Sraffian General Equilibrium

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Abstract

If the Sraffa system of equations is augmented by consumption demand equations of households and investment demand equations of industries the result is a complete system of general equilibrium having a unique positive solution for the relative prices, absolute levels of industrial outputs, the rates of growth and profit, the real wage rate, and the shares of ownership of the capital stock. The model has been generalized to include the government sector and determine the tax rate and public expenditure. Constructive algorithms for the computation of the general equilibrium have been presented. Empirical evidence from cross-country sources has been gathered in support of a central prediction of the model viz. the long-run convergence of industrial growth rates and rates of profit towards uniformity.

Keywords: General Equilibrium, Sraffa System

J.E.L. Classification: B 24, C 67, C 68, D 58

I Motivation

Sraffa (1960) presented his system of equations as determining the relative prices and one of the distributive variables given the levels of outputs and the other distributive variable. Sraffa was aware that this system of equations is incomplete in the sense of being indeterminate; there are not enough independent equations to determine all the unknowns whose solution is usually sought by economic theory. In the unpublished Sraffa papers he notes, (1)

> *"This is not proposed as a complete system of general equilibrium. The data assumed are not sufficient to determine either distribution or values. Only the effects of hypothetical, arbitrarily assumed extra data (such as wages, or the rate of profits) are discussed….. It is offered as a preliminary and there is no a priori reason why, on the basis of it, an equilibrium system should be built: there is some room left for it, as this confessedly indeterminate, but the question is whether there is room enough for the marginal system." [D/3/12/46:20 dated 3.4.1957].*

The present paper is based on the idea that the Sraffa system is indeterminate because it does not allow the circular flow of income to run its full course; that is to say, there are industries transacting with other industries and paying to the households wages and profits for the labour and capital that they receive but the manner in which the wages and profits are utilized by the households is not made a part of the system. At first glance it might appear that Sraffa consciously ignored this part of the loop in the circular flow because he thought that the manner in which wages and profits are utilized, made no difference to the economic outcomes of prices, distribution or outputs. But nowhere does Sraffa say such a thing. On the contrary, when the Indian economist Arun Bose sent him a draft of his review of *Production of Commodities* containing the remark that, "Consumers' demand plays a purely 'passive role' in the Sraffa system", Sraffa's reaction was one that can only be described as violent,

> *"Never have I said this … Nothing, in my view, could be more suicidal than to make such a statement. You are asking me to put my head on the block so that the first fool who comes along can cut it off neatly – Whatever you do please do not represent me as saying such a thing." [C32:3, 9/12/1964]*

Inspite of Sraffa's express warning to the contrary the literature in the Sraffian tradition has not allowed consumer demand to play a role. Neither has it allowed investment demand to play a role. More generally the question of the determination of outputs in Sraffa system has been left open. The general presumption has been that consumers would buy whatever they wish to, and can afford to, once the distribution and relative prices are established and their demands would be automatically met by the industries at the established prices. But the question of whether the given outputs actually suffice to meet the consumption and investment requirements and if not, the implications of the disequilibrium for the realization of profits has not been given serious consideration. The purpose of this paper is to study the effects of incorporating the manner in which wages and profits are utilised on the variables of the Sraffa system. Accordingly, this paper shall proceed to extend the Sraffa system by adding to it consumer and investment demand equations. It will be shown that the resulting system of general equilibrium is complete in the sense that it possesses a unique positive solution of the relative prices, absolute industrial outputs, the rate of growth, the rate of profit, the real wage rate, and the ownership shares of the capital stock. All value magnitudes such as the capital stock, the gross and net national incomes, the incomes of capitalists and workers, the values of their consumption and saving, etc. can be ascertained from the equilibrium solution.

The plan of the paper is as follows. Section II formulates the equations consisting of the Sraffa system, the consumption demand equations, the equations for saving and investment, the equation to determine the ownership shares and finally the growth-profit relationship and the dual system of equations to determine growth and outputs. This is a system that consists of simultaneous non-linear equations that can only be solved by a process of trial and error. Section III presents an algorithm for obtaining the equilibrium solution and section IV presents numerical illustrations. Section V articulates the caveats that must be kept in mind from the viewpoint of applying the system to obtain predictions that can be tested against actual data. Section V presents a generalization of the Sraffa system to include stocks of continuously replenished inventories and fixed capital which has all the important properties of the Sraffa system for circulating capital. Section VI presents the evidence on two predictions of the model, viz., the convergence in the long run of the growth rates of different industries and the convergence towards uniformity of the rates of profit earned by them. Expressed differently, these predictions establish that in the long run prices tend to gravitate towards the normal prices of production and outputs to the levels of demand for them. Section 7 contains concluding remarks.

II Supply and Demand

Consider an economic system in which the activities of production and distribution are described by the Sraffa system of equations,

$$(A_{11}p_1 + A_{21}p_2 + \ldots + A_{n1}p_n)\,(1+r) + wL_2 = p_1B_1$$
$$(A_{12}p_1 + A_{22}p_2 + \ldots + A_{n2}p_n)\,(1+r) + wL_2 = p_2B_2 \qquad \ldots 1(a)$$

$$(A_{1n}p_1 + A_{2n}p_2 + \ldots + A_{nn}p_n)\,(1+r) + wL_n = p_nB_n$$

The capital stock and the net national product measured in terms of the numeraire are,

$$K = \sum \sum A_{ji}\, p_j$$
$$Y = \sum p_i\, B_i - \sum \sum A_{ji} p_j$$
$$= wL_e + rK$$
$$= w(\sum L_i) + rK$$

where *Le* is the total labour employed in the economic system. In what follows we shall assume fixed technical coefficients and constant returns to scale.

The net national income is divided in a two-fold way, between wages and profits and between workers and capitalists. If *u* is the proportion of capital stock *K* owned by the capitalists, the incomes of the two classes are,

$$Y_k = urK \qquad \ldots 2(a)$$

$$Y_L = (1-u)rK + uL_e \qquad \ldots 2(b)$$

These will be used to consume and save. Final consumption behavior can be described by the linear expenditure system [Stone 1954] which has the properties of homogeneity, additivity and symmetry that are considered theoretically desirable and give good fits to observed consumption data. [Nothing prevents the use of the Almost Ideal Demand Systems of Deaton and Muellbauer (1980) if that is thought more desirable]. Accordingly the consumption demand equations of capitalists and workers may be specified as,

$$p_iC_{ik} = p_iC_{iKo} + a_i(Y_K - \sum p_iC_{iKo}) \qquad \ldots 3(a)$$

$$p_iC_{iL} = p_iC_{iLo} + b_i(Y_L - \sum p_iC_{iLo}) \qquad \ldots 3(b)$$

where, to use Stones' (1954) terminology, C_{iKo}, C_{iLo} are the 'committed quantities and the second terms represent 'supernumerary' quantities. Equation (3) are subject to the restrictions that $\sum a_i = a < 1\ (a_i \geq 0)$, $\sum b_i = b \leq 1 (b_i \geq 0)$ and for the overall stability of the system a < b [Kaldor (1956), Pasinetti (1962)]. The saving by the two classes are,

$$S_k = (1-a)(Y_k - C_{ko}) \qquad \ldots 4(a)$$

$$S_L = (1-b)(Y_L - C_{Lo}) \qquad \ldots 4(b)$$

Where $C_{ko} = \sum p_i C_{iKo}$ and $C_{Lo} = \sum p_i C_{iKo}$. Following Pasinetti (1962) the ownership ratio may be determined from,

$$\frac{u}{1-u} = \frac{S_k}{S_L} = \frac{(1-a)(Y_k - C_{ko})}{(1-b)(Y_L - C_{Lo})}$$

Substitution for Y_{dk} and Y_{dL} and cross-multiplying gives a quadratic equation to determine *s*,

$$\{(b-a)rK\}u^2 + \{(a-b)rK + (1-b)(wL_e - C_{Lo}) - (1-a)C_{ko}\}u + (1-a)C_{ko} = 0 \qquad \ldots(5)$$

Dividing through by the leading coefficient (which is necessarily positive considering that b > a) enables us to write equation (5) in the form

$$u^2 + (-1 + \propto - \beta)u + \beta = 0$$

To prevent complex roots the discriminant must be positive, i.e. $(-1 + \propto - \beta)^2 > 4\beta$ which leads to

$$1 + \propto^2 + \beta^2 > 2\,(\alpha + \alpha\beta + \beta) \qquad \ldots 6(i)$$

Since $\beta > 0$, to obtain at least one positive root the coefficients of equation (5) must alternate in sign so that

$$-1 + \propto - \beta < 0 \Rightarrow \propto - \beta < 1 \qquad \ldots 6(ii)$$

Of course if 6(ii) is satisfied the coefficients alternate in sign twice so both roots must be positive.

Finally, for $u < 1$, the condition is

$$\frac{-1+\propto-\beta+\sqrt{(-1+\alpha-\beta)^2-4}}{2} < 1$$

which leads to,

$$\propto > 0 \Rightarrow (1-b)wL_e > C_{LO} \qquad \ldots 6(iii)$$

The inequalities in (6) are the necessary and sufficient conditions to obtain a solution for u such that $0 < u < 1$. Of the two roots the greater one is the relevant solution. To see why suppose for a moment that $C_{ko} = 0$ in which case the two roots of (5) are $u_1 = 0$, $u_2 = 1-\propto$. Clearly the root $u_1 = 0$ is wrong – capitalists' share of ownership cannot be zero only because their 'committed' consumption is zero; for that to happen capitalists should not save, $a = 1$. But that is out of the question.

Substituting r, w and u in equation (3) and dividing by the commodity prices determines the quantities demanded for final consumption by capitalists' C_{ik} and workers C_{iL} respectively. Adding them gives the total quantity demanded of good i for final consumption. It remains to determine the quantities demanded for the purposes of new investment. The wherewithal for new investment is saving $S = S_k + S_L$ obtained in equation (4). Accordingly, the macroeconomic rate of growth will be,

$$g = \frac{S}{K}$$

Substituting from equations (2) and (4) gives

$$g = \frac{(1-a)[urK - C_{ko}] + (1-b)\,[\{(1-u)rK + wLe - C_{Lo}\}]}{K}$$

Solving for the rate of profit gives the growth-profit relation,

$$r = \frac{gK - (1-a)C_{ko} + (1-b)C_{Lo} - (1-b)wLe}{(1-a)uK + (1-b)(1-u)K} \quad \ldots(7)$$

Equation (7) is written in aggregative value terms. In order to obtain the investment demand for the individual capital goods in physical terms a disaggregative system is required. To obtain such a system the following considerations will be operative. Firstly, the value of new investment must equal the total saving.

$$S = I = \Delta K = \sum \sum (\Delta A_{ji})\, P_j$$

Secondly after the additions ΔA_{ji} are made to the A_{ji} the technical coefficients in each industry must be maintained intact, i.e.,

$$\frac{A_{ji} + \Delta A_{ji}}{B_i + \Delta B_i} = \frac{A_{ji}}{B_i} = a_{ji} => \frac{\Delta A_{ji}}{\Delta B_i} = \frac{A_{ji}}{B_i} => \frac{\Delta A_{ji}}{A_{ji}} = \frac{\Delta B_i}{B_i} = g \ \forall i$$

Suppose the total quantity of commodity j used as input in all industries is $A_j = \sum_i A_{ji}$.

To this quantity will be added $\Delta A_j = \sum_i \Delta A_{ji}$ which is equal to the gross output of commodity B_j less the quantity of it that is used for intermediate and final consumption C_j. The ratio $(B_j - A_j - C_j) / A_j$ may be called the "own rate of growth" of commodity j. Clearly, a situation in which the own rates of growth of commodities differ from one another is one of disequilibrium; commodities with higher own rates are in excess supply and commodities with lower own rates are in excess demand.(2) In effect at the arbitrary supply levels B_j the net physical surpluses available for new investment $B_j - A_j - C_j$ are such as would produce divergent own rates of growth. Industries producing commodities with high own growths would suffer heavy losses but those with low own growths make supernormal profits; the former set of industries will contract but the latter will expand relative to one another requiring reallocation of resources across the industries. (Realised rates of industrial growth are inversely related to the own rates of growth). The reallocations will cause the output levels to change. Therefore, if every item of the capital stock is to grow at a uniform rate (so that technical coefficients are not violated) all the own rates of growth must be equalized. But if this is to happen *across* the different industries the proportions in which the industrial activities take place relative to one another, that is to say the levels of input use and outputs produced by them, cannot be arbitrary. Instead, the proportions in which the industrial activities take place relative to one another must be such that it becomes possible to make additions to the inputs at a uniform growth rate within and across industries.

This leads us to the question of an appropriate dual system corresponding to the primal Sraffa price system (1). Clearly the choice of a dual system depends upon the nature of the commodities in the system in terms of the role that they perform as means of production and/or consumption. It also depends upon the purpose of the formulation. One example is Sraffa's standard system which is a dual system formulation. It is meant for the sole purpose of obtaining a standard commodity that can serve as an invariable measure of value. The standard commodity consists exclusively of basic goods. Another example is the Kurz and Salvadori (1995) formulation of a dual of the Sraffa price system. In our notation it reads as follows,

$AB(1+g)+C=B$

Then supposing that the vector C can be translated into an aggregative consumption per unit of labour and expressed as cL and supposing that the system uses a unit quantity of direct labour they derived an inverse consumption-growth relation to parallel Sraffa's inverse wage-profit relation.

The dual system that we are seeking should enable us to determine the market clearing outputs of all the intermediate goods. If we suppose that of the *n* commodities, *k* serve purely as capital goods, *m-k* serve in both capacities as means of production and as means of final consumption and *n-m* serve purely as consumption goods, then the condition of uniform growth across industries and intermediate commodities can be expressed as,

$$\frac{B_i x_i - C_i}{\sum A_{ij} x_i + A_{ic}} = 1 + g \quad \forall i = 1 \ldots m \qquad \ldots(8)$$

i.e. the net output of each commodity that is available for gross investment must be sufficient to meet the replacement demands of all the industries that use the commodity as an input and to meet new investment demand at a uniform growth rate. The x_i denote the process intensities or scale multipliers of the different industries which will ensure that $\frac{\Delta A_j}{A_j} = \frac{\Delta A_{ji}}{A_{ji}} = g$.This leads to the dual system formulation,

$$(A_{11}x_1 + A_{12}x_2 + \cdots + A_{1m}x_m + A_{1c})(1+g) = B_1 x_1$$

$$(A_{k1}x_1 + A_{k2}x_2 + \cdots + A_{km}x_m + A_{kc})(1+g) = B_k x_k$$

$$(A_{k+1,1}x_1 + \cdots + A_{k+1,m}x_m + A_{k+1c})(1+g) + C_{k+1} = B_{k+1}x_{k+1}$$

$$(A_{m1}x_1 + \cdots + (A_{mn}x_m + A_{mc})(1+g) + C_m = B_m x_m \qquad \ldots 9(a)$$

Where A_{ic} is the aggregate requirement of commodity i (1 ...m) in n-m industries producing the purely consumption goods. System 9(a) can only determine the growth rate g and relative outputs. To determine the absolute outputs add the equation

$$L_1 x_1 + \cdots + L_m x_m + L_{m+1} + \cdots + L_n = L_e \qquad \ldots 9(b)$$

Equation 9(b) ensures that the primary factor, labour, is reallocated between the industries in such a way that the outputs produced are equal to the demands for them while the labour coefficients of production of the industries remain intact. In matrix notation the equations in (9) can be written as follows,(3)

$$\begin{pmatrix} B_1 - A_{11}(1+g) & -A_{12}(1+g) & \ldots\ldots & -A_{1m}(1+g) & -A_{1c}(1+g) \\ -A_{21}(1+g) & B_2 - A_{12}(1+g) & \ldots\ldots & -A_{2m}(1+g) & -A_{2c}(1+g) \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ -A_{m1}(1+g) & -A_{m2}(1+g) & & B_m - A_{nm}(1+g) & -A_{mc}(1+g) \\ -L_1 & -L_2 & & -L_m & L_e - \sum_{m=1}^{n} L_i \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_m \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \\ \cdot \\ C_{k+1} \cdot \\ \\ C_m \\ 0 \end{pmatrix}$$

…(10)

This completes the general equilibrium system. There are $2n + 3$ equations viz. n price equations in 1(a), $m + 1$ equations in the dual system to determine the growth rate and intermediate goods' multipliers $n – m$ consumption demand equations (3), the growth-profit relation (7) and the ownership share equation (3) to determine as many unknowns, viz. $n – 1$ relative prices, n outputs, the rate of profit, the real wage rate, the rate of growth and the ownership share. Once these primary unknowns are determined the value magnitudes including the capital stock, real national income, etc. are readily ascertained.

It has been implicitly supposed in the above that the labour supply grows at the same rate as that of the economic system. This need not be supposed to happen in a Malthusian manner. Instead it may happen through adjustments in the length of the working day, the length of the working week, allowable entitlements to leaves of absence and/or vacations, changes in the retirement age, participation of women in the workforce, immigration rules, etc.

III Algorithm

The data requirements of the model are limited to (i) the size of the labour endowment L_e, (ii) technology of production as described by the input-output and labour coefficients (iii) consumer habits represented by a homogenous of degree zero function of real income and relative prices; in the specific case of a linear expenditure system the function is defined by the a_i, b_i, C_{iko} , C_{iLo} coefficients. The general equilibrium system contains equations that are interwoven into one another in a way that the unknowns of one set of equations appear as coefficients of the other sets of equations. Its solution can only be found by an iterative process for trial and error.

The algorithm to solve the equations is designed for bringing the supplies of the commodities in line with the demands for them – in each iteration the industries that produce commodities that are in excess supply are made to contract and those producing commodities in excess demand are made to expand until demand-supply gaps are reduced to the desired level of accuracy. The steps are as follows:

(i) Start with some trial value of $r = r_o$, such that $0 < r < R$ and using 1(a) obtain a tentative solution for the relative prices and the real wage in terms of the numeraire. Ascertain the value magnitudes $Y, K,\ C_{Lo}, C_{Ko}$.

(ii) From equation (5) obtain a tentative solution for the ownership share say s_0 and use equations (2) to find the incomes of capitalists and workers. Substituting them in equation (3) and dividing the consumption expenditures by the prices gives the consumption demands viz. $C_{id} = C_{ik} + C_{iL}$.

(iii) For the goods that serve in both capacities as means of production ($i = k + 1 \ldots m$) and as means of consumption substitute the consumptions demands C_{id} into (10). For those goods that serve exclusively as means of consumption, bisect the quantities demanded and the quantities that have been supposed as supplied, i.e. $C_{id} + B_{is}/2$ ($i = m + 1 \ldots n$) and calculate the inputs A_{ji} that are required to supply these new bisected output sizes. Add the inputwise requirements and substitute them in (10) as $A_{1c} \ldots A_{mc}$. Solve the system (10) to determine the corresponding (tentative) solution for g_o and x_{io}. Use equation (7) to find the corresponding solution of the rate of profit say r_1,. Multiply the price equations by $x_{io} \ldots x_{im}$ to obtain a tentative solution for the outputs $B_i x_{io}$.

(iv) Repeat steps (i), (ii), and (iii) until all the variables converge to the desired levels of accuracy, i.e. $r_t - r_{t-1} \approx 0,\ g_t - g_{t-1} \approx 0,\ B_t - B_{t-1} \approx 0, x_i \approx 1$, etc.

IV Illustration

Consider an economic system that produces five commodities of which the first three are capital goods and the remaining two are consumption goods. The technology of production is described by the following input-output and labour coefficients.

$$A^T = \begin{pmatrix} 0.10 & 0.25 & 0.15 & 0 & 0 \\ 0.20 & 0.28 & 0.20 & 0 & 0 \\ 0.033 & 0.133 & 0.033 & 0 & 0 \\ 0.05 & 0.033 & 0.0833 & 0 & 0 \\ 0.04 & 0.10 & 0.14 & 0 & 0 \end{pmatrix} \quad L = \begin{pmatrix} 0.25 \\ 0.20 \\ 0.33 \\ 0.166 \\ 0.20 \end{pmatrix} \quad L_e = 40$$

Consumption behaviour of the capitalists and workers is described by the following parameters of the linear expenditure system;

B_{iko} = [0, 0, 0, 0.2, 0.1] a_i = [0, 0, 0, 0.03, 0.07] a = 0.1

B_{iLo} = [0, 0, 0, 2, 1] b_i = [0, 0, 0, 0.4, 0.5] b = 0.9

It is quite possible that the consumption patterns of the capitalists and workers themselves are different, that is to say, some goods that find a place in the capitalists' consumption basket are absent from that of the workers and vice versa. In that case the corresponding coefficients of the linear expenditure systems will take zero values. Suppose the economic system to be in the following state of disequilibrium;

$$(2p_1 + 5p_2 + 3p_3 + 0p_4 + 0p_5)(1 + r) + \ \ 5w = 20p_1$$

$$(5p_1 + 7p_2 + 5p_3 + 0p_4 + 0p_5)(1 + r) + \ \ 5w = 25p_2$$

$$(1p_1 + 4p_2 + 1p_3 + 0p_4 + 0p_5)(1 + r) + 10w = 30p_3$$

$$(3p_1 + 2p_2 + 5p_3 + 0p_4 + 0p_5)(1 + r) + 10w = 60p_4$$

$$(2p_1 + 5p_2 + 7p_3 + 0p_4 + 0p_5)(1 + r) + 10w = 50p_5$$

13 23 21 40

The own rates of growth of the three capital goods, are $(B_j - A_j)/A_j$ are respectively 7/13, 2/23, and 9/21, that is to say, 0.5384, 0.0869 and 0.4285 respectively. Clearly this is a situation of disequilibrium; capital goods 1 and 3 are in excess supply relative to capital good 2 and capital good 1 is in excess supply relative to capital good 3. The first step towards correcting the disequilibrium would be for industries 1 and 3 to contract and for industry 2 to expand. This will be seen from the solution of scale multipliers x_1 and x_3 in Table 1 in the initial iteration; x_1 and $x_3 < 1$ but $x_2 > 1$ showing the contractions and expansion respectively.

Following the algorithm spelt out at the end of the previous section put into the price equations some arbitrary value for r say r_0=0.4 and iterate until convergence obtains. The greater the number of iterations the greater the numerical accuracy of the solution. The results of select iterations are shown in Table 1 below.

Table 1 (a): Iterative Convergence

Iteration	r	g	s	x_1	x_2	x_3
0	0.4	0.3082	0.7311	0.8978	1.237	0.9325
5	0.3317	0.2937	0.6528	0.9996	0.9997	1.0003
20	0.3251	0.2891	0.6437	0.9999	0.9999	1
50	0.3251	0.289	0.6436	1	1	1
100	0.3251	0.289	0.6436	1	1	1

Table 1 (b): Convergence of Prices and Outputs

Iteration	p_1	p_2	p_3	p_4	p_5	B_1	B_2	B_3	B_4	B_5
0	0.8317	0.9801	0.5822	0.3385	0.4979	20	25	30	60	50
5	0.7472	0.8676	0.5447	0.3154	0.4569	17.8993	30.3419	28.0703	58.4097	50.1021
20	0.7399	0.858	0.5415	0.3134	0.4534	17.8764	30.7961	28.1149	59.0078	50.8143
50	0.7399	0.8579	0.5415	0.3133	0.4533	17.8762	30.7957	28.1154	59.0145	50.8211
100	0.7399	0.8579	0.5415	0.3133	0.4533	17.8762	30.7957	28.1154	59.0145	50.8211

The solution for prices and outputs, letting w = 1 be the numeraire, is p_1 = 0.7399, p_2 = 0.8579, p_3 = 0.5415, p_4 = 0.3133, p_5 = 0.4533 and B_1 = 17.8762, B_2 = 30.7957, B_3 = 28.1154, B_4 = 59.0145, B_5 = 50.8211. The gross and net national products measured in units of-labour commanded are 96.4076 and 53.8400 and the capital stock is 42.5675. The economic system in the equilibrium state will look as follows.

$(1.787\ p_1 + 4.469\ p_2 + 2.681\ p_3)\ (1+r) + \quad 4.469w = 17.876\ p_1$

$(6.159\ p_1 + 8.622\ p_2 + 6.159\ p_3)\ (1+r) + \quad 6.159w = 30.795\ p_2$

$(0.937\ p_1 + 3.748\ p_2 + 0.937\ p_3)\ (1+r) + \quad 9.371w = 28.115\ p_3$

$(2.950\ p_1 + 1.967\ p_2 + 4.917\ p_3)\ (1+r) + \quad 9.835w = 59.014\ p_4$

$(2.032\ p_1 + 5.082\ p_2 + 7.114\ p_3)\ (1+r) + \quad 10.164w = 50.821\ p_5$

13.867   23.889   21.810   40.000

Observe that the gross output of each capital good is $1+g$ = 1.2890 times the total use made of that good in all the industries, e.g., (13.867) (1.2890) = 17.876, etc.; the own rates of growth are equalized.

That this solution is unique is readily verified by putting in a different arbitrary initial value of r say r_0 = 0.6, and noting that the system converges to the same point irrespective of r_0. The path of convergence is as shown in Table 2.

Table 2: Convergence with r_0= 0.6

		r	g	s	x_1	x_2	x_3
Iteration	0	0.6	0.3642	0.8807	0.907	1.255	0.9189
	5	0.355	0.311	0.6821	0.9985	0.9981	1.0019
	20	0.3254	0.2892	0.644	0.9999	0.9999	1
	50	0.3251	0.289	0.6436	1	1	1
	100	0.3251	0.289	0.6436	1	1	1

A different composition of consumer preferences would lead to a different result, eg. if a_i = (0, 0, 0, 0.05, 0.05) and b_i = (0, 0, 0, 0.7, 0.2) with a, b, B_{iko}, B_{iLo} remaining the same as before, the equilibrium obtained is r = 0.3875, g = 0.3455, u = 0.7165, p_1 = 0.8149, p_2 = 0.9577, p_3 = 0.5748, p_4 = 0.3339, p_5 = 0.4897 and B_1 = 19.3602, B_2 = 29.6710, B_3 = 27.6772, B_4 = 95.7373, and B_5 = 20.2188. The output of commodity 4 has risen and that of commodity 5 has fallen due to the shift in preferences. All the prices have risen (the real wage rate has fallen) due to the rise in the rate of profit.

If the government imposes say an income tax at a uniform rate on all incomes and uses the proceeds to provide a public good for which it purchases goods A_g = [2, 1, 1.5, 1, 0.75] of the five commodities and employs 1 unit of labour the resulting equilibrium is r = 0.5337, g = 0.4345, u = 0.8323, t = 0.0895, p_1 = 1.0687, p_2 = 1.2949, p_3 = 0.6879, p_4 = 0.4027, p_5 = 0.6118 and B_1 = 18.3622, B_2 = 31.1680, B_3 = 27.0474, B_4 = 44.0163, and B_5 = 36.6897 with gross and net national products being 132.4588 and 76.9516 and government expenditure of 6.3259. Of course the incorporation of the government budget will necessitate a restatement of the equations. Saving and consumption will be made out of disposable (after-tax) incomes and the government's budget equation $tY = \sum A_{ip} p_i + wLg$ will be added and the tax rate solved as an additional unknown.

To take only one more example suppose commodity 3 serves in a dual capacity as a capital as well as a final consumption good. Suppose the parameters of the linear expenditure system are as follows:

B_{iko}, = (0, 0, 0, 0.2, 0.1) a_i = (0, 0, 0.02, 0.04, 0.04) a = 0.1

B_{iLo} = (0, 0, 0, 2, 1) b_i = (0, 0, 0.1, 0.6, 0.2) b = 0.9

So far as the system of equations is concerned there is only one change; the equation for the third commodity in the dual system will now be,

$$(A_{31}x_1 + A_{32}x_2 + A_{33}x_3 + A_{3c})(1+g) + C_3 = B_3x_3$$

i.e., the left hand side contains replacement demand, new investment demand and final consumption demand for commodity 3. The equilibrium obtained is as follows,

$$\begin{array}{lllll}
(1.625\,p_1 & + 4.062\,p_2 & + 2.437\,p_3)(1+r) + & 4.062w & = 16.251\,p_1 \\
(5.084\,p_1 & + 7.118\,p_2 & + 5.084\,p_3)(1+r) + & 5.084w & = 25.424\,p_2 \\
(1.085\,p_1 & + 4.341\,p_2 & + 1.085\,p_3)(1+r) + & 10.852w & = 32.556\,p_3 \\
(4.646\,p_1 & + 3.097\,p_2 & + 7.744\,p_3)(1+r) + & 15.488w & = 92.929\,p_4 \\
(0.902\,p_1 & + 2.255\,p_2 & + 3.158\,p_3)(1+r) + & 4.511w & = 22.558\,p_5 \\
\hline
13.344 & 20.876 & 19.510 & 40.000 &
\end{array}$$

The solution of the other unknowns is; r = 0.2490, g = 0.2178, u = 0.4164, p_1 = 0.6851, p_2 = 7584, p_3 = 0.5085, p_4 = 0.2927, p_5 = 0.4168 with K = 34.630 and Y = 48.624. Of the gross output of commodity 3, 19.510 unit is replacement demand, 4.249 units is new investment demand and 8.797 is final consumption demand and (19.510) (1.2178) = 32.556 – 8.797 = 24.368.

V Empirical Considerations

It is only natural to extend the inquiry into the empirical significance of the Sraffian general equilibrium model. At first glance this might be considered a foolhardy venture because real world production conditions vary greatly from the simplistic description that is contained in the circulating capital model depicted in equations (1). To begin with profits in the real world are not earned on the value of produced inputs as described in (1). Moreover, cost-plus pricing rules suggest that prime input cost including the cost of goods consumed and the wage bill is used as a base to apply a gross profit markup to arrive at the market price net of indirect taxes, if any. These markup rates differ across industries. There are other ways in which the real world substantially differs from the model. In the real world profits are earned on owners' capital, not on the assets that serve as inputs in the production process. It is an implicit assumption of the Sraffa system that the value of produced inputs equals the equity capital of the owners. But, industries carry a variety of assets that are financed by means of a variety of liabilities including equity and debts from several sources carrying different interest rates. Moreover industries consist of companies which are usually diversified to different extents and are classified into particular industries depending on the proportion of sales of their dominant product. Further, real world economic systems are monetary economies that are open to international trade and finance whereas the model of section 3 pertains to a closed non-monetary economy. Finally, the real world is characterized by technical change and changes in consumption patterns. All these appear to be formidable barriers from the viewpoint of the empirical validation of the model's results.

However, if suitable generalisations can be made of the equations in (1) some of the barriers can become less formidable. Two such generalisations include a) incorporating stocks of inventories that are continuously carried by firms and b) incorporating fixed capital. The effect of these generalisations would enable the price equations of (1) to describe continuous input continuous output processes of production and to make them consistent with on-cost markup pricing methods that most industries use. Let S^T be an n x n matrix of stock items that are continuously replenished as soon as they are depleted in the course of being used up in production. The price equations that incorporate these stocks can be suitably written as

$$S^T Pr + A^T P + wL = P \qquad \ldots(11)$$

In equation (1) the A matrix consisted of elements that are carried as stock at the beginning of the period and used up during the period; the stock turnover rates are assumed to be 1 for all items of stock in all industries. But equation (11) allows them to be different in different industries and different for the different items of stock. Fixed capital can also be readily incorporated. By using the relationships between book-values of durable inputs like machinery all old machines can be suitably "reduced" to their new machine equivalents. The book values of machines of successive ages are

$p_0 = p$
$p_1 = (1+r-\varphi)p$

$$p_{kji} = \left[(1+r)^{k_{ji}} - \varphi\{(1+r)^{k_{ji-1}} + \cdots + 1\}\right]p \qquad \ldots(12)$$

Where $\varphi = \frac{r(1+r)^{kji}}{(1+r)^{kji}-1}$

All the old machines can be translated into their new machine equivalents by applying to them the book values and adding the machines of different ages.

Letting F^T be an n x n matrix. We may write the price equations and the dual output equations as

$$F^T P\varphi + S^T Pr + A^T P + wL = P \qquad \ldots 13(a)$$

$$FB\Omega + SBg + AB + C = B \qquad \ldots 13(b)$$

Where $\varphi = \frac{r(1+r)^{k_{ji}}}{(1+r)^{k_{ji}}-1}$, k_{ji} is the life of machine j in industry i and $\Omega = \frac{g(1+g)^{k_{ji}}}{(1+g)^{k_{ji}}-1}$

Equations 13(a) and 13(b) can be readily expressed as sums of convergent series of direct and indirect input requirements,

$$P = [I - (I - A^T)^{-1}(F^T(r)r + S^T r)]^{-1}(I - A^T)^{-1}wL$$

$$B = [I - (I - A)^{-1}(F(g)g + Sg)]^{-1}(I - A)^{-1}C$$

Where *F(r), F(g)* notation indicates that the elements of F are functions of the rate of profit and rate of growth,

The price equations (13) have all the important properties of the usual Sraffa system, viz. inverse wage-profit relation, possibility of reducing prices to dated labour terms, existence of a unique positive standard system, impossibility of measuring capital independently of distribution and prices, possibility of reswitching of techniques etc. Besides 13(a) is consistent with markup pricing because

$$P = (A^T P + wL)\ (1 + m_i) => (A^T P + wL)m_i = F^T P\varphi + S^T Pr \qquad \ldots(14)$$

i.e., the markup charged on full prime cost recovers the fixed cost (depreciation) plus net profit.

The simple relationship between the rate of growth and the rate of profit shown in equation (7) does not hold in the presence of fixed capital except in the extremely ideal situation of a balanced age distribution of all durable assets in all industries i.e. the number of new machines is ($1+g$) times the number of one-year old machines which in turn are ($1+g$) times the number of tow-years old machines, and so on. In general however growth rate could exceed the rate of profit because gross saving includes depreciation provision plus the saving out of wage and profit incomes; in a model with only working capital the growth rate is always equal to or lower than the rate of profit in equilibrium.

As mentioned earlier industries incur other types of expenses and also carry assets (and liabilities) other than stocks and fixed assets. Their role however is to smooth out the flow of the operations and/or transactions and to ensure a smooth flow of production and sale.

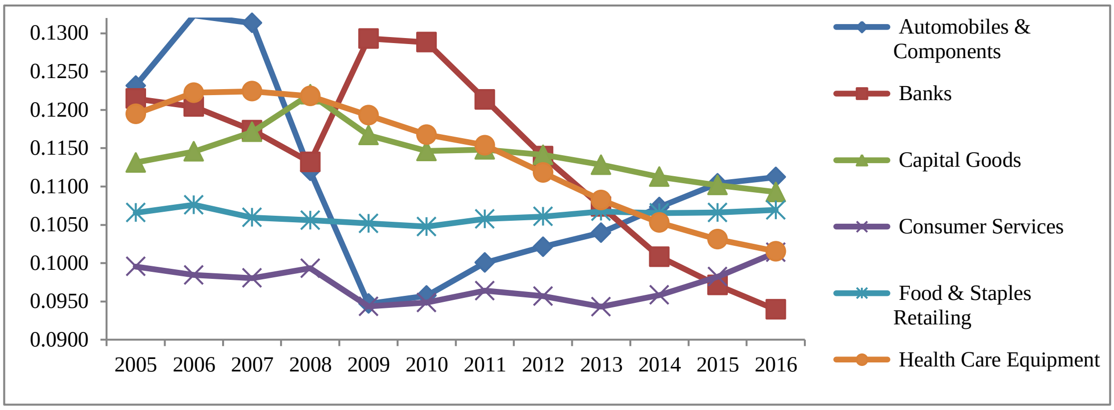
VI Evidence

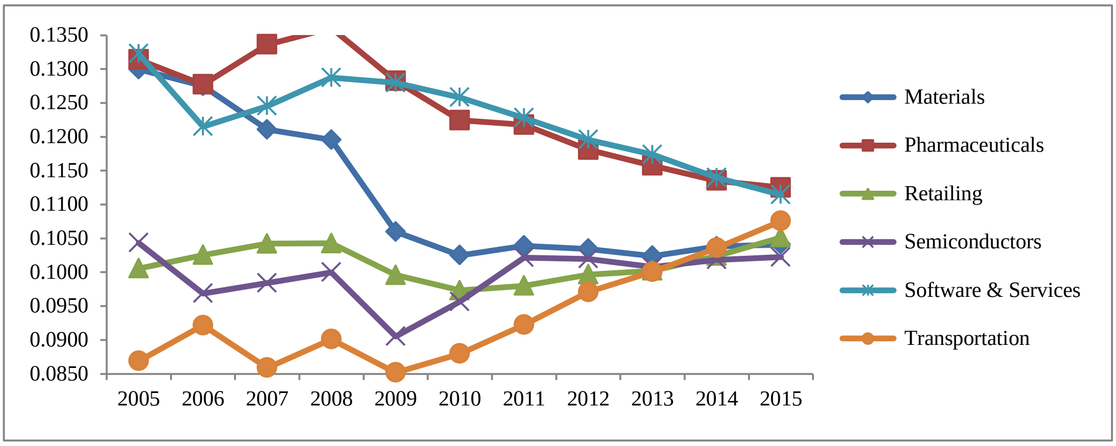
The prediction of the general equilibrium model that shall be put to test against the data is the following one: in the long run the rates of profit across industries would tend towards equality and so also the industrial rates of growth. This is equivalent to saying that prices tend to gravitate towards their normal values and so do outputs. The question is, "does this actually happen?" Before doing so it is necessary to dwell on the question of what exactly is meant by the long run or long period.

The idea of short and long periods belongs to logical instead of historical time. Short periods are slices of time in which (a) the industries cannot at their own will control all factors of production and (b) the effects of outside disturbances loom large in their effects on the decisions and ability to produce and sell. Long intervals of time which are themselves made up of short periods in each of which some limitations or disturbances loom large are not long periods. Instead, long periods are those in which the limitations and disturbances have a muted influence. Long period should therefore be understood as being a long unit of time, a quarter instead of a month, a year instead of a quarter, a decade instead of a year, and so on, over which the behaviour of variables is observed. A long period therefore does not mean averaging over a succession of short periods. As an example consider a situation in which a variable rises from a level of 10 to a level of 25 in two years. It might have done so by following two alternative paths such as 10 → 24 → 25 or 10 → 5 → 25 having annual growth rates of 140 per cent and 4.16 per cent in the two years for the first path and -50 per cent and 400 per cent for the second path. Their annual averages are 72.05 per cent and 175 per cent respectively. Neither can be considered the long period average. However, the compounded annualized growth rate of 58.11 per cent over the two-year time unit would better represent the long period average. The effect of the abnormal boom in the first year and a sharp recession in the next on the first path and the sharp recession in the first year and an abnormal boom in the next on the second path are both suppressed in the calculation of the long period average of 58.11 per cent.

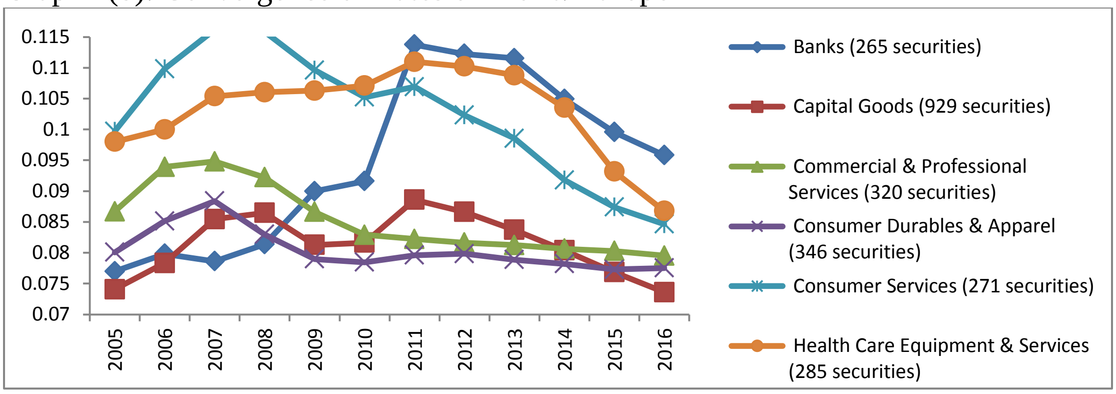
The data presented in this section are drawn from the Bloomberg database. Data on corporate profits, sales, assets and net worth have been grouped together into industries following the classification scheme of the data provider. A ten-year period from 2005 to 2016 has been covered. This decade has been one of great turbulence for all the economies whose data shall be presented, viz. USA, Europe, India and Japan. Graphs 1(a) to 1(d) show (a) show the rates of profit on net worth for some of the major industry groups in the four economic systems with respect to the length of the run. The long run rates of profit are $\sum(PAT)_t/\sum(NW)_t$ which means that they are obtained by cumulating the annual figures. Some industries that exhibit atypical behaviour (see Table 4 below) have not been shown in graphs 1. However, graphs 2 and 6 depict the behaviour of all industries whether typical or atypical. Graph 2(a) to 2(d) show the variance of individual companies' rates of profit from the long run mean rate of profit for all the industries in the economy across the length of the run. In reading the graphs it should be kept in mind that it is the length of the time unit of measurement that is being successively varied even though the numbers themselves are annualized. The convergence of the rates of profit and the rates of growth towards their respective means (i.e. gravitation to their normal values) is rendered *visible* when they are observed over longer intervals of time.

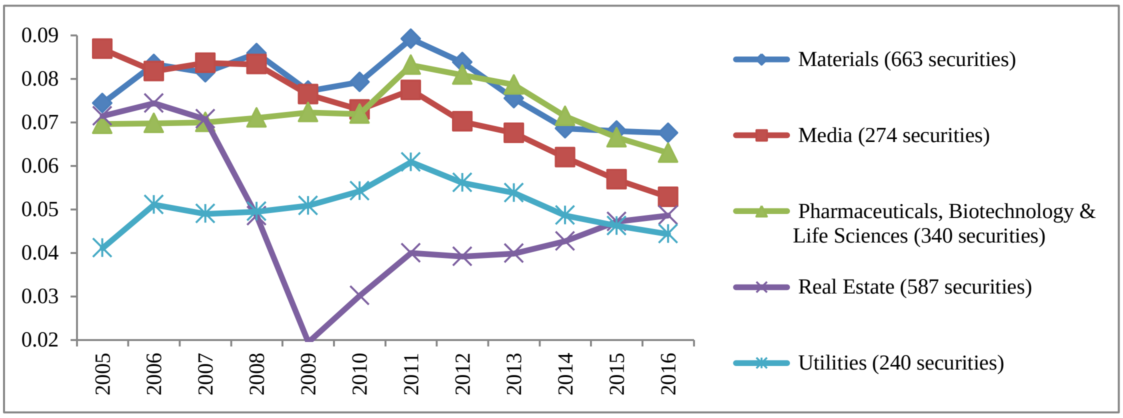
Graph 1(a): Convergence of Rates of Profit: USA



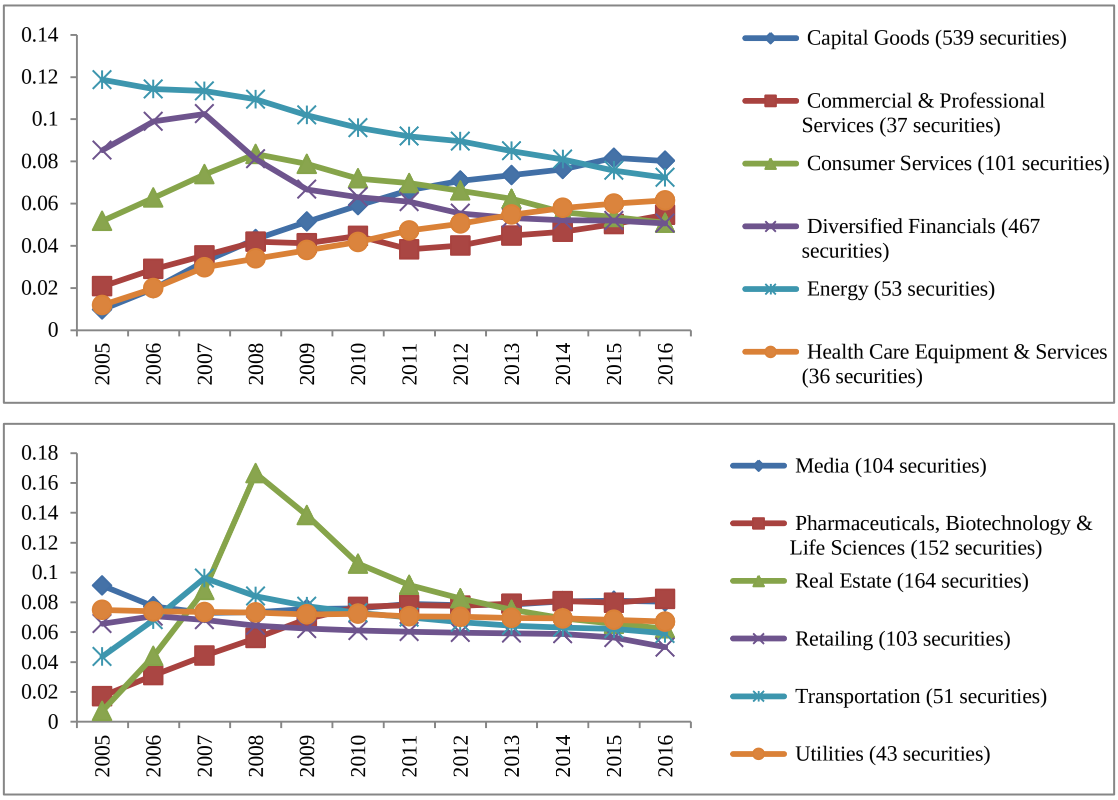


Graph 1(b): Convergence of Rates of Profit: Europe

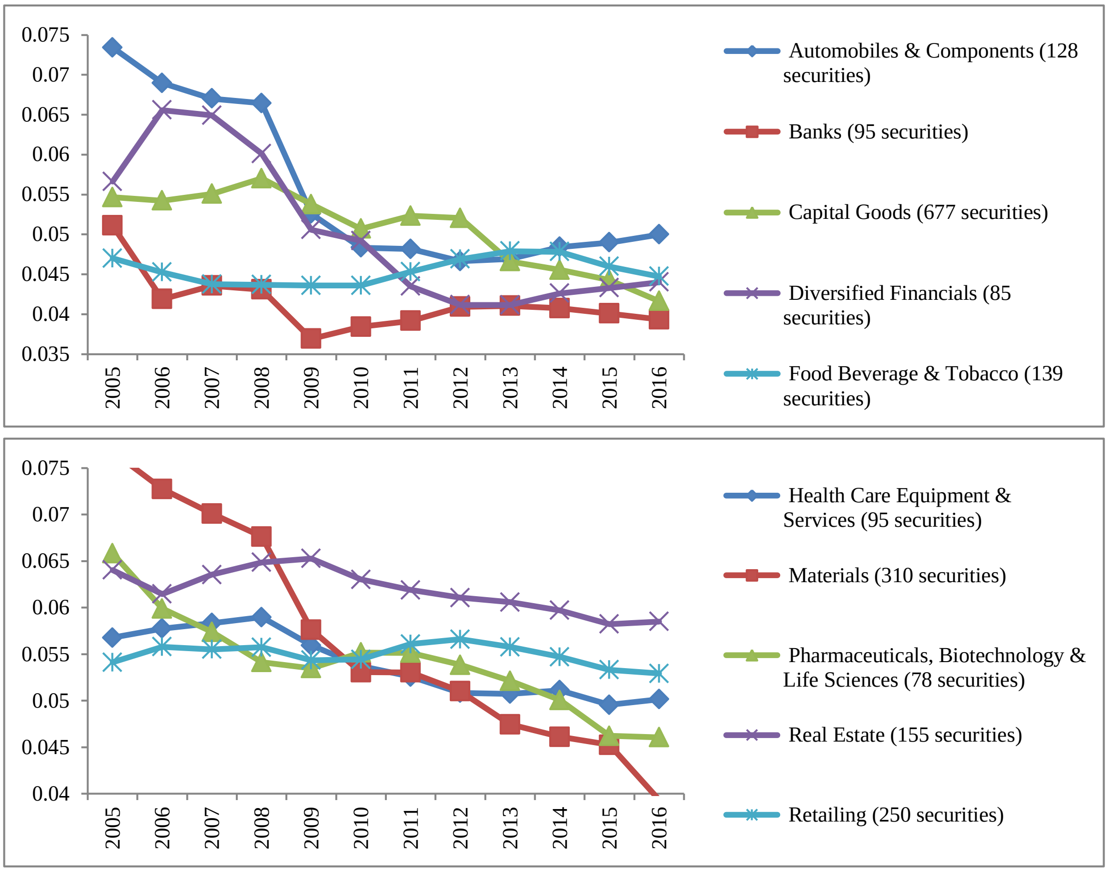




Graph 1(c): Convergence of Rates of Profit: India



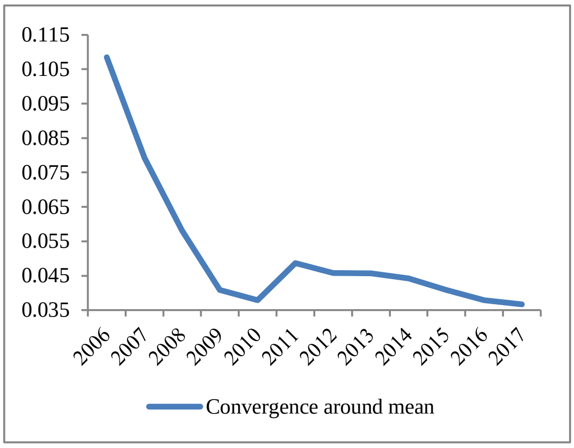
Graph 1(d): Convergence of Rates of Profit: Japan



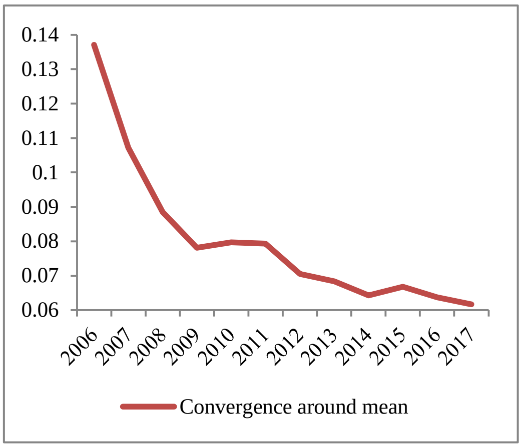
Graphs 3(a) to 3(d) present the frequency distributions of the rates of profit with respect to the length of the run ranging from 10 quarters to 40 quarters i.e. the entire decade. The rates of profit are successively cumulated net profit divided by the cumulated net worth's for each company. In order to obtain good plots for the frequency distribution it is necessary to have large numbers of observations in each interval. Therefore the grouping of companies into industries has been avoided in these graphs. Below each graph are the parameters of the distribution. Kurtosis has been measured as the height of the distribution divided by the base i.e. modal probability density divided by the standard deviation. The steady declines in standard deviation over time and the increases in kurtosis are visible. This shows the workings of the competitive process towards equalization of the rates of profit across companies and therefore industries as well. The declining mean and modal rate of profit reflects the recessionary trend in all four economies.

Graph 2: Convergence of Rates of Profit

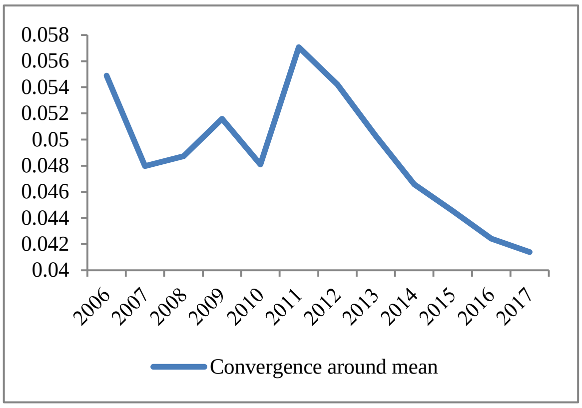
2(a) USA



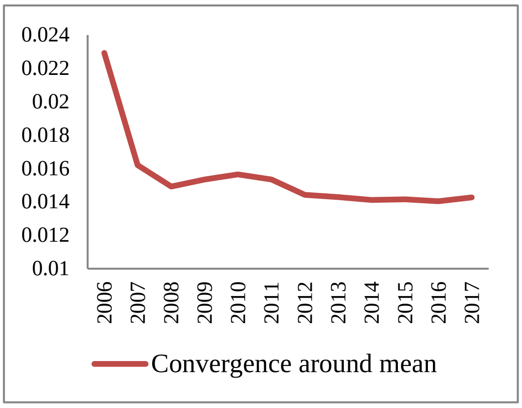
2(b) Europe



2(c) India



2(d) Japan



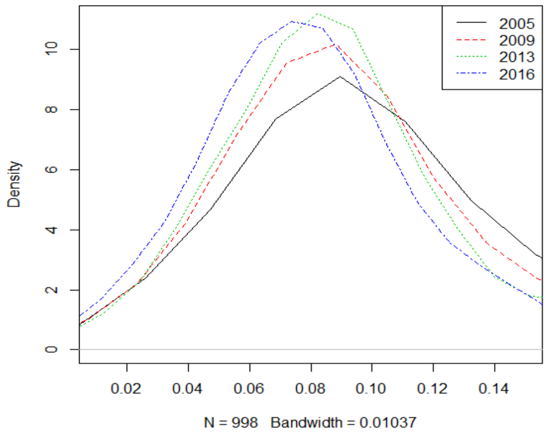
Graphs 3(a) to 3(d) depict the corresponding frequency distribution plots of the rate of growth of cumulative net sales. This is defined as

$$g = \exp\left(\frac{1}{t}\log\frac{S_t}{S_o}\right) - 1$$

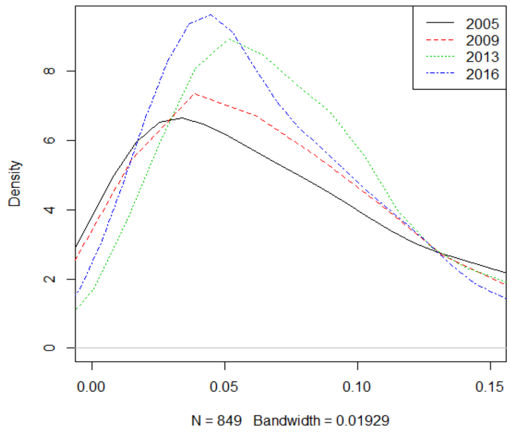
where $S_t = \sum_{j=o}^{t} S_j \qquad t = (1 \ldots 10)$

Graph 3: Frequency Distribution of Rates of Profit

3(a) – USA



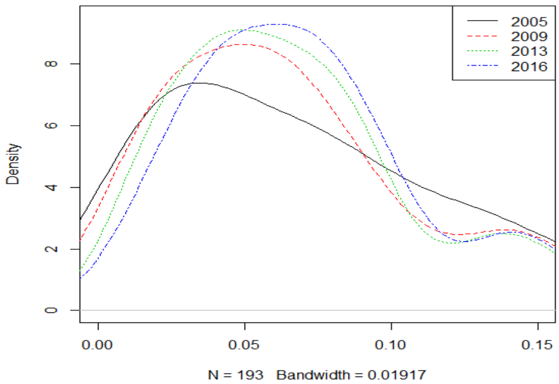
3(b) - Europe



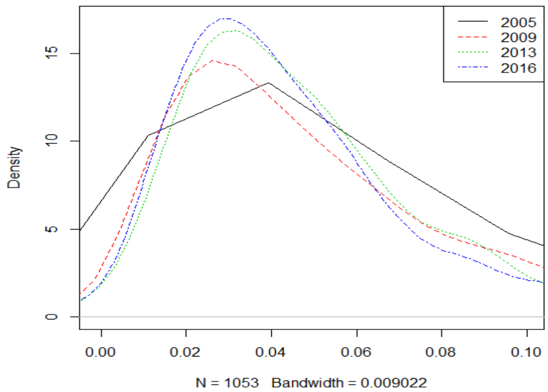
	2006	2010	2013	2016
Mean	0.1254	0.1231	0.1072	0.1003
std.dev	0.314112	0.382277	0.222544	0.193079
Kurtosis	28.94326	27.35477	50.29749	56.69409

	2006	2010	2013	2016
Mean	0.1177	0.08335	0.08426	0.0785
std.dev	0.210672	0.50082	0.21163	0.145768
Kurtosis	31.56138	13.31901	42.23014	66.15065

3(c) – India



3(d) - Japan

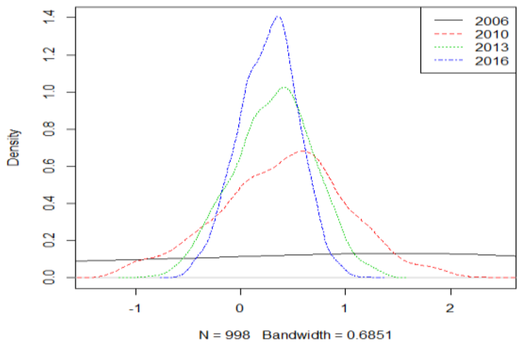


	2006	2010	2013	2016
Mean	0.0691	0.07049	0.07427	0.07759
std.dev	0.089067	0.081026	0.079562	0.071945
Kurtosis	82.94668	103.3894	114.39912	129.1791

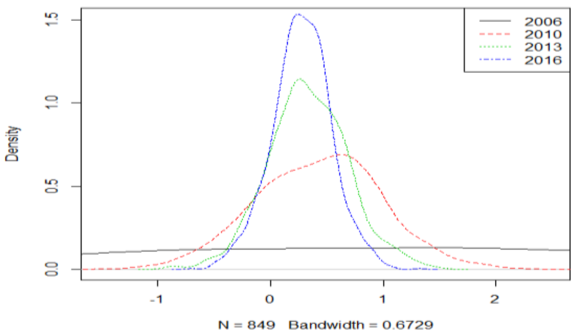
	2006	2010	2013	2016
Mean	0.05285	0.05528	0.04723	0.04646
std.dev	0.427168	0.101941	0.057682	0.049928
Kurtosis	31.25226	132.7378	282.23706	340.01124

Graph 4: Frequency Distribution of Rates of Growth of Net Sales

4(a) – USA



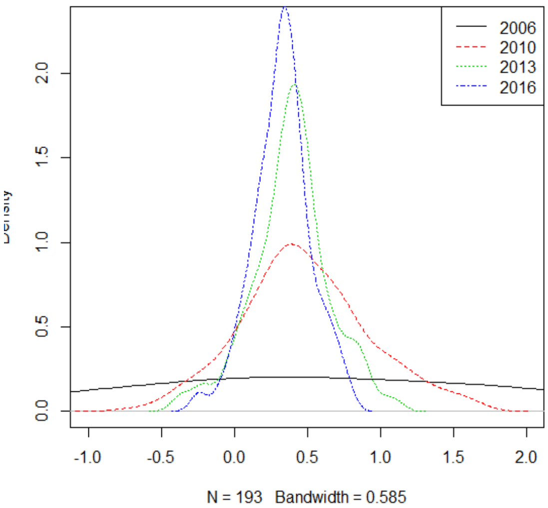
4(b) - Europe



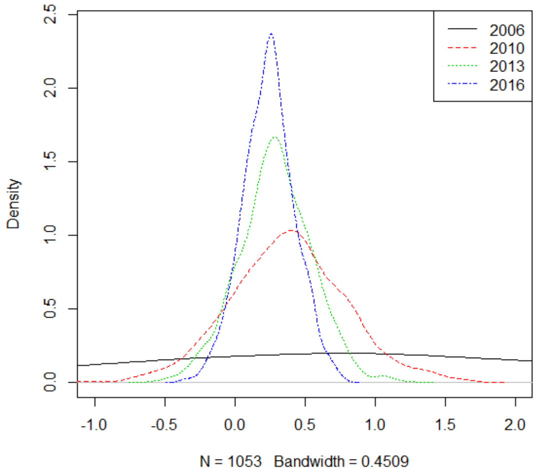
	2006	2010	2013	2016
Mean	0.7730	0.4553	0.3145	0.2617
std.dev	3.0291	0.7534	0.3837	0.2802
Kurtosis	0.0435	0.68881	2.67182	5.02005

	2006	2010	2013	2016
Mean	0.7986	0.4795	0.3325	0.2765
std.dev	2.9108	0.7341	0.3626	0.2673
Kurtosis	0.04535	0.7374	3.1567	5.7383

4(c) – India



4(d) - Japan

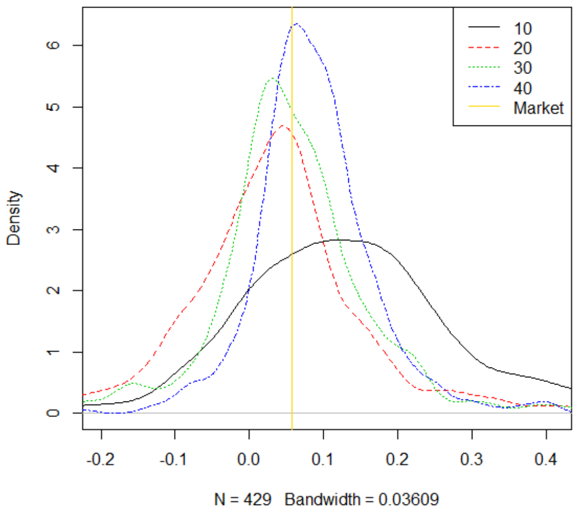


	2006	2010	2013	2016
Mean	0.8192	0.5393	0.3864	0.3270
std.dev	2.2372	0.5397	0.2668	0.1950
Kurtosis	0.0909	1.7270	7.2506	12.2765

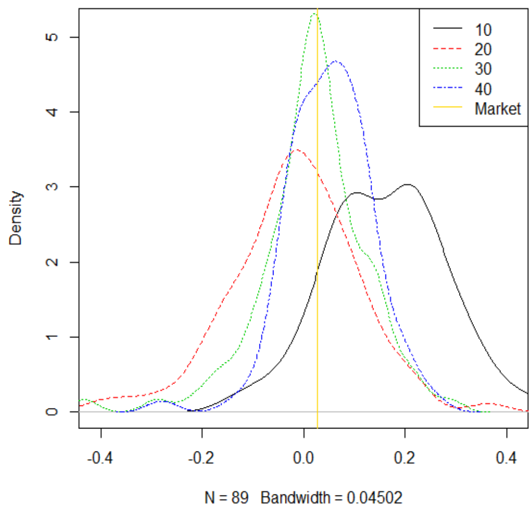
	2006	2010	2013	2016
Mean	0.7319	0.4314	0.2905	0.2419
std.dev	2.090	0.529	0.264	0.184
Kurtosis	0.09601	1.4965	6.3197	12.8316

Graph 5: Frequency Distribution of Stock Returns

5(a) USA



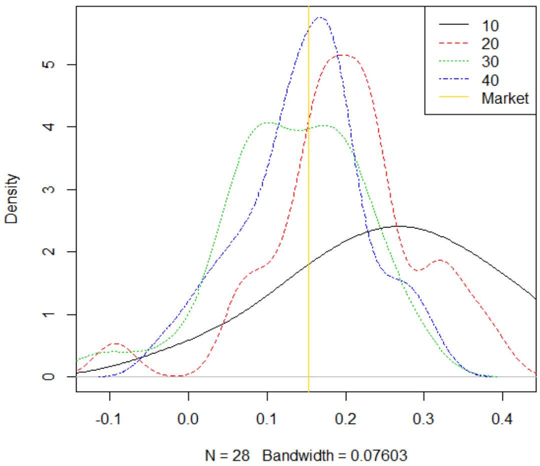
5(b) Europe



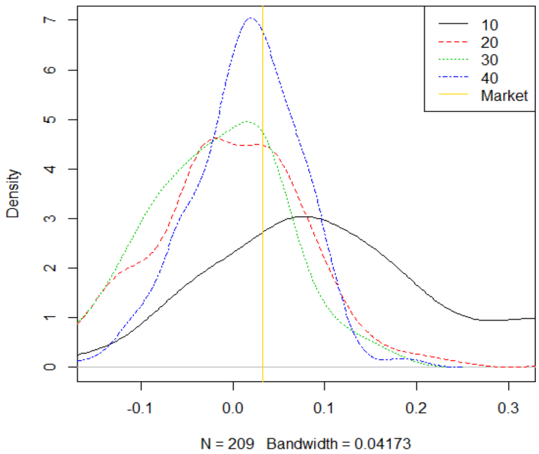
	2006	2010	2013	2016
Mean	0.1389	0.0280	0.0556	0.0871
std.dev	0.1588	0.1278	0.1029	0.0811
Kurtosis	17.7974	36.6715	53.0696	78.2911

	2006	2010	2013	2016
Mean	0.1642	0.0232	0.0182	0.05125
std.dev	0.1227	0.1384	0.1081	0.08225
Kurtosis	24.7375	25.2377	49.1671	56.87458

5(c) India



5(d) Japan



	2006	2010	2013	2016
Mean	0.3002	0.1975	0.1323	0.1468
std.dev	0.1843	0.0986	0.0872	0.0786
Kurtosis	13.0785	52.1887	46.6353	73.2400

	2006	2010	2013	2016
Mean	0.1170	0.0077	0.0243	0.0136
std.dev	0.1617	0.0942	0.0827	0.0615
Kurtosis	18.8137	49.0079	59.8010	114.3909

The same pattern is observed. As the length of the run increases the standard deviation declines and the distributions become more leptokurtic, i.e. the growth-rates of individual companies and industries huddle closer to the means. If profits and their growth rates tend to converge over time it is to be expected that stock markets must respond in the same manner. The probability distributions of stock returns with respect to longer holding periods in the four economies have been depicted in Graphs 5(a) to 5(d). All the probability distributions shown in graphs 3, 4 and 5 belong to the class of logistic distributions.

Attention may be called to some empirical regularities in the data. Table 3 shows the behaviour of the cross sectional standard deviations (across companies) of profitabilities and growth rates.

Table 3:Cross-Sectional Standard Deviations

Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
USA												
Rate of Profit	0.314	0.406	0.495	0.382	0.309	0.273	0.246	0.223	0.209	0.203	0.193	0.185
Growth Rate of Net Sales		2177.390	19.120	3.205	1.629	1.165	0.837	0.680	0.568	0.490	0.437	0.378
Holding Period Return		0.316	0.178	0.174	0.175	0.135	0.122	0.113	0.099	0.088	0.082	0.080
EUROPE												
Rate of Profit	0.211	0.341	0.235	0.501	0.388	0.312	0.250	0.212	0.181	0.162	0.146	0.134
Growth Rate of Net Sales		1425.329	13.994	3.970	1.840	1.113	0.874	0.656	0.569	0.484	0.419	0.373
Holding Period Return		0.241	0.146	0.143	0.194	0.145	0.131	0.122	0.102	0.089	0.082	0.082
INDIA												
Rate of Profit	0.089	0.083	0.082	0.081	0.073	0.073	0.085	0.080	0.076	0.074	0.072	0.070
Growth Rate of Net Sales		171.490	7.713	2.252	1.294	0.848	0.670	0.520	0.444	0.357	0.316	0.272
Holding Period Return		0.201	0.188	0.210	0.176	0.108	0.096	0.088	0.087	0.079	0.076	0.080
JAPAN												
Rate of Profit	0.427	0.199	0.132	0.102	0.084	0.073	0.063	0.058	0.054	0.052	0.050	0.061
Growth Rate of Net Sales		75.297	4.805	1.675	1.029	0.706	0.539	0.440	0.377	0.319	0.272	0.241
Holding Period Return		0.296	0.175	0.147	0.120	0.104	0.085	0.081	0.080	0.066	0.063	0.058

Firstly, the across-companies standard deviations of the rate of profit, the growth-rate and stock return decline over time as the competitive process drives them towards uniformity. Secondly, the rate of decline of the standard deviations of the profitability and growth tends to slow down as the length of the run increases which implies that complete uniformity which is the theoretical ideal may never be actually reached. Cross company standard deviations of the rates of profit are lower than that of the rates of growth but the rate of decline in the standard deviations of the growth rates is much faster than the rate of decline in the standard deviations of the rates of profit. [Visually graph 4 looks more appealing than graph 3]. Thirdly, the standard deviations of the stock returns also mirror the behaviour of those found for profitabilities and growth rates in all the four stock markets. In fact the growth rates of stock earnings and the rates of stock returns tend to approach one another as the length of the holding period increases. Fourthly the industrial rates of profits and growth also converge closer to one another as the length of the run increases but in view of equation (7) not towards complete equality. To see this the following distance measure has been employed,

$$[\sum(r_{it} - g_{it})^2]^{1/2}$$

where i denotes the industry group and reported in Table 4.

Table 4: Convergence of Industrial Profitablities towards Growth Rates

	USA	Europe	India	Japan
05-06	11.96	13.95	12.61	6.86
05-10	2.25	2.73	3.88	2.11
05-13	1.58	2.02	2.55	1.53
05-16	1.13	1.14	1.66	1.11

Fourthly, the correlations between the growth rates of assets, sales and net worth across companies over the ten-year period shown in Table 5 are fairly high and significant.

Table 5:

Correlation	USA	Europe	India	Japan
Assets & Sales	0.86	0.80	0.89	0.58
Assets & Net Worth	0.96	0.92	0.89	0.92

Fifthly, although the theoretical model is based on the assumption of complete absence of changes in technology and consumption behaviour, the data pertaining to four large economic systems over a ten-year period would surely contain several episodes of such changes. Therefore, some systematic departures from a smooth and harmonious convergence are bound to remain.

Finally it becomes possible to identify industries (and companies) that underperform and outperform the competitive middle even in the long run. Table 6 gives a summary of underperforming and outperforming sectors in the different countries. Received theories of market structure would have us believe that perfect competition defines one end of a spectrum that stretches on with successively increasing degrees of imperfection until the other end of an exclusive monopoly i.e. they lead us to expect that distributions shown in Graph 1 would have no sections to the left of the mode/mean but only have the section to the right stretching all the way to monopoly. In contrast, the picture that emerges from Graphs 3 to 5 and Tables 4 and 6 is that competition lies in the middle and encompasses the largest numbers of industries and the extremes are occupied by "imperfections" of a type that produce long-period gaps in performance with some industries being laggards and others being outperformers. In so doing it broadens our perspective of imperfect competition.

Table 6: Under- and Out- Performers

Underperformers	Outperformers
USA	
Energy (6.59%)	Insurance (23.90%)
Real Estate (8.12%)	Household Product (16.11%)
Telecommunication Services (3.57%)	Technology Hardware (15.22%)
Europe	
Food & Beverages (4.50%)	Retailing (14.95%)
Media (5.28%)	Software & Services (14.66%)
Real Estate (4.86%)	
Semiconductors (1.59%)	
Technology Hardware (2.27%)	
Transportation (2.11%)	
Utilities (4.43%)	
India	
Commercial & Professional Services (5.47%)	Auto & Components (11.78%)
Consumer Services (5.09%)	Consumer Durables (12.20%)
Diversified Financials (5.07%)	Food and Beverages (13.00%)
Retailing (4.99%)	Household Products (21.05%)
Technology Hardware (5.74%)	
Japan	
Commercial and Professional Services (2.61%)	Food and Staples (6.38%)
Consumer Durables (2.09%)	Household Products (6.57%)
Semiconductors (2.16%)	Software & Services (6.40%)
Transportation (2.83%)	Telecom Services (6.74%)
Media (3.2%)	Technology Hardware (6.18%)

Note: Figure in brackets are decadal profit rates earned by the industries to be compared to their decadal means.

VII Concluding Remarks

The incompleteness/indeterminacy of the Sraffa system is a result of keeping open one of the loops in the circular flow of income viz. the loop that describes the manner in which capitalists and workers utilise their incomes by way of consumption and saving. This paper has closed the loop by specifying consumption demand by means of a linear expenditure system and allowing saving to be invested in capital goods in accordance with the dictates of the technologies employed by the industries. This leads to a complete system of general equilibrium to determine all the unknowns of economic interest. Changes in technology and consumption patterns can easily be incorporated provided they can be described in terms of the coefficients of the model, i.e. the input output and labour coefficients, the fixed and working capital coefficients and the parameters of the linear expenditure system. Recall from the first quotation of Sraffa's unpublished notes that has been cited in the introductory part of this paper that Sraffa himself wondered whether, in the endeavour to obtain a complete system, "there is room enough for the marginal system." I think this paper can give a firm answer: No! No reference, directly or tacitly, has been made to anything even remotely resembling marginal productivity, cost or revenue. None of the adjustments towards the establishment of equilibrium or thereafter are of a ‘marginal’ nature; on the contrary equilibrium is only established by a process of simultaneous structural adjustment in which the input and output levels of all the industries are realigned to one another in their totalities.

The predictions of a theoretical model that has abstracted from several features of the real world are not *prime facie* expected to be empirically validated except in broad and roughly indicative ways. The paper has presented evidence from various parts of the world on one of central predictions of the model viz. the tendency for growth rates and rates of profit of the industries to converge towards their long run equilibrium values. This prediction is largely vindicated by the data presented in Tables 3 & 4 and Graphs 1, 2, and 3. Cross-sectional standard deviations of growth and profit rates decline with increases in the length of the run and a systematic increase in the leptokurtosis of the probability distributions is observed. However, the consistency manifested by the empirical evidence suggests that the predominant and persistent forces that govern the relationships between economic variables have been adequately captured by the model. The movement towards equality of the long run rates of growth in net profits, net worth, net sales and total assets serves as an additional vindication of the equilibrating process. That the same processes should be operative around the world and that too in the aftermath of a great crisis is an assurance of the general validity of the predictions of the model. In this context it is well to point out that the convergence in the rates of profit, rates of growth and stock returns is not a peculiarity of the chosen period of study viz. 2005-2016. This period has been chosen only because it is recent. Any long period would throw up essentially the same results.

In closing this paper it should suffice to make a couple of remarks of a methodological nature. The first remark concerns a crucial methodological aspect that starkly contrasts the Sraffian general equilibrium model from the dominant neoclassical and neo-Walrasian intertemporal equilibrium models. It has been noted [e.g., Mongiovi 1999, Petri 1999, Eatwell and Milgate 1999] that if capital is treated as consisting of initial endowments in physical terms then, except by a fluke, a solution with a uniform rate of profit is generally not possible. Neoclassical theory requires that the size of the capital stock be specified exogenously and in value terms in order to obtain a uniform rate of profit. This is true of the neo-Walrasian model as well. The Sraffian model does not require this – all value magnitudes are determined only after the equilibrium is ascertained. Indeed the sizes and composition of the capital stocks employed within individual industries and across the industries and the size and composition of the real national income are themselves determined in the process of equilibration, and that process is driven by the forces of competition operating in a setting of interdependent industries. The second remark concerns the computability of the general equilibrium. It is now well-known that the neo-Walrasian models in their full generality are essentially non-computable, i.e. there are no algorithms that will compute their fixed point solution in finite time. In contrast the Sraffian model has been shown to possess an algorithm that gives an explicit numerical solution.

Notes

1. References to Sraffa's unpublished papers follow the catalogue prepared by the archivist, Jonathan Smith. The papers are kept in the Trinity College Library, Cambridge.
2. Commodities having high own rates of growth being in excess supply, their prices fall relative to those having low own rates of growth which will be in excess demand. The profits realized by their producers will decline and so also the outputs produced while those of the latter set of commodities will rise.
3. Unfortunately the use of a dual system in which all goods serve both as means of production and consumption does not give a unique equilibrium solution – it gives an equilibrium solution for every rate of profit. The reason is that the growth-profit relation (7) becomes dependent on the dual equations (10). So if we put an exogenous rate of profit into the price system, then equation (7) solves for a growth rate but the more elaborate dual system solves for the same growth rate. This situation is avoided by having some goods purely performing the role of consumption goods. They allow an independent consideration in the form of A_{ic} to be introduced in the dual system. The result is that the growth rate obtained from the dual system (10) differs from the one implied by the tentative values of the rate of profit, so that the system does not cycle around its arbitrary initial value.

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