Definitions:

- **Quadratic function**: is a function that can be written in the form

$$f(x) = ax^2 + bx + c$$

where a, b, and c are real numbers and $a \neq 0$.

- **Parabola**: The graph of a squaring function is called a parabola. It is a U-shaped graph.
- **Vertex of a parabola**: The point on the parabola where the graph changes direction. It is the lowest point if $a > 0$, and it is the highest point if $a < 0$.

Important Properties:

- **Standard form of a quadratic function**: A quadratic function $f(x) = ax^2 + bx + c$ can be expressed in the standard form

$$\boxed{f(x) = a(x-h)^2 + k}$$

by completing the square.

- Once in standard form, the vertex is given by (h, k).
- The parabola opens up if $a > 0$ and opens down if $a < 0$.

Steps to put quadratic function in standard form:

1. Make sure coefficient on x^2 is 1. If the leading term is ax^2, where $a \neq 1$, then factor a out of each x term.
2. Next, take one-half the coefficient of x and square it. In other words,

$$\left(\frac{1}{2} \cdot \text{coefficient of } x\right)^2.$$

3. Add the result of step 2 inside the parenthesis.
4. In order not to change the problem you must subtract ($a \cdot$ result of step 2) outside the parenthesis.
5. Factor the polynomial in parenthesis as a perfect square and simplify any constants.

Common Mistakes to Avoid:

- When performing Step 4 above, do NOT forget to multiply the result of step 2 by the a that was factored out.

PROBLEMS

Express the quadratic function in standard form. Identify vertex.

1. $f(x) = x^2 + 4x - 5$

$$f(x) = x^2 + 4x - 5$$

$$\left(\frac{1}{2} \cdot 4\right)^2 = 2^2 = 4$$

$$f(x) = (x^2 + 4x + 4) - 5 - 4$$
$$= (x+2)^2 - 9$$

$$\boxed{f(x) = (x+2)^2 - 9}$$

$$\boxed{\text{Vertex} = (-2, -9)}$$

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2. $f(x) = x^2 - 6x + 1$

$$f(x) = x^2 - 6x + 1$$

$$\left(\frac{1}{2} \cdot -6\right)^2 = (-3)^2 = 9$$

$$f(x) = (x^2 - 6x + 9) + 1 - 9$$
$$= (x-3)^2 - 8$$

$$\boxed{f(x) = (x-3)^2 - 8}$$

$$\boxed{\text{Vertex} = (3, -8)}$$

3. $f(x) = -x^2 + 10x - 2$

Before we complete the square, we need to factor -1 from each x term.

$$f(x) = -x^2 + 10x - 2$$
$$= -(x^2 - 10x \quad) - 2$$

$$\left(\frac{1}{2} \cdot -10\right)^2 = (-5)^2 = 25$$

$$f(x) = -(x^2 - 10x + 25) - 2 - (-1 \cdot 25)$$
$$= -(x^2 - 10x + 25) - 2 - (-25)$$
$$= -(x^2 - 10x + 25) - 2 + 25$$
$$= -(x-5)^2 + 23$$

$$\boxed{f(x) = -(x-5)^2 + 23}$$

$$\boxed{\text{Vertex} = (5, 23)}$$

4. $f(x) = 2x^2 + 8x - 1$

Before we can complete the square, we need to factor a 2 from each x term.

$$f(x) = 2x^2 + 8x - 1$$
$$= 2(x^2 + 4x \quad) - 1$$
$$\left(\frac{1}{2} \cdot 4\right)^2 = 2^2 = 4$$
$$f(x) = 2(x^2 + 4x + 4) - 1 - (2 \cdot 4)$$
$$= 2(x^2 + 4x + 4) - 1 - 8$$
$$= 2(x + 2)^2 - 9$$

$$\boxed{f(x) = 2(x + 2)^2 - 9}$$

$$\boxed{\text{Vertex} = (-2, -9)}$$

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5. $f(x) = 3x^2 - 12x - 10$

Before we complete the square, we must factor 3 out of each x term.

$$f(x) = 3x^2 - 12x - 10$$
$$= 3(x^2 - 4x \quad) - 10$$
$$\left(\frac{1}{2} \cdot -4\right)^2 = (-2)^2 = 4$$
$$f(x) = 3(x^2 - 4x + 4) - 10 - (3 \cdot 4)$$
$$= 3(x^2 - 4x + 4) - 10 - 12$$
$$= 3(x - 2)^2 - 22$$

$$\boxed{f(x) = 3(x - 2)^2 - 22}$$

$$\boxed{\text{Vertex} = (2, -22)}$$

6. $f(x) = -4x^2 - 8x + 3$

Before we can complete the square, we need to factor -4 from each x term.

$$f(x) = -4x^2 - 8x + 3$$
$$= -4(x^2 + 2x \quad) + 3$$
$$\left(\frac{1}{2} \cdot 2\right)^2 = 1^2 = 1$$
$$f(x) = -4(x^2 + 2x + 1) + 3 - (-4 \cdot 1)$$
$$= -4(x^2 + 2x + 1) + 3 - (-4)$$
$$= -4(x^2 + 2x + 1) + 3 + 4$$
$$= -4(x + 1)^2 + 7$$

$$\boxed{f(x) = -4(x + 1)^2 + 7}$$

$$\boxed{\text{Vertex} = (-1, 7)}$$

7. $f(x) = 5x^2 + 5x + 8$

Before we can complete the square we need to factor 5 from each x term.

$$\begin{aligned} f(x) &= 5x^2 + 5x + 8 \\ &= 5(x^2 + x \quad) + 8 \end{aligned}$$

$$\left(\frac{1}{2} \cdot 1\right) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\begin{aligned} f(x) &= 5\left(x^2 + x + \frac{1}{4}\right) + 8 - \left(5 \cdot \frac{1}{4}\right) \\ &= 5\left(x^2 + x + \frac{1}{4}\right) + 8 - \frac{5}{4} \\ &= 5\left(x + \frac{1}{2}\right)^2 + \frac{27}{4} \end{aligned}$$

$$\boxed{f(x) = 5\left(x + \frac{1}{2}\right)^2 + \frac{27}{4}}$$

$$\boxed{\text{Vertex} = \left(-\frac{1}{2}, \frac{27}{4}\right)}$$