173.157	21.803	0.995	173.242
22	11.989	3.992	11.989	11.989	3.992	11.996	22.801	0.970	176.394	22.801	0.971	176.456
23	11.961	3.982	11.961	11.961	3.982	11.938	21.583	0.934	161.068	21.584	0.935	161.156
24	12.008	4.011	12.008	12.008	4.011	12.026	22.439	0.977	175.099	22.439	0.978	175.186

Table 9. Scenario analysis ($d = 0$): means (MVs) and variances (Vs) for both the $\overline{\mathcal{S}}_1^*$ and $\overline{\mathcal{S}}_2^*$ systems. Note that in gray are represented the scenarios where $\overline{\mathcal{S}}_1^*$ largely differs from $\overline{\mathcal{S}}_2^*$.

scenario	$d > 0$			$d < 0$			$d = 0$		
	δ	β_2^c	β_2^r	δ	β_2^c	β_2^r	δ	β_2^c	β_2^r
1	0.208	0.151	0.451	0.184	0.053	0.373	0.217	0.092	0.408
2	-0.173	-0.076	-0.240	-0.201	-0.132	-0.464	-0.206	-0.088	-0.417
3	0.097	0.183	-0.639	-0.121	0.143	-0.884	0.009	0.162	-0.803
4	0.089	-0.133	0.811	-0.087	-0.176	0.673	0.006	-0.154	0.823
5	-0.168	0.064	0.281	-0.220	0.121	0.626	-0.172	0.086	0.391
6	0.205	-0.105	-0.651	0.194	-0.061	-0.336	0.210	-0.106	-0.342
7	0.083	0.135	-0.812	-0.115	0.184	-0.614	0.006	0.154	-0.812
8	0.101	-0.181	0.662	-0.084	-0.130	0.845	0.001	-0.163	0.825
9	0.078	0.156	0.290	-0.079	0.075	0.725	0.009	0.132	0.620
10	0.085	-0.066	-0.754	-0.084	-0.155	-0.341	-0.005	-0.128	-0.631
11	0.209	0.157	-0.694	0.171	0.073	-0.413	0.190	0.115	-0.513
12	-0.179	-0.079	0.385	-0.204	-0.148	0.640	-0.208	-0.107	0.592
13	0.085	0.064	0.743	-0.073	0.151	0.338	0.007	0.121	0.662
14	0.073	-0.154	-0.351	-0.084	-0.069	-0.732	0.007	-0.135	-0.594
15	-0.165	0.068	-0.416	-0.204	0.141	-0.670	-0.215	0.132	-0.520
16	0.214	-0.146	0.721	0.194	-0.078	0.421	0.195	-0.105	0.564
17	0.193	-0.076	0.865	0.204	-0.019	0.495	0.211	-0.041	0.778
18	-0.227	0.020	-0.533	-0.224	0.088	-1.043	-0.214	0.040	-0.790
19	-0.202	-0.018	0.501	-0.215	-0.082	1.069	-0.216	-0.042	0.793
20	0.213	0.092	-1.055	0.217	0.019	-0.508	0.212	0.041	-0.783
21	0.205	0.226	-0.421	0.211	0.106	-0.098	0.214	0.166	-0.198
22	-0.215	-0.107	0.094	-0.210	-0.203	0.395	-0.207	-0.164	0.197
23	-0.216	0.109	-0.101	-0.201	0.216	-0.391	-0.209	0.161	-0.195
24	0.233	-0.231	0.411	0.219	-0.109	0.095	0.209	-0.164	0.184

Table 10. Scenario analysis: regression parameters for the equation $\mathbf{y^r}$ for both the $\overline{\mathcal{S}}_1^*$ and $\overline{\mathcal{S}}_2^*$ systems. Note that in gray are represented the scenarios where $\overline{\mathcal{S}}_1^*$ largely differs from $\overline{\mathcal{S}}_2^*$.