NEW LIGHT ON THE BABYLONIAN TONAL SYSTEM

Leon Crickmore

One of the most significant developments in recent musicology has been the transcription and interpretation of a number of musical cuneiform tablets dating from the second millennium B.C. It has been established that Old Babylonian music was diatonic and based on seven heptachords, corresponding to the first seven tones of the ancient Greek octave species. But a problem remains about the direction of these scales. This paper will suggest a resolution of the 'dilemma' reached by Kilmer in her pioneering research. It will also argue that the theoretical musicians of ancient Mesopotamia are likely to have quantified their scales, using sexagesimal arithmetic and numbers from their standard tables of reciprocals. The resulting tuning would therefore have been Just rather than Pythagorean.

During the second half of the last century, our understanding of the history of music was significantly extended as a result of the transcription and interpretation of a number of musical cuneiform texts dating from the second millennium B.C. For musicians - and possibly for the general reader, too - the most accessible and succinct summary of this research is to be found in Kilmer's article under the heading 'Mesopotamia', in the New Grove Dictionary of Music and Musicians. According to Kilmer[1] 'from the Old Babylonian to the Seleucid periods a standard corpus of Akkadian terms was used to describe seven heptatonic diatonic tuning sets or scales.' The archaeological evidence for our knowledge of the Mesopotamian tuning system, she continues: 'derives from nearly 100 cuneiform tablets'. Of these, three main texts will be crucial to my argument: namely, CBS 10996, UET VII 74 and CBS 1766. However, before commenting on each of these, for the benefit of those who are familiar with modern musical notation by letter-names, Kilmer's transcription of the Mesopotamian heptachords is presented (fig. 1).

1	*išartu*						Dorian	
	E	F	G	A	B	C	D	
2	*kitmu*						Hypodorian	
	E	F#	G	A	B	C	D	
3	*embūbu*						Phrygian	
	E	F#	G	A	B	C#	D	
4	*pītu*						Hypophrygian	
	E	F#	G#	A	B	C#	D	
5	*nīd qabli*						Lydian	
	E	F#	G#	A	B	C#	D#	
6	*nīš* GABA.RI*						Hypolydian	
	E	F#	G#	A#	B	C#	D#	
7	*qablītu*						Mixolydian	
	E#	F#	G#	A#	B	C#	D#	

Fig. 1. * Read *niš tuḫri*.†

Musicians will note that Kilmer and the musicologists with whom she worked have assumed that the scales were rising and corresponded to the ancient Greek octave species, the names of which appear on the right. Moreover, to be even more technical for a moment, the scales have been notated chromatically within a single octave - that is thetically, rather than dynamically - a point to be considered further. The *išartum* mode is the only scale expressed exclusively by means of letters corresponding to the white keys of a piano. The orthographically trained will have noticed that Kilmer gives the string-pair or scale names without mimation.

Commentary and Interpretation

The aim of this paper is to complement the work of archaeologists and textual scholars, by providing, from a musicological perspective, a commentary on and interpretation of the content of three cuneiform texts in particular: CBS 10996, UET VII 74 and CBS 1766.

CBS 10996 is a Neo-Babylonian text, published by Kilmer.[2] UET VII 74 is Old Babylonian. It was originally published by Gurney,[3] but later revised.[4] CBS 1766 is a badly damaged tablet of uncertain provenance and date. It was only published as recently as 2006.[5] In addition to a table of numbers, the text includes an unusual geometrical structure. The inscription above the numerical columns remains largely unintelligible, although recent work by a team at the British Museum suggests a link with the Middle-Assyrian song-list KAR 158.

† The Old Babylonian equation of the pseudo ideogram GABA.RI has recently been rendered as *niš tuḫrum.* See Krispijn-Mirelman, *Iraq* (forthcoming).

Basic tuning	Fine tuning	Heptachordal name
1-5	7-5	*nīš* GABA.RI*
2-6	1-6	*išartu*
3-7	2-7	*embūbu*
4-1	1-3	*(nīd qabli)*
5-2	2-4	*(qablītu)*
6-3	3-5	*(kitmu)*
7-4	4-6	*(pītu)*

Fig. 2. * Read *niš tuḫri.*

CBS 10996 lists fourteen pairs of integers between one and seven. The logogram 'SA', preceding the numbers, means a 'string', and suggests a tuning procedure for a seven-stringed instrument. If this is so, the odd-numbered lines from 11-24 refer to pairs of strings defining musical intervals of fifths and fourths. Modern string players still tune their instruments by fifths and fourths, although, unlike their Babylonian counterparts, modern musicians trained to think in terms of relationships between musical pitches rather than between named string-pairs, exclude the 'unclear' interval of the tritone (the diminished fifth or augmented fourth) from an integral role in the procedure. On the other hand, as will emerge later in the discussion of UET VII 74, the Babylonian tuning system could be construed as a cyclic procedure for the correction of tritones. Kilmer[6] interprets the seven 'dichords' (pairs of strings) in my left-hand column as a description of a method for tuning seven strings to each of seven modes or heptachords, with the outcome I have already indicated in figure 1. Smith and Kilmer[7] interpret the dichords of the even-numbered lines between 11 and 24 - that is, those in the righthand column of figure 2 - as a means of 'fine-tuning' the thirds and sixths in each of the seven scales, usually through the adjustment of the common string whose number is underlined in the figure. They consider the likely function of this procedure would be to make the thirds and sixths sound 'sweeter'. This would imply bringing the basic Pythagorean tuning closer to what acousticians call Just tuning - another matter to be considered in greater detail later. The dichords in the even-numbered lines have their own textual descriptions.

UET VII, 74

Kilmer[8] states that it was this text (which she refers to as U. 7/80, its field number) which convinced scholars that heptatonic diatonic scales must be the correct interpretation of the tuning tablets. Unfortunately, it has also left her own pioneering research work 'on the horns of a dilemma.'[9]

For in the secondary literature concerning CBS 10996 and UET VII 74, a difference of opinion emerges about whether the heptachordal scales should be interpreted as rising or falling. Musicologists have been uncertain about whether the word '*qudmu*' ('foremost string') in CBS 10996, refers to the string sounding the highest or the lowest pitch. When Gurney first published UET VII 74 in 1968, everyone assumed that the scales defined in the tablet were ascending. However, some years later, the musicologist, Vitale,[10] argued that the string descriptions 'thin' and 'small' in UET VII 126 must refer to higher-pitched strings, and in consequence the scales in UET VII 74 ought to be descending. Then the Assyriologist, Krispijn,[11] proposed an improved reading of the twelfth line of UET VII 74 which supported Vitale's view. The relevant portion of line 12 originally read: 'NU SU', 'no more', that is, 'end of sequence'. Krispijn considered that damaged signs were compatible with *'ḫu-um'*, and suggested *'nusuḫ(u-um)'*, the infinitive of the verb *'nasaḫum'*, 'to tighten'. Gurney[12] therefore, issued a revised transliteration, as a result of which most textual scholars and musicologists have accepted that (with regard to UET VII 74 at least) the scales defined must be falling. Such a consensus, however, created a problem for Kilmer, for while it is true that the tuning procedures she had derived from CBS 10996 can be applied in either an upward or a downward direction, the change of direction results in different names for the scales. The only scale which retains the same name whether rising or falling is *embūbum*. Fig. 3 indicates the anomalies in nomenclature.

Vitale	Kilmer
išartum	*nīd qablim*
embūbum	*embūbum*
nīd qablim	*išartum*
qablītum	*nīš* GABA.RI*
kitmum	*pītum*
pītum	*kitmum*
nīš GABA.RI*	*qablītum*

Fig. 3. *Read *niš tuḫrim.*

Kilmer frankly admitted this dilemma, but at the same time expressed her belief that 'we have not arrived at the end of the discussions of this subject' and 'perhaps the answer will lie in our eventual ability to understand how 'pitch sets' could work either up or down'.[13] A possible escape route out of this dilemma, was published earlier this year.[14] [15] The musicologists who assisted in the recovery of the Mesopotamian tuning system were perhaps too eager to relate its scales to the octave species of ancient Greece. Kilmer[16] notes that no-one has yet identified a Sumerian or Akkadian word for 'octave'.

The octave may not have been thought of as a unit in its own right, but rather by analogy like the first day of a new seven-day week. Nicomachus, writing in the second century A.D., devotes the whole of the fifth chapter of his *Manual of Harmonics* to the thesis that 'Pythagoras, by adding the eighth string to the seven-stringed lyre, instituted the attunement of the octave' (for full text and commentary see Levin[17]). The pioneering musicologists were not comparing like with like, but seven-note scales (heptachords) with eight-note scales (octachords). Thus, for example, when defined as a series of tones (t) and semitones (s), the heptachord *išartum* would be stttst, corresponding to the first seven tones of the ancient Greek Dorian scale, rising. But the first seven notes of the falling Dorian octave, starting from the octave above the original note, displays a different pattern: ttsttt - the pattern of the heptachord with the alternative name in figure 3, that is '*nīd qablim*', corresponding to the Lydian octave species and our modern major scale. Each of the heptachords forming a pair in figure 3 are in fact the mirror image of each other. *'embūbum'* is the only scale which keeps the same name in both columns. This is because the pattern of tones and semitones in the octave to which it belongs (the Phrygian) is palindromic: tstttst. If one were to quantify the Babylonian heptachords mathematically, using tone-numbers to express ratios of string-length, the pairs of scales carrying the same name in both columns of figure 3 would be the inverse or reciprocal scales of each other. The Greek octave species and our modern scales consist of ladders of musical pitches. It is these pitches which remain unchanged when the direction of the scale is reversed. The names of the Babylonian scales, however, may be taken to represent specific modal patterns of tones and semitones, and it is these patterns which remain identical whether the heptachord is rising or falling. If my proposed solution to the problem of nomenclature is correct, it seems likely that a remnant of the Babylonian system may have survived in our modern melodic minor scales.

The upper tetrachord of such scales rises and falls in an identical modal pattern: tts, and although the pitches of the scale-ladder change when its direction is reversed, the name of the scale does not. Figure 4 displays the modal patterns of the seven Babylonian heptachords by name.

By focussing on the direction of the scales - a perennial problem in musicology - the musicological significance of UET VII 74 has not yet been explained. The tablet as a whole comprises a cyclical method of tuning and re-tuning a nine-stringed instrument through seven modes in an upward and a downward series. Each of the quatrains of the text follow a similar pattern along the following lines: (1) when the instrument is tuned to scale A, (2) the 'unclear interval' (assumed to be the tritone) falls between strings x and y, (3) tighten string x by a semitone (or, in part 2, tune down string y by a semitone) and (4) the instrument will be tuned to scale B. The names ('*išartum*', '*qablītum*' and so on) refer initially to pairs of strings (the dichords in CBS 10996). The heptachords are called after the dichord which in the previous scale of the series sounded a tritone, but which by the sharpening or flattening of one of its members has now become a perfect fifth. Dumbrill,[18] has elucidated the text succinctly. Figures. 5 and 6 tabulate the tuning procedure. For the construction of these figures. I have used '*išartum*' in its descending form. Figure 5 demonstrates the cycle of tuning by 'tightening', as described in the first part of UET VII 74.

In the '*išartum*' heptachord the tritone lies between the fifth and the second string. The player is instructed to tighten the fifth string in order to tune the instrument to the heptachord '*qablītum*'. Subsequently, in turn, the c, g, a and e are similarly sharpened until the heptachord '*kitmum*' is reached. If, finally, the b in '*kitmum*' is sharpened, the instrumental tuning returns to the original '*išartum*' tuning, but now transposed up a semitone.

Figure 6 shows the tuning procedure by 'loosening', explained in the second part of the text. I have notated this tuning-cycle, beginning from the white-key version of '*išartum*' used in figure 5. It could just as well have started with the transposed version of the scale with which figure 5 ends. This would simply have reversed the tuning procedure in figure. 5, until it returned to the initial white-key scale of '*išartum*'.

In figure 6, however, the b, e, a, d, g and c of

1		2		3		4		5		6		7		String number
Modal Pattern (string intervals)														Name
	s		t		t		t		s		t			*išartum*
	t		s		t		t		t		s			*embūbum*
	t		t		s		t		t		t			*nīd qablim*
	s		t		t		s		t		t			*qablītum*
	t		s		t		t		s		t			*kitmum*
	t		t		s		t		t		s			*pītum*
	t		t		t		s		t		t			*nīš* GABA.RI*

Fig. 4. * Read *nīš tuḫrim.*

No	1		2		3		4		5		6		7	Tritone	Retuning
Name	*išartum*														
	c”		b’		a’		g’		f’		e’		d’	5-2	5♯
		s		t		t		t		s		t			
Name	*qablītum*														
	c”		b’		a’		g’		f♯’		e’		d’	1-5	1♯,8♯
		s		t		t		s		t		t			
Name	*nīš* GABA.RI*														
	c♯”		b’		a’		g’		f♯’		e’		d’	4-1	4♯
		t		t		t		s		t		t			
Name	*nīd qablim*														
	c♯”		b’		a’		g♯’		f♯’		e’		d’	7-4	7♯
		t		t		s		t		t		t			
Name	*pītum*														
	c♯”		b’		a’		g♯’		f♯’		e’		d♯’	3-7	3♯
		t		t		s		t		t		t			
Name	*embūbum*														
	c♯”		b’		a♯’		g♯’		f♯’		e’		d♯’	6-3	6♯
		t		s		t		t		t		s			
Name	*kitmum*														
	c♯”		b’		a♯’		g♯’		f♯’		e♯’		d♯’	2-6	2♯,9♯
		t		s		t		t		s		t			

Fig. 5. *Read *niš tuḫrim*.

‘*išartum*’ (the twin partners of the member of the tritone sharpened in figure 5) are each, in turn, flattened, until the heptachord ‘*qablītum*’ is reached. The loosening of the fifth string (f) in this scale would return the tuning of the instrument to ‘*išartum*’, but this time tuned a semitone lower than at the start.

Mespotamian Music Theory

Assyriologists accept that the Mesopotamians must have had their own system of music theory. The interpretation of the relevant evidence is a matter for musicologists. Before, therefore, dealing with the third cuneiform text (CBS 1766), two further questions need to be considered:

No	1		2		3		4		5		6		7	Tri-tone	Retun-ing
Name	*išartum*														
	c”		b’		a’		g’		f’		e’		d’	5-2	2♭,9♭
		s		t		t		t		s		t			
Name	*kitmum*														
	c”		b♭’		a’		g’		f		e’		d’	2-6	6♭
		t		s		t		t		s		t			
Name	*embūbum*														
	c”		b♭’		a’		g’		f’		e♭’		d’	6-3	3♭
		t		s		t		t		t		s			
Name	*pītum*														
	c’		b♭’		a♭’		g’		f’		e♭’		d’	3-7	7♭
		t		t		s		t		t		s			
Name	*nīd qablim*														
	c”		b♭’		a♭’		g’		f’		e♭’		d♭’	7-4	4♭
		t		t		s		t		t		t			
Name	*nīš* GABA.RI*														
	c”		b♭’		a♭’		g♭’		f’		e♭’		d♭’	4-1	1♭,8♭
		t		t		t		s		t		t			
Name	*qablītum*														
	c♭”		b♭’		a♭’		g♭’		f’		e♭’		d♭’	1-5	5♭
		s		t		t		s		t		t			

Fig. 6. *Read *niš tuḫrim.*

(1) did the theoretical musicians of ancient Mesopotamia define their musical scales mathematically? and

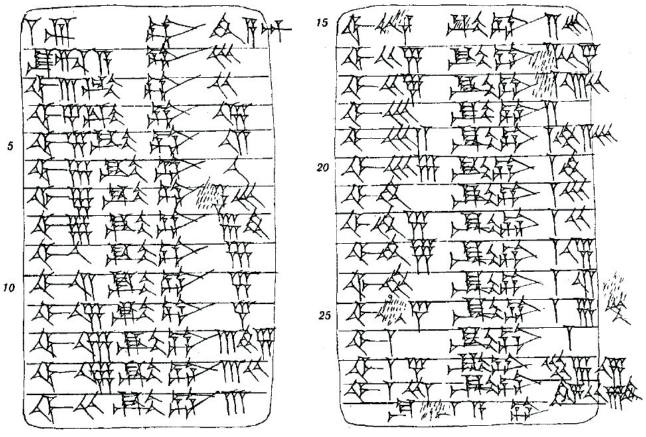
(2) if so, what was the nature of their idealized tuning model? The picture of the Babylonian tonal system that has emerged so far, will also be summarised as a single diagram.

Quantification

The musicologist Crocker,[19] suggests that the Babylonians could have quantified their scales, adding: ‘they certainly had the mathematical capacity - indeed the needed numbers are there in the mathematical texts’. But Gurney and West[20] respectively an Assyriologist and a classical scholar, retorted: 'since there is no evidence that the Babylonians had any notion of this, there is little point in speculating that they might have done, or that such evidence might yet turn up'. Differences of opinion of this kind are only exacerbated if scholars insist on restricting their research rigorously within a single discipline and a single set of academic criteria. If we seriously wish to increase our understanding of questions of this kind, assyriologists and textual scholars need to have honest dialogue with musicians and historians of mathematics. Interdisciplinary teamwork has become essential. Numerous examples exist of mathematical cuneiform tablets from the scribal schools of nineteenth and eighteenth century Larsa, Ur and Nippur, which contain thirty standard pairs of numbers with their reciprocals, encompassing all the sexagesimally regular numbers from 2-81. It was the musicologist, Ernest McClain[21] who first suggested that these numbers, all in the form $2^p3^q5^r$ seem 'perfectly engineered to fit the specific needs of mathematical harmonics'. It is unfortunate that scholars have paid so little attention to McClain's ideas,' though, happily, more recently the mathematician Kappraff[22] has devoted the entire third chapter of a recent book to the arithmetic of 'Harmonic Law', supporting McClain's suggestions. Elsewhere Crickmore[23] has provided a re-evaluation of the cultural significance of this ancient science of harmonics.

In his article on the 'Musicality of Plato'[24] he further indicated how Plato's 'sovereign geometrical number' (Republic, 546c), namely 60^4 - an intrinsic component of sexagesimal arithmetic - together with certain of its factors, which Plato refers to as 'two harmonies', can be used to quantify the seven Babylonian heptachords.

Figure 7 from Robson[25] shows the text MLC 1670, a typical example of a standard table of reciprocals. Each of its numbers could be used to define a musical pitch. The range 24-60, highlighted, would sound a continuous scale.



Two thirds of 1 is 0;40.
Its half is 0;30.
The reciprocal of 2 is 0;30.
The reciprocal of 3 is 0;20.
The reciprocal of 4 is 0;15.
The reciprocal of 5 is 0;12.
The reciprocal of 6 is 0;10.
The reciprocal of 8 is 0;07 30.
The reciprocal of 9 is 0;06 40.
The reciprocal of 10 is 0;06.
The reciprocal of 12 is 0;05.
The reciprocal of 15 is 0;04.
The reciprocal of 16 is 0;03 45.
The reciprocal of 18 is 0;03 20.
The reciprocal of 20 is 0;03.

The reciprocal of 24 is 0;02 30.
The reciprocal of 25 is 0;02 24.
The reciprocal of 27 is 0;02 13 20.
The reciprocal of 30 is 0;02.
The reciprocal of 32 is 0;01 52 30.
The reciprocal of 36 is 0;01 40.
The reciprocal of 40 is 0;01 30.
The reciprocal of 45 is 0;01 20.
The reciprocal of 48 is 0;01 15.
The reciprocal of 50 is 0;01 12.
The reciprocal of 54 is 0;01 06 40.
The reciprocal of 1 00 is 0;01.
The reciprocal of 1 04 is 0;00 56 15.
The reciprocal of 1 21 is 0;00 44 26 40.
<Its half>

Fig. 7

Within this, 27-48 defines the Babylonian heptachord '*embūbum*'. The same scale, with an added octave note (54), was known to the ancient Greeks as the Phrygian octave species; in the Christian church it became the first ecclesiastical mode; by musicians of the renaissance and in modern times it was known as the Dorian mode. Thus the diatonic scale has survived intact for at least four thousand years.

Figure 8 shows a transcription of the range 24-60 of figure 7 as tone-numbers, representing hypothetical musical pitches: on the extreme right, the falling '*embūbum*' heptachord is indicated; on the left side, the first three columns show the corresponding rising scale, including, within its octave, Kilmer's rising '*išartum*' from figure 1.

The ratios between the tone-numbers represent ratios and reciprocal ratios of string-length. The tone-numbers 25 and 50 are redundant with regard to defining the heptachords. But they become crucially relevant in determining the nature of the thirds and sixths produced by the fine-tuning procedure of CBS 10996. The tuning system which these tone-numbers generate is known technically as 'Just' - a matter that calls for further explanation.

A Case for Just Tuning

Modern science measures pitch in terms of frequency of vibration (Herz). However, since the ancients were unable to measure frequency accurately, they relied instead on ratios of string-length, assuming that all other factors such as the tension and thickness of strings remained constant. The earliest surviving Greek treatise on tuning the musical scale, '*The Division of the Canon*', may date from as early as the turn of the fourth century B.C., and is often attributed to Euclid[26]. The first explicit description of an extended diatonic scale expressed in tuning ratios occurs in Plato's dialogue *Timaeus* (34-7), where it appears in the form of a creation myth as the 'World Soul'. When transcribed musically, the first octave of the 'World Soul' turns out to be the ancient Greek Dorian mode. All the tones are 9:8. This means that the semitone has to be that which is left over (Greek *leimma*) when a diatonic third $(9/8)^2$ is taken from a perfect fourth (4/3), that is 4/3 divided by 81/64 = 256/243. This tuning system is known as

Pythagorean. Mathematically, all its tonenumbers are in the form2^p3^q. 'Just' tuning, on the other hand, also uses the prime number 5 as a generator. Its commonest semitone is 16:15. There are two kinds of tone: 9:8 and 10:9. Its major third is pure, 5:4, as in the harmonic series produced by a natural trumpet. The difference between a Pythagorean diatonic third $(9/8)^2$ and a pure third (5:4) is called by modern acousticians the 'syntonic *comma*'. Its value is 81/80. Friberg[27] cites a mathematical problem in the Seleucid text AO 6484:7. 'In this exercise', he writes, 'the terms 'igi' and 'igi.bi' denote a reciprocal pair of (positive) sexagesimal numbers such that their product is equal to '1' (any power of 60)'. This sounds uncannily like an exercise in the symmetry of harmonic arithmetic. For instance, if we express a perfect fifth (3:2) as 90/60, and its reciprocal (2:3) as 40/60, then 90 x 40 = 60^2.

Also, any musical ratio, expressed as a fraction and multiplied by its reciprocal equals unity (for example, 4/1 x 1/4 = 1; 4/3 x 3/4 =1), and unity (60^0 =1) serves as their geometric mean.

Unity functioned as the fulcrum of ancient mathematics, which, unlike modern mathematics lacked both zero and negative numbers. Friberg gives the solution to the problem as 'igi' = 81/80 and 'igi.bi' = 80/81.

Both 80 and 81 are regular numbers and appear in the reciprocal tables. Could this problem, then, possibly represent a calculation of the syntonic *comma*, thus providing a theoretical underpinning for the practical fine-tuning procedures, the 'sweetening' by ear of the thirds, described in CBS 10996? While this idea must remain pure speculation, the very existence of 'igi'-'igi.bi' mathematical problems, together with the musical connotation of the ratios between the regular numbers in the standard tables of reciprocals, may be taken to suggest that the arithmetic of Just tuning was known at an early date to the theoretical musicians in the temples of Mesopotamia.

On the basis of tablets K170 and Rm 520, Livingstone[28] lists numbers associated with the Babylonian gods: Anu (60), Enlil (50), Ea (40) and Sin (30). The ratios between these numbers also define the main intervals of Just tuning. The perfect fifth (60:40) and the perfect fourth (40:30) are the intervals required for the 'rough' tuning described in CBS 10996; the major third and sixth (50:40 and 50:30), the minor third (60:50) and, (if we allow the octave double of 40) even the minor sixth (80:50), all intervals needed for the subsequent 'fine-tuning'. In UET VII 126, a string listed as 'fourth, small string' in Sumerian, is called 'Ea-created' in Akkadian. If the 'normal' heptachord tuning (*išartum*) is defined in tone-numbers taken from the tables of reciprocals, starting at 30 (Sin), the tone- number of the fourth string will be 40, the number of Ea, patron of music. Thus the four main 'god' numbers (60,50,40,30) occur in the 24-60 range of the reciprocal tables, as do the ratios for the two kinds of tone (9:8 and 10:9) and also the three kinds of semitone (16:15, 27:25, 25:24) needed for the Just tuning of a chromatic scale in the octave 360-720.

Dumbrill[29] has drawn attention to a seal in the British Museum depicting a female musician with a lute. This seal (BM 141632) belongs to the Uruk period some eight hundred years before any previously known representation of the instrument. The lute is a fretted instrument. Dumbrill hypothesizes that the procedure of fretting could have been the origin of the use of ratios and proportional arithmetic to define musical tuning. Fretting involves the proportional shortening of a string. If the necessary measurements were expressed as tone-numbers, the ratios between them would represent 'inferred vibration' - or, as we would call it 'frequency'. The earliest scales would therefore have been rising, as Kilmer originally assumed. It seems possible that with the increasing sophistication of mathematics and tuning theory, ratios of string-length, and consequently falling scales, may have become the norm by the Old Babylonian period, and remained so until after the era of Classical Greece.

If this is so, the tuning ratios could have remained unchanged: they would only require a different interpretation. For we now know that frequency varies in inverse proportion to string-length, but it remains uncertain when the ancients intuitively recognised this fact. In practice, of course, musicians, ancient or modern, tune their instruments by ear, taking account of the acoustics of their particular instrument and of the place of performance.

This inevitably results in some slight degree of diversity between tunings, usually only discernable by a trained ear. Nevertheless, each age tends to favour a particular norm as its own, and all actual tunings approximate to this model. In ancient Greece, for example, the model was Pythagorean tuning; in modern times it is equal temperament.

In the light of all the evidential hints cited, therefore, we may conclude, as a working hypothesis, that the theoretical musicians of ancient Mesopotamia quantified their heptachordal scales in sexagesimal arithmetic, using numbers from their standard tables of reciprocals, and that, as a result, their tuning system would have been 'Just', rather than Pythagorean, as has so far been assumed. In the West the earliest documentary evidence for Just tuning is in the harmonics of Ptolemy[30], as late as the second century A.D. But McClain[31] reports on a remarkable piece of archaeological evidence from China, where, in ancient times, there were seven heptachords similar to the Babylonian scales[32]. The tuning of a carillon of 65 bronze bells recovered from a tomb dating from 433 B.C. is 'Just'. Bearing in mind that, for example, historians of mathematics have now established that Pythagorean triples were known in Mesopotamia a thousand years before Pythagoras, is it not conceivable that mathematicians understood the arithmetic of Just tuning, and that their practical musicians used it, as far back as 1500 years before the Chinese carillon? Figure 9 summarizes all that has been

Regular Number in tables of reciprocals	Rising pitch	Ratio of string length rising	Ratio of string length falling	Falling pitch
24	C^6			E^4
25	$C\sharp^6$			$E\flat^4$
27	D^6	e.t.c. → 9:10	10:9 ← e.t.c.	D^4
30	E^6	15:16	16:15	C^4
32	F^6	8:9	9:8	B^3
36	G^6	9:10	10:9	A^3
40	A^6	8:9	9:8	G^3
45	B^6	15:16	16:15	F^3
48	C^7	8:9	9:8	E^3
50	$C\sharp^7$			$E\flat^3$
54	D^7	9:10	10:9	D^3
60	E^7			C^3
		Reciprocals		

embūbum (left, rows 27–48) · išartum (centre, rows 30–54) · embūbum (right, rows 30–48)

Fig. 8. Regular numbers as tone-numbers (C4 =middle C) with ratios of string length.

inferred so far, in the light of evidence from archaeology, musicology and mathematics, about the Babylonian tonal system. For the sake of simplicity, in this figure the heptachords are notated using letters representing the white keys of the piano only.

Although a modern piano is tuned to equal temperament - that is, all twelve semitones are equal in size, and their mathematical expression involves irrational numbers and a logarithmic spiral which would have been beyond the capacity of ancient Mesopotamian mathematicians - nevertheless, for the purpose of practical explanation, the use of the white keys, though approximate, is quite adequate.

Technically, this form of presentation is described as 'dynamic' in contrast to the 'thetic' notation of previous examples. Also, in view of the current state of the debate about Kilmer's work, the scales are presented initially as rising, starting from Kilmer's original transcription of '*išartum*'. The corresponding falling scales have been added in dotted lines. Presented in this manner, the symmetry between the black and the dotted is stunning. Musically, it is accurate in terms of the patterns of tones and semitones. But it will be noticed that in '*išartum*', for example, the highest tone of the rising scale (9:8) differs from the lowest tone in the falling scale (9:10). To remedy this would require us to place the scales in the octave 144-72, the smallest integers capable of corresponding to the reciprocals of 30-54. Figure 9 shows the string numbers, pitches, tone-numbers ratios and intervals for each of the seven Babylonian heptachords. The numbers in the reciprocal tables have been extended beyond 81 to include 96 and 108, the octave doubles above 48 and 54. Capital letters in the columns on the extreme left and right, relate to cuneiform tablet CBS 1766, on which I shall comment next.

CBS 1766

CBS 1766 was published by Horowitz.[33] The tablet is unusual in that it is headed by a seven-pointed star within two concentric circles, (fig. 10). Below this are columns of seven integers between one and seven. Horowitz reads the figures in pairs horizontally and proposes a mathematical interpretation.

Waerzeggers and Siebes[34] propose an alternative musical interpretation. They read the figures in pairs by column. Thereby they relate the numbers to the seven-pointed star, which they interpret as a visual tuning-chart for a seven-stringed instrument, supplementing the numerical and verbal instructions contained in CBS 10996. The musical interpretation is supported by the research team here at the British Museum.[35]

Figure 11 indicates how the text, with three emendations, might have originally been intended to be understood. The names of the heptachords produced by Kilmer's tuning procedure are listed and identified by capital letters which relate them both to Horowitz's transcription of the tablet and to my figure 9.

In the original, the last four columns on the right are empty, except for line one. The numbers in columns E, F, G and H are extrapolations from Waerzeggers and Siebes's reading of the first line as 5, 4, 3, 2. However, a team at the British Museum has recently suggested an improved and extended reading: 5, 2, 5, 2, 5, 2. Since 5-2 is the tritone in the '*išartum*' scale (column A), its triple use over empty columns may be a kind of musical shorthand to indicate the application of the tritone procedure from UET VII 74 (figs 5 and 6) to the scales defined in detail in columns A-D, with a view to generating heptachords for columns E-H. The application of such a procedure would, in effect, produce an identical musical result to my extrapolation in figure 11. Alternatively, the integers 5 and 2 may refer to the heptachords '*qablītum*' (5-2) and '*išartum*' (2-6) as appropriate modes for certain classes of incantation which the British Museum team think might be listed in the textual heading to the geometrical figure. However, as long as the heading remains almost indecipherable, and the overall context remains musical, we may believe that Waerzeggers and Siebes's interpretation adequately represents the most likely intention of the author of the tablet.

Textual scholars may helpfully be able to throw more light on whether these incantations are imprecations of the scribe, or of a musician, or whether they correspond to items in the song-list KAR 158, which sometimes indicates the appropriate musical modes for certain classes or styles of music (fig. 12).

For this figure, the information in figure 9 is compressed into a single octave, and projected onto the seven-pointed star from CBS 1766. At the centre is information derived from CBS 10996. The heptachords are displayed as falling to the right in black, and rising to the left in red. The initial 'rough' tuning by fifths and fourths is indicated in red; while the subsequent 'fine-tuning' of the thirds is shown by added lines in green. Thus, for instance, the tuning algorithm for *išartum* is represented by 2-6 (red), followed by 1-6 (green). The modal patterns of the scales are identical with those generated by the tuning procedure in UET VII 74 (figs 5 and 6), except that here they are notated dynamically rather than thetically. Thus CBS 10996, UET VII 74 and CBS 1766 are all musically compatible with each other. Next, moving outwards from the centre, the respective tone-numbers from the standard tables of reciprocals, noting the ratios between them. Finally, around the outer circle, I have listed the modern pitch equivalents by letter name, together with alternative numberings of the modes, which correspond to Kilmer's interpretation (red), and to Vitale's (black).

The two concentric circles surrounding the seven-pointed star in CBS 1766 may carry some geometrical and associative meaning. Or, the entire picture may perhaps be a design for some kind of rotating mechanism, like an astrolabe, but for the purpose of tuning an instrument to the appropriate mode for an incantation as listed in the cuneiform tablet KAR 158.

But such possibilities are matters for others to consider. The question which the tablet poses for musicologists is this: could CBS 1766 be the earliest known example of a tone-circle?

At present, the earliest known reference to a tone-circle occurs in the Harmonics of Ptolemy.[36] In the passage in question, Ptolemy bends round the two-octave scale of the ancient Greek Greater Perfect System into a circle to match the ecliptic. In Ptolemy's tone-zodiac, the circle is divided into twelve equal parts as can be done with compasses.

Geometrically, at least, this could correlate with a double octave whole-tone scale in equal temperament.

But mathematicians are right to be sceptical about the validity of ancient tone-circles for which the mathematical expression requires logarithms. Besides, in the text, Ptolemy explicitly associates his tone-circle with the Greater Perfect System, a diatonic scale in which some intervals are tones and others semitones. Ptolemy is probably simply drawing an analogical, rather than a quantitative parallel between a circle, the Greek tonal system and the ecliptic. Similarly, in CBS 1766 the circle is divided into seven approximately equal segments which do not represent equal measures of distance: heptachordal scales comprise two sizes of tones and one of a semitone.

Ancient diagrams of symbolic geometry, such as we find in CBS 1766, may have never been intended to be construed as precisely accurate with regard to particular measurements, but rather understood as approximations of the kind later known as Diophantine. These are entirely adequate for the purposes of analogical philosophy and primitive cosmological thinking. Modern science undoubtedly achieves greater accuracy when it measures musical intervals in cents, or ancient temples and mediaeval cathedrals in metres, but the price of such accuracy is that the proportional integer ratios essential to the structures of musical scales and sacred buildings are thereby concealed.

The Nature of the Evidence

Finally, although much of the evidence supporting my reconstruction of the Babylonian tonal system is circumstantial and dependent on musicological interpretation of such archaeological evidence as has survived, the case I have presented has mathematical consistency and is compatible with what we now know about Babylonian mathematics. It may be considered, therefore, worthy to be treated as plausibly credible. As we continue to unravel Middle Eastern cultures of the past there is likely to be an increasing need for closer co-operation and tolerant understanding between archaeologists, Assyriologists, musicologists and historians of mathematics. Professor Finley[37] once wrote: 'there is eminent authority for the view that questions about the past can be answered at least approximately, through the imagination, provided it is disciplined by an underpinning of sound scholarship'. It is for the reader to decide how far this paper has managed to meet Professor Finley's criterion.

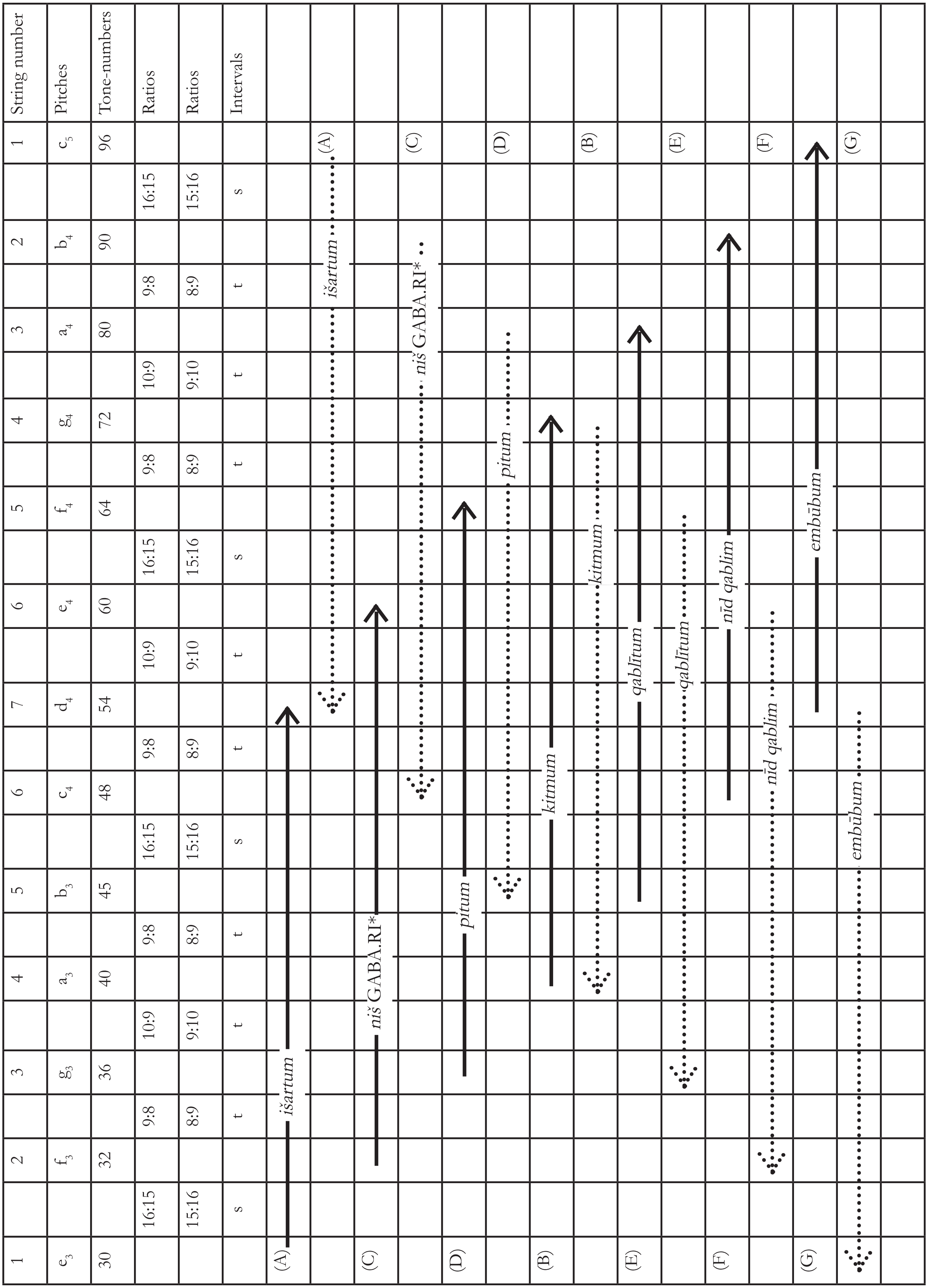
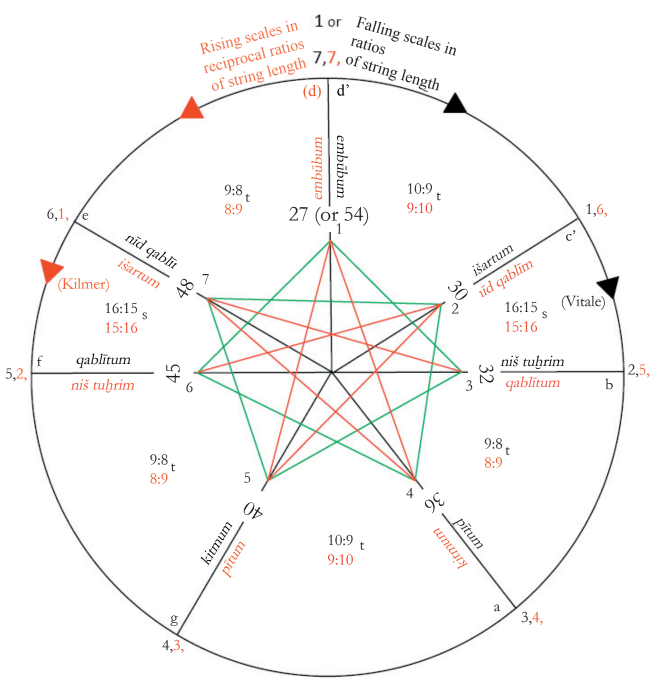


Fig. 9. * Read *niš tuḫrim.*

Fig. 10



CBS 1766 as a Tone Circle

Notes and Key:

The tritonic tuning procedire of UET VII 74
can be applied to falling scales
t = Tone
s = Semitone
c' = Middle c

Red and green lines = Dichords in CBS 10996
Red lines = Initial Tuning 5ths/4ths
Green lines = Fine tuning (3rds and 6ths)

Figures in red indicate reciprocal (inverse) scales

Fig. 12

(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
2	6	1	7	5	4	3	2
6	3	5	4	2	1	7	6
3	7	2	1	6	5	4	3
7	4	6	5	3	2	1	7
4	1	3	2	7	6	5	4
1	5	7	6	4	3	2	1
5	2	4	3	1	7	6	5

Key		
(A)	*išartum*	(6 emended to 5)
(B)	*kitmum*	
(C)	*nīš* GABA.RI*	(5 emended to 4)
(D)	*pītum*	
(E)	*qablītum*	
(F)	*nīd qablim*	
(G)	*embūbum*	(7 emended to 3)
(H)	Return to *išartum*	

Fig. 11. * Read *niš tuḫrim.*

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