Adding and Subtracting Mixed Numbers and Improper Fractions

Just like our counting numbers (1, 2, 3,…), fractions can also be added and subtracted. When counting **improper fractions** and **mixed numbers**, we are counting the number wholes and parts.

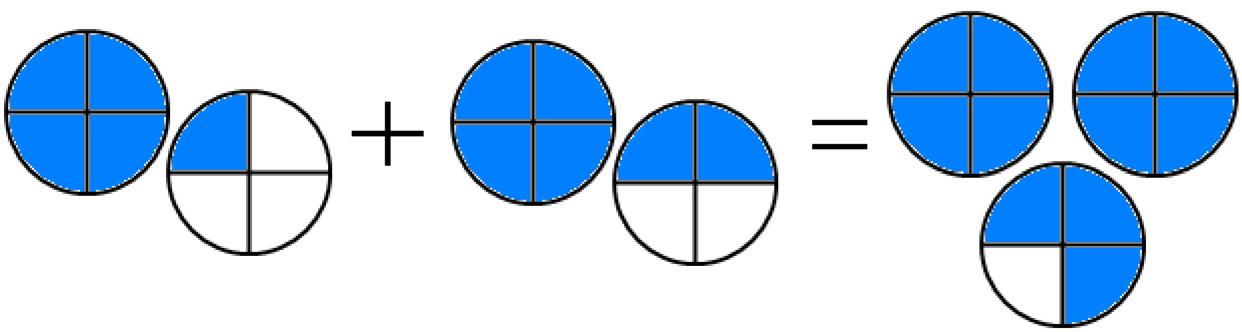
Note: The rules for adding and subtracting improper fractions are the same as working with proper fractions.

Case 1: Adding and Subtracting Improper Fractions with **Common Denominators**

Step 1: Keep the denominator the same.
Step 2: Add or subtract the numerators.
Step 3: If the answer is an improper form, reduce the fraction into a mixed number.

Exercise 1: Add the fractions, $\frac{5}{4}+\frac{6}{4}$.
Let's draw a picture to see what this looks like.

The 4 in the **denominator** tells us that each whole is cut into **4 equal** portions. By adding the fractions we are grouping the total number of wholes and parts.



We have **5** slices and each whole is made up of **4** slices, $\frac{5}{4}$.

We have **6** slices and each whole is made up of **4** slices, $\frac{6}{4}$.

Altogether, we have **2 wholes** and 3 **quarters**, $2\ \frac{3}{4}$

How does the math work?

Step 1: Since the two fractions have equal sized slices, keep the denominator the same, $\frac{?}{\mathbf{4}}$.

Step 2: Add the numerators, $\frac{\mathbf{5}}{4}+\frac{\mathbf{6}}{4}=\frac{\mathbf{5+6}}{4}=\frac{\mathbf{11}}{4}$.

Step 3: Thus, we have $2\frac{3}{4}$ wholes.

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Case 2: Adding and Subtracting Improper Fractions with **Different Denominators**

Step 1: Find the Lowest Common Multiple (LCM) between the denominators.
Step 2: Multiply the numerator and denominator of each fraction by a number so that they have the LCM as their new denominator.
Step 3: Add or subtract the numerators and keep the denominator the same.
Step 4: If the answer is an improper form, reduce the fraction into a mixed number.

Exercise 2: Subtract the fractions, $\frac{7}{6} - \frac{3}{8}$.

Step 1: List the multiples of 6 and 8.

Multiples of 6: 6, 12, 18, **24**, 30, 36, 48…
Multiples of 8: 8, 16, **24**, 32, 40, 48, 56…

The Lowest Common Multiple between 6 and 8 is **24**.

Step 2: a) We need to find a number that when multiplied to the top and bottom of $\mathbf{\frac{7}{6}}$, we get the LCM (24) as the new denominator.

$$\frac{7 \times ?}{6 \times ?} = \frac{?}{24}$$

Since $6 \times \mathbf{4} = 24$, we need to multiply the numerator and the denominator by **4.**

$$\frac{7 \times \mathbf{4}}{6 \times \mathbf{4}} = \frac{\mathbf{28}}{\mathbf{24}}$$

Thus, $\frac{7}{6}$ is equivalent to $\frac{28}{24}$.

b) We need to find a number that when multiplied to the top and bottom of $\mathbf{\frac{3}{8}}$, we get the LCM (24) as the new denominator.

$$\frac{3 \times ?}{8 \times ?} = \frac{?}{24}$$

Since $8 \times \mathbf{3} = 24$, we need to multiply the numerator and the denominator by **3.**

$$\frac{3 \times \mathbf{3}}{8 \times \mathbf{3}} = \frac{\mathbf{9}}{\mathbf{24}}$$

Thus, $\frac{3}{8}$ is equivalent to $\frac{9}{24}$.

Step 3: Since our fractions now have equal sized slices, we can subtract their numerators. Thus, we now have, $\frac{28}{24} - \frac{9}{24} = \frac{19}{24}$ of a whole.

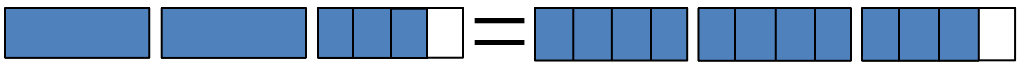
Adding and Subtracting Mixed Numbers and Improper Fractions

Case 3: Adding and Subtracting Mixed Numbers **Method 1**

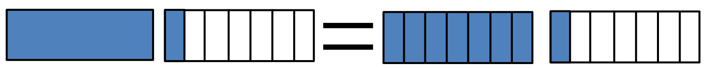
Step 1: Convert all mixed numbers into improper fractions.
Step 2: Check! Do they have a common denominator? If not, find a common denominator.
Step 3: When necessary, create equivalent fractions.
Step 4: Add or subtract the numerators and keep the denominator the same.
Step 5: If the answer is an improper form, reduce the fraction into a mixed number.

Exercise 3: Subtract the fractions, $2\frac{3}{4} - 1\frac{1}{7}$.

Step1: Convert both mixed numbers into improper fractions.



$$\mathbf{2\frac{3}{4}} = \frac{(2 \times 4) + 3}{4} = \mathbf{\frac{11}{4}}$$



$$\mathbf{1\frac{1}{7}} = \frac{(1 \times 7) + 1}{7} = \mathbf{\frac{8}{7}}$$

Step 2: List the multiples of 4 and 7.

Multiples of 4: 4, 8, 12, 16, 20, 24, **28**…
Multiples of 7: 7, 14, 21, **28**, 35…

The Lowest Common Multiple between 4 and 7 is **28**.

Step 3: a) We need to find a number that when multiplied to the top and bottom of $\mathbf{\frac{11}{4}}$, we get the LCM (28) as the new denominator.

$$\frac{11 \times ?}{4 \times ?} = \frac{?}{28}$$

Since $4 \times \mathbf{7} = 28$, we need to multiply the numerator and the denominator by **7.**

$$\frac{11 \times \mathbf{7}}{4 \times \mathbf{7}} = \mathbf{\frac{77}{28}}$$

Thus, $\frac{11}{4}$ is equivalent to $\frac{77}{28}$.

b) We need to find a number that when multiplied to the top and bottom of $\mathbf{\frac{8}{7}}$, we get the LCM (28) as the new denominator.

$$\frac{8\ \times ?}{7\ \times\ ?} = \frac{?}{28}$$

Since $7 \times\ \mathbf{4} = 28$, we need to multiply the numerator and the denominator by **4.**

$$\frac{8\ \times\ \mathbf{4}}{7\ \times\ \mathbf{4}} = \frac{\mathbf{32}}{\mathbf{28}}$$

Thus, $\frac{8}{7}$ is equivalent to $\frac{32}{28}$.

Step 4: Since our fractions now have equal sized slices, we can subtract their numerators. Subtracting their numerators we have, $\frac{77}{28} - \frac{32}{28} = \frac{45}{28}$ of a whole.

Step 5: Thus, we have $1\frac{17}{28}$ wholes.

Case 4: Adding and Subtracting Mixed Numbers **Method 2**

In this second method, we will break the mixed number into *wholes* and *parts*.

Step 1: Add or subtract the *whole number part*.
Step 2: Check! Does the *fraction part* share a common denominator? If not, find one.
Step 3: When necessary, create equivalent fractions.
Step 4: Add or subtract the numerators of the *fraction part* and keep the denominator the same.
Step 5: If the answer is an improper form, reduce the fraction into a mixed number.

Exercise 4: Jessica is $19\frac{1}{2}$ years old today. How old was she $2\frac{1}{4}$ years ago?

Since we are looking at the difference between her current and past ages, our equation will look like, $\mathbf{9}\frac{\mathbf{1}}{\mathbf{2}} - \mathbf{2}\frac{\mathbf{1}}{\mathbf{4}}$.

Step 1: Subtract the **whole number part**, $\mathbf{19} - \mathbf{2} = 17\ wholes$.

Step 2: List the multiples of 2 and 4.

Multiplies of 2: 2, **4**, 6, 8…
Multiplies of 4: **4**, 8, 12…

The Lowest Common Multiple between 2 and 4 is **4**.

Step 3: a) We need to find a number that when multiplied to the top and bottom of $\frac{\mathbf{1}}{\mathbf{2}}$, we get the LCM (4) as the new denominator.

Since $2 \times\ \mathbf{2} = 4$, we need to multiply the numerator and the denominator by **2.**

$$\frac{1 \times \mathbf{2}}{2 \times \mathbf{2}} = \frac{\mathbf{2}}{\mathbf{4}}$$

Thus, $\frac{1}{2}$ is equivalent to $\frac{2}{4}$.

b) Since $\frac{\mathbf{1}}{\mathbf{4}}$ already has the LCM (4) as the denominator, we leave the fraction as it is.

Step 4: Since our **fraction part** now has equal sized slices, we can subtract their numerators. Subtracting their numerators we have, $\frac{2}{4} - \frac{1}{4} = \frac{1}{4}$ of a whole.

Step 5: Combining our **whole number** and **fraction** parts we get,

$$17 \; wholes \; and \; \frac{1}{4} = 17\,\frac{1}{4}\,.$$

Exercise 5: Subtract the fractions, $3\frac{1}{4} - 1\frac{3}{4}$.

Step 1: Subtracting the **whole number part**, we get $\mathbf{3} - \mathbf{1} = 2 \; wholes$.
Step 2: Subtracting the **fraction part**, we get $\frac{\mathbf{1}}{4} - \frac{\mathbf{3}}{4} = ?$ of a whole.

<u>Since we cannot take 3 away from 1, we need to ***borrow*** a whole from the first fraction.</u>

Given $\mathbf{3\frac{1}{4}}$, let's ***borrow*** a whole by following the steps below:

$(\mathbf{2} + \mathbf{1})\frac{1}{4}$	Rewrite *3 wholes* into *2 wholes* + *1 whole*.
$2\,\frac{1 + \mathbf{4}}{\mathbf{4}}$	Since each whole has 4 slices, add the four slices from the borrowed whole into the numerator of the fraction part.
$2\frac{5}{4}$	Thus, we have created an equivalent fraction where $3\frac{1}{4} = 2\frac{5}{4}$.

Step 3: Now we are able to subtract the fractions, $2\frac{5}{4} - 1\frac{3}{4}$.

Subtracting the **whole number part**, we are left with, $\mathbf{2} - \mathbf{1} = 1 \; whole$.
Subtracting the **fraction part**, we are left with, $\frac{5}{4} - \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$ of a whole.

Combining our **whole number** and **fraction** parts we are left with,

$$1 \; whole \; and \; \frac{1}{2} = 1\frac{1}{2}\,.$$

Adding and Subtracting Mixed Numbers and Improper Fractions

Exercises:

1. Add or subtract the following improper fractions and mixed numbers. Remember to reduce where possible.

a) $6\frac{5}{8} - 4\frac{3}{8}$ b) $6\frac{3}{8} + 9\frac{1}{24}$ c) $9\frac{9}{10} + 6\frac{7}{10}$ d) $\frac{10}{7} + \frac{11}{7}$

e) $\frac{9}{5} + \frac{14}{7}$ f) $1\frac{2}{4} - \frac{4}{3}$ g) $\frac{11}{8} + 3\frac{2}{3}$ h) $3 - \frac{6}{5}$

i) $1\frac{3}{5} - 1\frac{4}{9}$ j) $4\frac{1}{7} + 2\frac{1}{3} - \frac{3}{4}$ k) $5\frac{4}{5} + 8\frac{1}{3} - \frac{23}{4}$ l) $5\frac{4}{7} - 4\frac{6}{7}$

2. Each week Fred works 3½ hours on Monday, 3 hours on Tuesday, 2 hours on Wednesday, 2¼ hours on Thursday, and 4 hours on Friday. How many hours does he work per week?

3. During a workshop, the English Tutors ate 3½ pizzas and the Math Tutors ate 5⅔ pizzas. How many pizzas were ordered? (**Hint**: Pizzas are ordered in wholes.)

4. The fourth floor of the D building has 600½ ft^2 of space to house the TLC (Tutoring Learning Centre), SLC (Student Learning Centre), and PAL (Peer Assisted Learning). If the TLC uses 120¼ ft^2 and the PAL uses 115⅓ ft^2, how much space does SLC use?

5. It takes 2⅔ hours to travel to Toronto from Waterloo while travelling with the GO. However, driving takes 1⅛ hours. How much time do you save by driving?

Solutions:

1.

a) $\mathbf{2\frac{1}{4}}$ b) $\mathbf{15\ \frac{5}{12}}$ c) $\mathbf{16\frac{3}{5}}$ d) $\mathbf{2\ \frac{3}{7}}$

e) $\mathbf{3\ \frac{4}{5}}$ f) $\mathbf{\frac{1}{6}}$ g) $\mathbf{5\frac{1}{24}}$ h) $\mathbf{1\ \frac{4}{5}}$

i) $\mathbf{\frac{7}{45}}$ j) $\mathbf{5\frac{61}{84}}$ k) $\mathbf{8\frac{23}{60}}$ l) $\mathbf{\frac{5}{7}}$

2. $\mathbf{14\ \frac{3}{4}}$ 3. **10 pizzas** 4. $\mathbf{364\ \frac{11}{12}}$ 5. $\mathbf{1\ \frac{13}{24}}$