An Expert's Approach to Solving Physics Problems

A problem is a question to which you do not immediately see how to obtain the answer. If you do see how to obtain the answer – well, then the problem is not a problem for you, is it? An expert approaches problems differently than a novice. A novice may search for an equation to apply to the problem that matches the given variables, or the novice may try to solve a problem using the solution to a similar problem they have done before. There are many other novice problem solving techniques that may suffice for easy homework problems in an introductory class. But these approaches are not universal, and will not allow you to solve every problem you encounter. An expert is able to apply the physics concepts she knows to solve any problem, which may combine multiple separate physics concepts, even one that is entirely new and unique. She does this through several heuristics, or "rules of thumb", based on experience solving many problems. The purpose of this short essay is to impart some of those heuristics to you without requiring you to actually accumulate the years of experience that generated them.

When to use this Expert's Approach

If you have ever said to yourself, "I just don't know where to start," then THIS is how you start. The steps below are general enough to apply to any problem and will help organize your approach when you cannot immediately see how to obtain the answer to a problem.

If you have ever said to yourself, "I understand the problem conceptually, but I can't do the math," then THIS is how you approach the math. In fact, you may have diagnosed your difficulty backwards: you *can* do the math, but you have not understood the concepts well enough to set up the *correct* math. "Doing the math" is usually the easiest part of solving any problem: it almost always boils down to algebra and calculus, with which you likely have a strong ability already. *The truly difficult part of solving a problem is determining how to describe it mathematically.* How can you describe the relevant concepts using equations? After this step is done, combining the equations to determine the answer is usually straightforward algebra or calculus.

Steps in the Expert's Approach

The order of these steps is purposeful. The earlier the step, the more critical. For example, you cannot "plan a solution" when you do not have a "clear mental image of the problem."

1. **Focus on the Problem.** Establish a clear mental image of the problem.
   A. Visualize the situation and events by sketching a useful picture.
   B. Identify physics concepts and approaches that might be useful to reach a solution.
   C. In your own words, precisely state the question to be answered in terms you can calculate.
2. **Describe the Physics.** Refine and quantify your mental image of the problem.
   A. Draw any necessary diagrams with coordinate systems that are consistent with the approaches you have chosen.
   B. Name and define consistent and unique symbols for any quantities that are relevant to the situation. Consistency here will avoid problems later.
   C. Identify the target quantities that will provide the answer to the question.
3. **Plan a Solution.** Turn the concepts into math.
   A. Construct specific equations to quantify the physics concepts and constraints identified in your approach.

B. Outline a plan either leading backwards from the target quantities to quantities that are known, or leading from known quantities to the target quantities.
4. **Execute the Plan.** This is the easiest step – it's just the algebra/calculus/etc.
    A. Arrive at a formula for your target quantities by following the solution steps.
    B. Check the units of your final formula before putting in numbers.
    C. If quantities have numerical values, substitute them in your final equation to calculate a value for the target quantity.
5. **Evaluate the Answer.** Be skeptical. Ask yourself what a wrong answer would look like.
    A. Is the answer properly stated as an answer to the question you identified?
    B. Is the result reasonable?
    C. Is the answer complete?

What does a good solution look like?
Those grading exams are expert problem solvers, so they know the components that a good solution will have. A good solution is not a spray of equations across the page. A good solution not only obtains the answer to the problem, but importantly it also demonstrates that you understand the physics concepts being tested and can apply them to an unfamiliar situation to answer a non-trivial question.

A good solution progresses in an orderly and logical way from (1 & 2) a description of the concepts in the problem and definitions of relevant quantities, to (3) mathematical representations of the concepts (e.g., equations and formulae), to (4) manipulation of the equations to obtain the answer, and finally to (5) a clear statement of the answer so the grader does not have to hunt for it. The grader will be looking for all of these components to a good solution, so the clearer these steps are, the better for your grade. Therefore, neatness counts, if only indirectly.

An Example
An example problem, its solution, and annotations on the process of solving the problem.
The solutions to the problems from past exams will help you see what a good solution looks like. But seeing the solution alone may not illustrate the *general method* that could be used to solve other problems. Here is provided a problem from the fall 2016 Quantum Mechanics exam, and its solution. Alongside the solution are annotations related to the above Expert's Approach to problem solving.

Problem: Consider the spin degrees of freedom of the proton and electron in a hydrogen atom. They are coupled by the hyperfine interaction, whose Hamiltonian is

$$H_{\mathrm{hf}} = \frac{A}{\hbar^2} \vec{S} \cdot \vec{I}$$

where A has units of energy, and $\vec{S}$ and $\vec{I}$ are the spin operators for the electron and proton, respectively. The electron and proton total spin quantum numbers are $s = 1/2, I = 1/2$. The operator for the total spin angular momentum of the atom is $\vec{F} = \vec{S} + \vec{I}$. What are the possible quantum numbers associated with F^2 and the z-component F_z? Demonstrate that the basis states identified by those quantum numbers are the eigenstates of the hyperfine Hamiltonian. Determine which spin state is the ground state.

Solution

The electron and proton spin quantum numbers are

$$s = 1/2, m_s = \pm 1/2$$
$$\text{and}$$
$$I = 1/2, m_I = \pm 1/2$$

respectively. The total spin angular momentum quantum number is:

$$F = |s + I|, \ldots, |s - I| = 1, 0.$$

The total spin z-component quantum number depends on F:

$$M_F = -F, -F + 1, \ldots, F.$$

The basis states are $|F, M_F\rangle$.

We can demonstrate that the $|F, M_F\rangle$ states are eigenstates of H_{hf} by expressing the dot product in the Hamiltonian using the total angular momentum operator $\vec{F} = \vec{S} + \vec{I}$.

$$F^2 = \left(\vec{S} + \vec{I}\right)^2$$
$$= S^2 + I^2 + 2\vec{S} \cdot \vec{I}$$

Rearranging the above equation, we can obtain another expression for the Hamiltonian:

$$H_{\text{hf}} = \frac{A}{2\hbar^2}(F^2 - S^2 - I^2)$$

The effect of this Hamiltonian operating on one of the basis states $|F, M_F\rangle$ is:

$$H_{\text{hf}}|F, M_F\rangle = \frac{A}{2}[F(F + 1) - s(s + 1) - I(I + 1)]|F, M_F\rangle$$

Notes on the method

A picture might not be that helpful here, but naming the concepts involved is. It is not done explicitly in this solution, but a partial list of relevant concepts might be:

- Spin quantum numbers (i.e. space quantization)
- Addition of angular momentum
- Eigenstates of angular momentum operators
- The "coupled basis" of angular momentum states
- The hyperfine interaction

In answering the first part of the problem, we don't particularly need all the steps of the Approach. If we understand how quantum numbers behave in space quantization and addition of angular momentum, then we can almost just write down the answers. Here, the concept of addition of angular momentum is embodied in the equations $F = |s + I|, \ldots, |s - I|$ and $M_F = -F, -F + 1, \ldots, F$. Then values defined earlier are substituted in, and the answers are obtained.

We start again by identifying the relevant concepts:

- Eigenstates and eigenvalues, in general; i.e., the idea of the eigenvalue equation
- Addition of angular momentum
- Eigenstates of angular momentum operators
- Simultaneous eigenstates and commuting operators

We "plan the solution" by constructing the mathematical representations of these concepts. First, the goal is to show that $|F, M_F\rangle$ satisfies an eigenvalue equation with H_{hf}. We know from the angular momentum operators that $|F, M_F\rangle$ are eigenstates of the operators F^2, S^2, and I^2. And we know that commuting operators have simultaneous eigenstates. So, if we can show that H_{hf} commutes with F^2, S^2, and I^2, then $|F, M_F\rangle$ will satisfy the energy eigenvalue equation.

Now we "execute the plan." To the left, the physics concepts are not explicitly stated, but the mathematical manipulations being done are accomplishing the very goals outlined in the conceptual plan above.

All the basis states have the same values for the quantum numbers s and I, so the above equation simplifies to:

$$H_{\text{hf}}|F, M_F\rangle = \frac{A}{2}\left[F(F+1) - \frac{3}{2}\right]|F, M_F\rangle$$

This equation is clear an eigenvalue equation, showing that the basis states $|F, M_F\rangle$ are eigenstates of H_{hf}.

To determine which spin state is the ground state, we evaluate the eigenvalues (a.k.a. eigenenergies) for the possible values of $F = 0, 1$.

$|F, M_F\rangle$	Energy
$|1, 1\rangle$	$A/4$
$|1, 0\rangle$	$A/4$
$|1, -1\rangle$	$A/4$
$|0, 0\rangle$	$-3A/4$

We can see that the spin singlet state $|0,0\rangle$ has the lowest energy and is thus the ground state, while the spin triplet states $|1, M_F\rangle$ are all degenerate (at zero magnetic field).

The remaining step is to substitute in numerical values previously defined. Both the quantum numbers $s = 1/2$ and $I = 1/2$ given in the problem, and the derived quantum numbers $F = 0, 1$ and $M_F = -F, ..., F$ are necessary. The result allows us to answer the final part of the problem regarding the ground state.

Concluding comments

As you can see by following both the solution above and the thought process underlying it, the actual math involved is nearly trivial – it is only algebra. The difficult part is always determining how to "mathematize" the physics concepts. Thus, the Expert's Approach prioritizes getting a clear mental image of the problem and identifying the relevant concepts. If you go right for the math, you will probably do a lot of work with no result. If you try to reach for the answer before preparing the foundation of the right concepts, it will remain beyond your grasp. Don't search for the answer, develop understanding first. "Walk around the problem," view it from all sides, and you will be able to reveal the answer.