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How to Evaluate GDP-Linked Warrants: Price and Repayment Capacity

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How to Evaluate GDP-Linked Warrants: Price and Repayment Capacity

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Abstract

Following a brief review of the recent history of GDP-linked instruments, this paper proposes a set of tools to examine the quantitative properties of GDP-linked warrants. It argues that trigger conditions should be clearly identifiable and payment amounts easily calculable. Based on a design that includes these features and historical data for the main EMBI countries, the paper provides an assessment of the issuer's capacity to service GDP-linked warrants, comparing payments with tax revenues stemming from contemporaneous growth. The price of the GDP-linked warrants are then estimated from the point of view of both domestic and foreign investors.

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I. INTRODUCTION

Recent research in the sovereign debt literature has focused on identifying mechanisms that could improve debt sustainability for sovereigns in times of economic downturns. In this context, debt instruments that link capacity to service debt to economic performance, as a way to share the economic growth risks between an issuer and its creditors have received considerable attention. For instance, Froot, Scharfstein, and Stein (1989) maintained that linking debt payments to the issuer's GDP performance would cushion the impact of negative growth shocks on the ability to service debt. Shiller (1993) argued that the use of GDP-linked debt would allow a sovereign to buy insurance against growth uncertainty, and, thus, help smooth the revenue loss from adverse economic performance. Obstfeld and Peri (1989) and Borensztein and Mauro (2002) suggested that government would be able to reduce their idiosyncratic GDP risks by issuing GDP-linked warrants (GLWs), a derivative security, the payments of which are linked to a sovereign's GDP performance.

While the academic literature has focused on the theoretical discussion of the pros and cons of GLWs until recently, this paper develops a pricing model for GLWs and evaluates numerically the risk profile of a proposed specific design of GLWs from the point of view of both investors and sovereign issuers. In particular, the paper focuses on cash-flow considerations to develop both a pricing mechanism for GLWs and a framework for sustainable payments on these instruments. More specifically, payments on GLWs are designed to be proportional to the tax revenues available to make these payments in each period. By requiring that payments related to GLWs are proportional to these fiscal cash resources, these instruments are assured to be always fully financed. This approach differs from a more general evaluation of sovereign issuers' overall solvency in terms of the relative size of their debt with respect to GDP as discussed by Borensztein and Mauro (2002), and Chamon and Mauro (2005). This paper therefore provides sovereign issuers that are considering issuing GLWs a framework to analyze cash requirements for debt-service purposes, which could be part of a broader fiscal financing strategy.

This paper focuses on two aspects of the GLWs, namely the projected flow of debt payments and the price of the securities, by using Monte Carlo methods calibrated to different sets of parameter values and data for the average GDP growth experienced by a group of emerging market countries. In a first stage, the paper estimates the flow of future debt payments, and compares these payments with the expected revenues available to service such payments. This is critical since, in any design of GLWs, these payments need to be matched by the contemporaneous tax revenues, as GLW payments are contingent on the issuer's GDP growth. In a second stage, the paper estimates the price of such warrants on the basis of the present value of the expected future cash flows. The absence of a systematic rule to determine the parameter values in the calibration of the model implies that there will be significant discrepancies in the estimated price across individual investors.

The structure of this paper is as follows. Section II reviews the history and characteristics of GLWs. Section III analytically evaluates GLWs, assuming a specific design, through the estimation of the price of GLWs and a comparison of the debt payments with the resources available to service the debts. Section IV concludes.

II. History and Characteristics

A. Origin

Interest in GLWs emerged both in the context of the discussion about the possibility of sharing risk between the issuer and the investor, and, more specifically, in the context of the debt restructurings following the debt crisis in the 1980s. The sovereign debt literature portrayed GLWs' benefits in the context of international risk sharing. Individual investors could diversify the growth risk of a country by trading securities that link payments to this country's growth (Shiller, 1993; and Athanasoulis and Shiller, 2001), as financial markets do not offer opportunities to hedge the risks associated with conventionally nontraded components, such as noncorporate business income and wages, which typically represent a large share of GDP. The market for GLWs therefore is expected to bridge this gap by allowing investors to take positions on countries' GDP growth. This would allow countries to handle idiosyncratic national macroeconomic shocks better (Obstfeld and Peri, 1998).

A set of studies analyzed the properties of GLWs in the aftermath of the debt crisis in the 1980s. Governments and creditors were seeking ways to increase the issuer's resilience to negative growth shocks by linking debt payments to the issuer's capacity to service debts. As growth is uncertain in the long run[2] and growth slowdowns have been documented to raise the frequency of debt rescheduling (Easterly, 2001), GLWs, by restricting the variation of the ratio of coupon payments to revenues, were expected to reduce the incidence of defaults. It is possible to envisage that such securities could generate moral hazard problems if GLWs indeed reduced the issuer government's incentive to undertake growth-oriented policies. However, since GDP is the sum of efforts made by many economic agents, it is unlikely to be solely under control of the government.

Initial proposals to avoid moral hazard problems suggested linking debt payments to exogenous variables (Bailey, 1983; and Krugman, 1988). Froot, Scharfstein, and Stein (1989), however, argued that, although GDP may be an endogenous variable to the issuer government, asymmetric information creates situations where coupon payments should be indexed to the issuer's GDP in order to facilitate debt-service relief.[3] An indexation scheme that links repayments to a country's future output could help determine the appropriate amount of relief on a case-by-case basis: because a country that produces more now would be more likely to take actions to promote future growth, output serves as an indicator of a country's willingness to repay its debt. Most recently, Borensztein and Mauro (2002) argued that GDP-indexed bonds could help prevent future debt crises, and Tabova (2005) argued for the introduction of GDP-linked debt arrangements in the context of the World Bank's concessional lending.

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[2] Numerical examples are provided in, for instance, Athanasoulis and van Wincoop (2000), and Athanasoulis and Shiller (1999).

[3] See Borensztein, Chamon, Jeanne, Mauro, and Zettelmeyer (2004) for a comprehensive survey of index-linked securities issued in the past.

In addition to moral hazard issues, GLWs could be subject to other problems. The sources of reference may not be defined with precision. GDP statistics often do not reflect all economic activities. The measurement of GDP is a complex and error-prone process, which could make investors wary of using GLWs. An issuer may have an incentive to misreport GDP to reduce the payment of GLWs.[4] For example, the issuer may set the payments on the basis of initial low levels of GDP, and publish the true estimate only later. Unless properly designed, the issuer may not incur punishment. The inclusion of population statistics in the trigger conditions may also increase the uncertainty with the reference value. In addition, as the experience of countries that have issued GLWs has shown, warrants that are callable by the issuer appear inconsistent with the objective of having debt-service payments that change in line with GDP performance.

Several articles published recently focused on the pricing of GDP-linked products, although with a different emphasis than this paper. Kruse, Meitner, and Schröder (2005) used a Black-Scholes model to price GDP-linked bonds and showed that, without expectation errors on future GDP, differences in their performance compared to straight bonds should stem from risk premia associated with factors such as liquidity or uncertainty. Chamon and Mauro (2005) showed that, using an equation describing debt dynamics, the price of "plain vanilla" bonds would increase and default frequency would fall as a country increases its shares of GDP-linked bonds in total debt. While these papers have made significant contributions to the literature, they include rather limited number of pricing exercises, and they do not consider the issuer's repayment capacity as a factor in designing formula of GDP-linked bonds. This paper fills these gaps, and, thorough simulations of price sensitivity to different parameter values, derives the risk profile from the investor's point of view. It also takes into account the issuer's repayment capacity in designing GDP-linked warrants.

B. Recent Experiences with GLWs

As part of the Brady restructurings, Bulgaria, Bosnia and Herzegovina, and Costa Rica[5] issued GLWs—the only cases where GLWs were issued for sovereign funding purposes.[6] To a certain extent, the limited development of GLW markets to date can be attributed to the problems associated with the GLWs issued by these countries. However, the recent case of Singapore's growth-linked shares emphasizes that GLWs can be managed successfully when designed properly and accompanied by operational skills. Most recently, Argentina included 30-year GLWs in its recent debt restructuring exchange in June 2005.

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[4] It is argued that the issuance of GLWs would enhance the reliability of statistics in emerging market countries.

[5] The case of Costa Rica is not surveyed.

[6] In the emerging markets, domestic securities linked to commodity and inflation, among other products, were issued in Brazil, Chile, Turkey and elsewhere, while the Brady deals for Mexico and Venezuela offered a standard option for value recovery rights tied to oil prices.

Bulgaria[7]

The payment of the GLWs is triggered if (a) Bulgaria's GDP reaches 125 percent of its level in 1993 and (b) there is a year-to-year increase in GDP. In the years when such conditions are met, 50 percent of the year's GDP growth rates are paid on underlying "plain vanilla" bonds, in addition to the "plain vanilla" coupons. In the design of Bulgaria's GLWs, the definition of the source of reference data and GDP measurement units is ambiguous in that the sources could be either the World Tables of the World Bank or any other publications in which the World Bank publishes GDP data. In addition, the term sheet does not specify the units of measurement. To complicate matters further, the last published edition of the World Tables includes four measures of GDP in current and constant prices using market and factor prices, all in local currency units. The World Bank's replacement of the World Tables with the World Development Indicators complicated the definition of trigger events even further because GDP is measured in both constant and current prices and in U.S. dollars and Bulgarian leva. Had the GDP measure been defined in current-value U.S. dollar or leva terms, payments would have been triggered. Instead, the Bulgarian government decided to use constant-value local currency units and the GLW payments never triggered.

Moreover, the GLWs are callable, a feature that would appear inconsistent with the objective of the warrants because a call option offers the opportunity to reduce the share of debt that is resilient to GDP shocks. Bulgaria has the right to call the bond at par, and the option to call, together with the practice of refinancing with lower-yielding vanilla bonds, has reduced both the insurance against growth slowdowns and the opportunity for investors to benefit from the upside potential of the country.

Bosnia and Herzegovina[8]

The GLWs pay if (a) GDP hits a predetermined target level and remains at such a level for two years and (b) GDP per capita rises above US$2,800 in 1997 units, adjusted for German consumer price inflation. The GLWs of Bosnia and Herzegovina have been plagued both by poor security design and poor data quality. In Bosnia and Herzegovina, the share of the gray economy not included in official statistics may be as much as 37 percent. Although the recent data collected by the central bank reflect an attempt to account for informal sector activity, a large share of economic activity remains unaccounted for. Moreover, as the trigger conditions refer to per capita GDP, they involve population statistics, the reliability of which has often been questioned. The price of Bosnia and Herzegovina's GLWs, which mature in December 2017, is rarely published, and at least recent trading activity appears to have been very limited.

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[7] Sources of information: *Bear Stearns Sovereign Eastern Europe Update* (March 18, 2004 and March 25, 2004) and *EXOTIX LTD. Fixed Income Research* (April 2004).

[8] Source of information: *Bear Stearns Sovereign Eastern Europe Update* (April 19, 2004).

Singapore[9]

The Singapore government issued to low-income citizens two sets of shares that link payments to GDP growth. Unlike the Brady restructuring cases, the Singapore shares are distributed to eligible citizens, thereby perhaps creating less of a conflict of interest between the issuer and holder than issuance during a debt restructuring. Moreover, these shares are neither tradable nor transferable and can be exchanged only for cash with the government.

The first share, the New Singapore Shares (NSS), was introduced in 2001 to help the lower-income group during economic downturns. The NSS will earn annual dividends in the form of bonus shares, which will be calculated at a rate of 3 percent plus the real GDP growth rate of the preceding calendar year, with a guarantee of at least 3 percent. The second share,, Economic Restructuring Shares (ERS), was meant to subsidize Singapore citizens following the increase in the goods and services tax (GST). The bonuses will be calculated in a similar way as the NSS. Dividends on outstanding shares are paid on every March 1 from 2002 to 2007 for the NSS, and on every March 1 from 2004 to 2008 for the ERS, respectively. In 2005, the dividend rate was 11.4 percent.

Argentina

The Argentine authorities included 30-year GLWs in the debt-restructuring exchange completed in June 2005. The payments on peso principal will be converted into U.S. dollars, euros, or yen at the time of payment. The GLWs annually pay 5 percent of excess cash flows, defined as the difference between actual GDP and threshold GDP in nominal terms, when the following two trigger conditions are satisfied: (a) actual GDP, expressed in constant peso terms as of the reference date, exceeds threshold GDP, and (b) the annual growth rates of actual GDP, expressed in constant peso terms as of the reference date, exceed 3 percent. The first payment is scheduled for December 15, 2006. The value of payments will be calculated on November 1, about a month before the annual payment occurs. The corresponding reference date is December 31 of the year preceding the calculation date, reflecting the significant time requirement associated with the calculation of GDP value. Argentina's GDP-linked warrants are detachable from the plain vanilla bond and have been traded separately since the end of November 2005, unlike the inseparable warrants in the case of Bulgaria and Bosnia and Herzegovina.[10]

[9] Source of information: the Singapore government official website: http://www.nss.gov.sg/

[10] The on-line version of *Buenos Aires Herald* reported that GDP-linked warrants have shown strong growth (January 31, 2006).

III. The Model

A. Specific Design of GLWs

On the basis of the discussion of the benefits of GLWs and the problems encountered in the past issuances, the ideal GLWs woud be designed such that the trigger conditions are clearly identifiable and the payment amount easily calculated. In this respect, it is possible to design GLWs with the following characteristics:

- The terms are sufficiently long to allow a smoothing of payments over a series of economic cycles, such as 20 years.
- The trigger conditions are the function of GDP in real value and in domestic currency terms, so that nominal factors, such as inflation and exchange rates, do not dictate the trigger events.
- The trigger for the repayments reflects (a) the actual real GDP levels exceed the potential levels determined upon issuance, and (b) the actual growth rates of real GDP are positive. Condition (a) ensures that a country recovering from a severe GDP contraction by growing above potential will not pay on the warrants until the actual GDP level has finally recovered and exceeded its long-term potential trend.
- Each reference date is the year preceding the calculation date, reflecting the significant time requirement associated with the calculation of GDP value.
- All payments on the GLWs are capped to ensure that, in the event of extreme growth surprises, the cash-flow payments do not exceed the country's capacity to service debt.
- The difference between the actual and potential real GDP growth rates is multiplied by nominal GDP levels in the period before to calculate the payment amount.

B. The Model

To estimate the expected flows of debt payments from the proposed GLWs, GDP expressed in domestic currency is assumed to follow a stochastic process, while trigger conditions are modeled using Monte Carlo methods. The amount of cash flows payable to the investor is calculated according to the coupon formula, incorporating the characteristics described above. The model assumes a single good, GDP, in each period $t = 0,1,..., 21$. Although the maturity of GLWs is 20 years, the twenty-first period needs to be modeled because of the one-year lag in payments—the payable amount observed in period $t-1$ will be paid in period t. This model focuses on the estimation of cash flows and does not consider an optimizing consumer who would evaluate welfare gains from the coupon payments, or the production sector that would affect the value of GDP.

The payable amount of cash flows is used to measure the issuer country's capacity to service its debt, as GLWs are particularly designed to match the amount of debt payments with the resources available to the issuer country. It is also be used to estimate the price of GLWs, which will be calculated as the present value of payable cash flows.

The ability of the issuer to service the GLW payments with its resources is evaluated by matching, period by period, payable cash flows with the revenue available for servicing this instrument. The issuer's capacity to service debts is defined as the difference between the incremental payments of GLWs and the increases in tax revenues due to economic growth. Such tax payments are defined as incremental tax revenues.[11] The issuer's capacity to service debt payments is then measured by the ratio of the warrant payments (W) to incremental tax revenues (I), which is defined as the WI ratio.

Furthermore, the price of GLWs is estimated on the basis of payable cash flows. As it is envisaged that GLWs are held by both foreign and domestic investors, their price is estimated both in terms of the domestic currency and foreign currency. Initially, the value of cash flows is estimated in domestic currency terms; it is then converted into the foreign currency terms using implied forward exchange rates. Forward exchange rates are estimated on the basis of the interest rate parity condition, using both the domestic and foreign interest rate term structures.

To introduce risks in the computation, disturbances are applied to the two parameters: domestic inflation rates and rates of change in exchange rates. Two kinds of disturbances are applied in every period to both parameters: random shocks and parallel shifts. Random shocks are generated on the basis of random draws from $\phi(0,1)$, while parallel shifts refer to the cases where parameter values are changed by a constant throughout the periods.

While a detailed numerical description of the model and parameter definitions can be found in Section I of the Appendix, four main equations are presented here. Excess cash flows are defined as the difference between real growth rates of realized and potential GDP in period t, multiplied by realized nominal GDP levels:

$$\begin{aligned} ECF^{i}{}_{t,t+1} &= (g_t^i - g)V_t^i \\ &= \sigma\varepsilon_t^i v_0 \prod_{N=0}^{t-1}(1+g+\sigma\varepsilon_{t-N}^i)(1+\pi_{t-N}^i). \end{aligned} \tag{1}$$

Warrant payments are excess cash flows capped at C percent of trend GDP levels in nominal terms in the period before:

[11] In this paper, incremental tax revenues collected in period t are assumed to be available for servicing warrant payments only in period t, and cannot be put aside for period $t+1$ and beyond.

$$W^i{}_t = \min(ECF^i_{t-1,t}, Q_{t-1} * C). \tag{2}$$

A cap, defined as a fraction of trend GDP levels, serves to smooth warrant payments. The present value of warrant payments in period t is

$$PV[W^i_t] = \frac{W^i_t}{\prod_{N=0}^{t-1}(1+R_{t-N})(1+\pi^i_{t-N})}. \tag{3}$$

Forward exchange rates are derived on the basis of interest rate parity conditions:

$$E^i_t = \prod_{N=0}^{t-1}(1+h^i_{t-N}). \tag{4}$$

In these four equations, $V^i{}_t$ is realized nominal GDP, Q_t is nominal trend GDP, v_0 is the initial value of real GDP, g is average growth rates of real GDP, σ is volatility of growth rates of real GDP, $\varepsilon^i{}_t$ is random shocks to growth rates of real GDP, $\pi^i{}_t$ is realized inflation rates, $W^i{}_t$ is the warrant payments, C is cap rates, R_t is the issuer country's domestic real interest rate, E_0 is the initial value of exchange rates (the amount of domestic currency per U.S. dollar), and $h^i{}_t$ is realized rates of change in exchange rates.

C. Parameter Value

The absence of markets for GLWs imposes particular challenges for the choice of parameter value and, hence, the estimation of prices.[12] The main objective of this paper is, however, to propose a pricing method, and the choice of parameter value is not central to the numerical exercises. For the purpose of a numeric computation, growth rates and volatility of the issuer country's real GDP, by far the most critical parameters, are taken from the historical value for the main EMBI countries.[13] [14] For all other parameters that turned out to have less impact

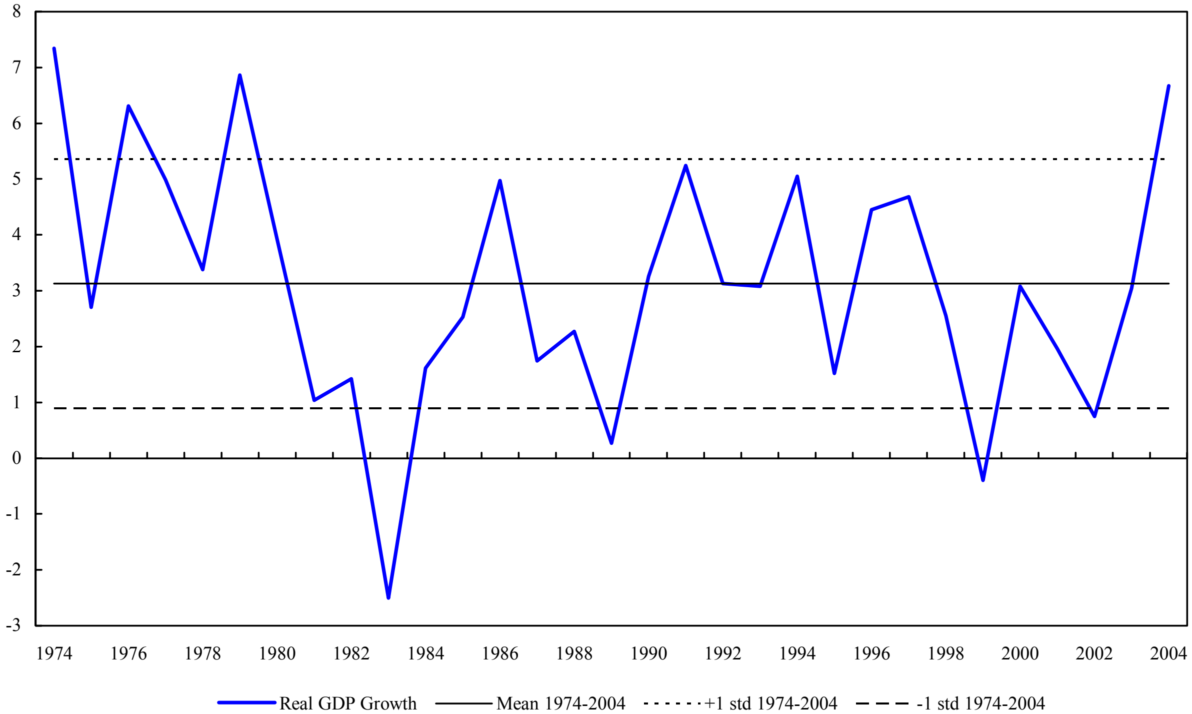
[12] Although some GDP-linked products have been traded through brokers, the market has remained illiquid.

[13] The Emerging Markets Bond Index (EMBI), published by JPMorgan, tracks total returns for traded external debt instruments in the emerging markets.

[14] Simple average over the last three decades, 1974–2004, for the following countries: Argentina, Brazil, Colombia, Ecuador, Mexico, Morocco, Nigeria, Panama, Peru, Philippines, and Venezuela. Bulgaria, Egypt, Poland, and Turkey are excluded due to data limitations.

on estimation results, value is chosen rather arbitrarily to construct a hypothetical scenario in which a developing country issues GLWs to both the domestic and foreign investors, the latter based in developed countries.

Figure 1. Growth Rates of Real GDP, Average of Main EMBI Countries, 1974-2004
(In percent)



Sources: JPMorgan; and IMF, *International Financial Statistics* .
Note: Average growth rates and standard deviation in the figure are 3.1 percent and 2.2 percent, respectively.

The baseline parameter values were chosen as follows: the values of average growth rates (g) and volatility (σ) for the baseline scenario were set at 3.1 percent and 2.2 percent, respectively (Figure 1); the ratio of total tax revenues to GDP (κ) and the initial value of GDP (v_0) were set at 20 percent and 100, respectively; spot exchange rates as the issuer's currency per the foreign currency (E_0) was set at 1; the volatility of shocks to the inflation rates (υ) and to rates of change in exchange rates (μ), parallel-shift parameters for the inflation rates (s), and rates of change in exchange rates (ψ), as well as the value of the pass-through of growth shocks to inflation (δ), were all set to be equal to zero;[15] the real interest rate for the foreign country ($r_t = r$) was set at 3 percent; and the issuer country's domestic real interest rate ($R_t = R$) and foreign rate[16] ($\chi_t = \chi$) were set at 8 and 14 percent,

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[15] These parameters take non-zero values during alternative simulation exercises.

[16] Discount rates for the issuer country's sovereign bonds issued in foreign currency.

respectively. The foreign inflation rates ($\lambda_t = \lambda$) were set equal to 2 percent, and the issuer's domestic inflation rates (γ_t) were assumed to fall from 6.5 to 4 percent over the periods.[17]

Table 1. Prices of GLWs and WI Ratios at Maturity Under Growth Surprises (g = 5 percent) [1/]

Cap	∞	5 percent	4 percent	3 percent	2 percent	1 percent
Price	29.2	26.3	24.0	20.4	15.2	8.3
WI Ratio	1.23	1.00	0.89	0.74	0.54	0.29

1/: Mean growth rates are assumed to be 5 percent, compared with the baseline value of 3.1 percent. All other parameters take the baseline values.

To limit cash-flow payments on the warrants in case of extreme growth scenarios, a cap (C), equal to 3 percent of trend GDP in the year before (Q_{t-1}), is added in order to contain peaks in warrant payments. Warrant payments could represent a large amount of tax revenues if significantly strong growth is to continue for an extended period, so this would raise warrant payments and thus contradict the spirit of GLWs. A cap of 3 percent can somewhat contain such warrant payments; hence, the WI ratio, with minimal impact on the price of GLWs (Table 1).

D. WI Ratio

This section quantitatively analyzes how the WI ratio for an issuer country varies with respect to changes in the initial parameter value. Subsequently, the sensitivity of the path of the WI ratio is analyzed with respect to changes in both the average and standard deviation of the rate of real GDP growth and the degree of disturbances.

Baseline economy

The following three variables were calibrated on the basis of the baseline parameter value: (a) the value of the WI ratio, (b) the probability that the value of the WI ratio would exceed one, and (c) the average amount of cash shortfalls, given that the value of the WI ratio would exceed one.

The value of the WI ratio was estimated to remain within 0.31 and 0.42 in the baseline scenario (Table 2, row 2) ─ with an estimate of 0.42 in the initial period, its value would gradually fall to 0.33 toward maturity. The probability that the value of the WI ratio exceeds one formally is

$$\text{prob}(WI_t^i > 1). \qquad (5)$$

[17] Both the rates of inflation and interest rates are exogenously given to the model and, therefore, do not have an effect on the state-dependent risk premia.

Table 2. WI Ratios and Other Variables — Baseline Scenario

Periods	2	3	4	5	6	7	8	9	10	11
WI Ratio	0.42	0.37	0.35	0.33	0.36	0.37	0.36	0.35	0.40	0.39
Eq. (5) 1/	20.00	17.80	16.80	15.10	18.40	19.00	16.80	15.90	18.20	18.20
Eq. (6) 2/	-0.87	-1.12	-1.30	-1.41	-1.39	-1.64	-1.91	-1.95	-2.30	-2.31

Periods	12	13	14	15	16	17	18	19	20	21
WI Ratio	0.36	0.33	0.37	0.31	0.35	0.37	0.35	0.36	0.35	0.33
Eq. (5) 1/	16.20	15.50	17.50	15.30	15.00	16.80	15.80	16.50	15.90	15.10
Eq. (6) 2/	-2.55	-2.46	-2.86	-2.88	-3.67	-3.63	-3.89	-4.01	-4.44	-4.21

1/: Equation (5) represents the probability that the value of the WI ratio exceeds one.

2/: Equation (6) represents the average amount of cash shortfalls given the value of the WI ratio exceeds one. Average amount of shortfalls are expressed in current terms (billions of domestic currency units).

When the value of equation (5) exceeds one, the issuer government will have to service part of its GLW payments with tax revenues that are likely to be earmarked for specific expenses.[18] The value of equation (5) is relevant because the WI ratio is silent about the likelihood of the consequences of such extreme cases. Calibrated results in the baseline scenario show that, the probability that the WI ratio exceeds one remains moderate, at around 15 to 20 percent, throughout all periods (Table 2, row 3).

The expected amount of cash shortfalls in the event the issuer government cannot fully service the warrants in a given year can be formally written as

$$E(\Delta T_t^i - W_t^i \,|\, WI_t^i > 1), \tag{6}$$

where $\Delta T^i{}_t$ is incremental tax revenues.[19] While the probability that the value of the WI ratio exceeds one remains low and stable over time, the expected value of cash shortfalls increases sharply (Table 2, row 4). The expected shortfalls in current value (billions of domestic currency units) rise from 0.9 in period 2 to 4.2 in period 21. In such fat-tailed events, these shortfalls would have to be covered with tax revenues already earmarked for other expenses.

[18] One might also set much lower levels of the WI ratio as the ceiling value above which the issuer government would have to withdraw cash from other specific expenses.

[19] See Section I of the Appendix for the derivation.

Sensitivity analysis

To analyze how the results discussed above change with respect to the parameter value, simulations were run for the value of the WI ratio in scenarios constructed from a set of alternative parameter values:

- **Average growth rates were assumed to range between -0.9 and 7.1 percent**. The surface plot in Figure A1 in the Appendix shows how the value of the WI ratio can rise steeply even at shorter maturities when growth rates in the scenario are increased beyond 4 percent, while the WI ratio tends to fall to zero under low-growth scenarios. However, the 3 percent cap starts binding and thus successfully limits the WI ratio below 0.9, even in the worst-case scenario.

- **The volatility of real GDP growth was assumed to range between 0 and 20 percent, with mean growth rates held at the baseline value of 3.1 percent**. The value of the WI ratio, and hence, the frequency of trigger events, increases as volatility was increased up to some point, beyond which the cap on warrants started binding. The surface plot in Figure A2 in the Appendix shows how the value of the WI ratio is kept relatively low—generally below 0.5, except at the short ends, where the ratio approaches 0.6.

- **Average growth rates were above the baseline value (5 percent > 3.1 percent), while the volatility of GDP growth was held as in the second set of assumptions**. In these scenarios, the frequency of trigger events falls as volatility increases, thus reducing the value of the WI ratio. Figure A3 the Appendix illustrates how the value of the WI ratio with zero volatility reaches 1 across all maturities. However, with a positive value of volatility, the WI ratio starts declining dramatically, implying that volatility risks are minimal under a plausible volatility value.

- **Average growth rates were assumed to be 1.5 percent and, thus, below the baseline value, and the range of volatility was left as in the third set of assumptions**. The shape of Figure A4 in the Appendix becomes similar to that of Figure A2, thus showing how high volatility can compensate low growth. The only difference between the two figures is the level of surface; it is lower in Figure A4 simply because average growth rates are lower. The value of the WI ratio can increase when volatility is increased close to 20 percent, while only up to 0.62 at the short ends and much lower at the longer ends.

- **The value of the pass-through of growth shocks to the inflation rates was varied between -2 and 2 (compared with a baseline value of zero)**. [20] Figure A5 in the

[20] The inflation rates are assumed to be affected by innovations in real GDP growth—the difference between realized and average growth rates of real GDP. The effect of such innovations on realized rates of inflation is controlled by the parameter defined as the pass-through of growth shocks to inflation.

Appendix illustrates how changes in the value of the pass-through of growth shocks to inflation have a limited impact across all maturities, with the ratio remaining within the range of 0.31 and 0.42.

- **Random shocks υ between 5 and 15 percent were applied to inflation rates (compared with a baseline value of zero)**. Such shocks are applied to the inflation rates in each period, and Figure A6 in the Appendix illustrates how the impact of such random shocks on the value of the WI ratio is limited.[21]

E. Price of GLWs in Domestic Currencies

The present value of cash flows was estimated in terms of the issuer country's currency as viewed by domestic investors. The estimated value of GLWs on the basis of equation (3) would be 6.36, compared with an initial GDP value of 100 in the domestic currency. For the purpose of performing a sensitivity analysis on the initial price estimates, the following parameters were changed: real growth rates of GDP and volatility, random shocks and parallel shifts to the inflation rates, and the value of the pass-through of growth shocks to inflation.

- **Average growth rates were assumed to take values of 1.5, 3.1 (baseline value) and 5 percent, while volatility is assumed to take values of 1, 2.2 (baseline value) and 10 percent**. Table 3[22] shows the price of GLWs in terms of the domestic

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[21] Simulations with alternative average growth rate assumptions do not alter these conclusions.

[22] For robustness checks, the value of the WI ratio was estimated on the basis of a binomial model, where average growth rates, volatility, and prices were calibrated to match the values in Table 3. The binomial model assumes two alternative paths at each nodes: up (*U*) and down (*D*). Due to computing-capacity limitations, nodes were modeled only every three years: year 1, year 4, .., year 19. Between the nodes, real GDP was assumed to grow at the same rates chosen at the prior node. The growth rates of real GDP were assumed to follow a symmetric transition matrix:

$$\text{prob}\begin{pmatrix} UU & UD \\ DU & DD \end{pmatrix} = \begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix},$$

where UD stands for the state where GDP increases (*U*) in period t given GDP fell (*D*) in period $t-1$. The binomial model is solved for the value of p, assuming the growth rates associated with U (g_U) and those associated with D (g_D) are constant over time. The binomial model simultaneously solves for the value of g_U and g_D for nine combinations of g and σ, as in Table 3. Even though the paths of the WI ratio are "kinky" as the nodes are modeled only every three periods, the binomial model's results are similar to those of Monte Carlo methods.

currency under different combinations of average growth rates and volatility. When volatility was held constant, the price of GLWs increased significantly when growth rates outpaced threshold growth rates of 3.1 percent. When growth rates were held below (above) threshold growth rates of 3.1 percent, the price increased (decreased) with volatility. Note that the extent to which the price of GLWs changes depends on the growth assumptions: at the baseline parameter value, the corresponding baseline price is 6.36, while it increases by 1.5 times when volatility is changed from its baseline value of 2.2 percent to 10 percent. When average growth rates are increased to 5 percent from their baseline value of 3.1 percent, while volatility is held constant at the baseline value, the price increases by a factor of three.

Table 3. Price Sensitivities — Case I (changing g and σ)

	g = 1.5 percent	g = 3.1 percent	g = 5.0 percent
σ = 1.0 percent	0.03	3.29	23.24
σ = 2.2 percent	0.59	6.36	20.35
σ = 10.0 percent	6.05	9.26	14.03

- **Shocks were applied to the inflation rates; (a) parallel shifts, by both increasing and lowering the value of s by 5 and 10 percent (compared with a baseline value of zero), and (b) random shocks, by changing the value of υ to 10 and 20 percent (compared with a baseline value of zero)**. Table 4 summarizes estimated prices of GLWs. When parallel shifts are applied, higher inflation reduces the real value of payments, albeit modestly, as an effect of the lag between the timing of observations and payments of GLWs. When random shocks are applied, the price increases as the volatility of random shocks increases, although to a limited extent.

Table 4. Price Sensitivities — Case II (changing s and υ)

	s = -10 percent	s = -5 percent	s = 0 percent	s = 5 percent	s = 10 percent
υ = 0 percent	7.03	6.68	6.36	6.07	5.81
υ = 10 percent	7.13	6.76	6.43	6.13	5.86
υ = 20 percent	7.43	7.01	6.65	6.32	6.02

- **The value of the pass-through of growth shocks to inflation is changed to 1, 2 and 3 above and below from its baseline value of zero**. Table 5 shows how changes in such parameter values only have a limited impact on the price of GLWs. In addition, the baseline parameter value of zero appears to be a somewhat aggressive choice from the issuer's point of view, as the estimated price of GLWs is one of the lowest in value.

Table 5. Price Sensitivities — Case III (changing δ)

δ = -3	δ = -2	δ = -1	δ = 0	δ = 1	δ = 2	δ = 3
6.4	6.43	6.37	6.36	6.36	6.36	6.38

F. Price of GLWs in Foreign Currencies

As foreign investors are also expected to buy the warrants, the price of GLWs was estimated in foreign currency terms, and a sensitivity analysis was performed on the model's parameter values. Exchange rate risks assumed by foreign investors were modeled through both parallel shifts to rates of change in exchange rates and random shocks. In this context, forward exchange rates were assumed to be determined on the basis of the interest rate parity condition, while scenarios under random shocks to the inflation rates were analyzed. By using equations (3) and (4), the price of GLWs under the baseline scenario is 3.06 in foreign currency terms, compared with an initial value of 100.[23] To analyze price sensitivity to different parameter values, both rates of change in exchange rates and the inflation rates were changed as follows.

- **Rates of change in exchange rates were shifted in all periods by 10 percent above and below the baseline value of zero**. Table 6 shows how such parameter changes have a significant effect on the price of GLWs; the price would be halved if forward exchange rates depreciated throughout the periods by 10 percent more than what the interest rate parity condition implied, without changing interest rates and rates of inflation. If forward exchange rates appreciated by 10 percent, the price would more than triple.

Table 6. Price Sensitivities — Case IV (changing ψ)

ψ = -10 percent	ψ = 0 percent	ψ = 10 percent
6.79	3.06	1.79

- **Different magnitudes of random shocks were applied to the inflation rates by increasing the value of υ (a parameter that governs the impact of random shocks on the inflation rates) from the baseline value of zero to 10 and 20 percent in each period. Different magnitudes of random shocks were also applied to rates of change in exchange rates by changing the value of μ, which controls for the impact of random shocks on exchange rates, from the baseline value of zero to 10 and 20 percent in each period**. Table 7 reports estimated prices under different combinations of shock parameters. Shocks to the inflation rates have a limited impact on the price—when the value of υ is increased to 20 percent while value of μ is set to zero, the price changes modestly from 3.06 to 3.20. Shocks to rates of change in exchange rates have larger effects on the price: when the value of μ is raised from the baseline value of zero to 20 percent while keeping the value of

[23] When forward exchange rates are assumed to be constant throughout the periods at the initial exchange rates of 1, the estimated price is 5.58 in foreign currency terms. Clearly, the assumption of constant forward exchange rates leads to an overvaluation of the present value in favor of the issuer.

υ equal to zero, the price increases from 3.06 to 4.10. In this respect, it appears that the foreign investor would benefit from random shocks both through the inflation rates and rates of change in exchange rates.

Table 7. Price Sensitivities — Case V (changing μ and υ)

	μ = 0 percent	μ = 10 percent	μ = 20 percent
υ = 0 percent	3.06	3.28	4.1
υ = 10 percent	3.09	3.33	4.18
υ = 20 percent	3.2	3.47	4.46

G. Indexation Premium[24]

Because GLWs are likely to be held along with other financial products in a portfolio, the premium over and above the risk-free rate that investors would require for holding GLWs in their portfolios, as opposed to as a stand-alone product, needs to be estimated.[25] The indexation premium is expected to be low if the return to GLWs is not highly correlated with the return to the investors' existing portfolio, because GLWs would reduce the volatility of the portfolio. In that case, the price of GLWs estimated as a stand-alone product would be a reasonable approximation of the market price.

To this extent, both the mean-variance Capital Asset Pricing Model (CAPM) and Consumption Capital Asset Pricing Model (CCAPM) were estimated, suggesting that the insurance premium on GLWs would likely be low. The β was estimated for foreign investors, assuming they are U.S. dollar-based investors and their existing portfolio is highly correlated with GDP growth of the U.S. or the return to S&P stock index . Because domestic assets for this hypothetical issuer country are not well defined, the β for the domestic investors was not estimated. The estimated β turned out to be very small on the basis of the CAPM, ranging from 0 to 0.06, and CCAPM estimations further confirmed such findings.[26] Thus, the investor would appear to require little additional value from the insurance premium, at least from a theoretical point of view.

[24] The estimation procedure and the implication of the CAPM are described in Section II of the Appendix.

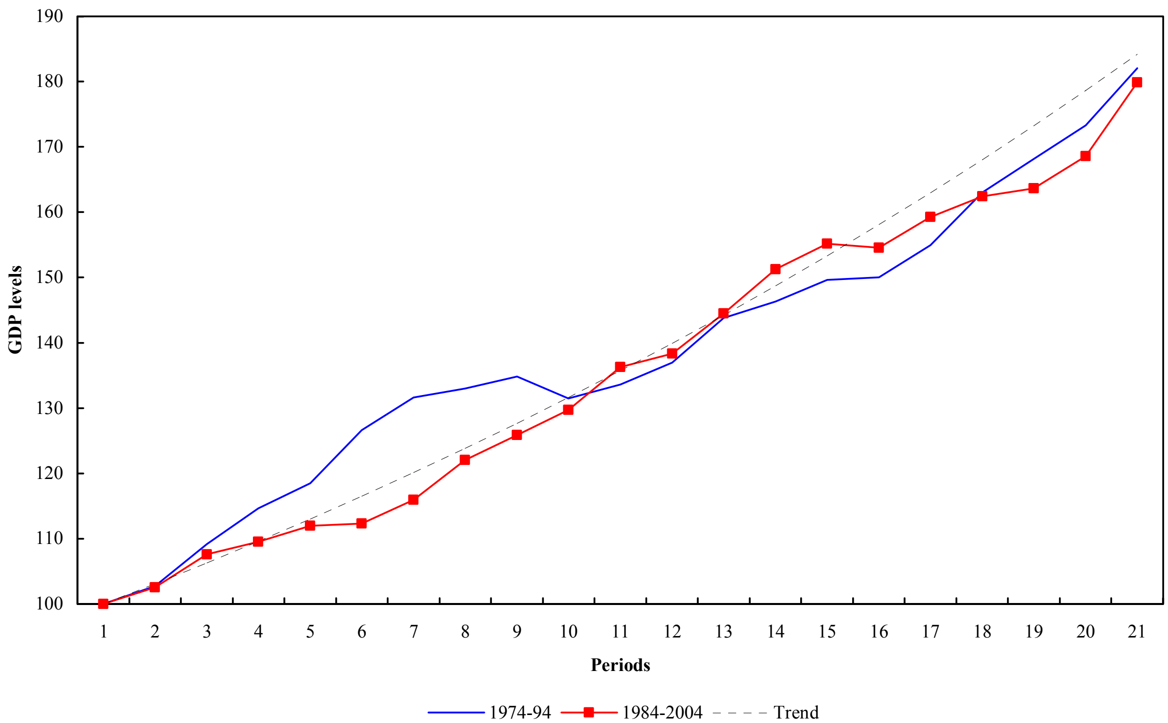
[25] If the required indexation premium is positive and high, the market price of GLWs would be lower than the price estimated as a stand-alone product.

[26] On the basis of a large number of countries, Borensztein and Mauro (2002) concluded that the risk premium implied by the CAPM would be low on GDP-indexed bonds.

H. Mean-Growth Assumptions

This section evaluates the validity of mean growth assumptions used in Monte Carlo methods. While the mean growth rates for real GDP in the sample design are set at 3.1 percent based on historical data, the issuer country's actual historical path of GDP may be significantly different for the same mean growth rates. Thus, the expected payouts of GLWs would be significantly altered from what the model predicts. Figure 2 illustrates the evolution of three paths of real GDP: (a) real GDP levels following the historical path during 1974–94, (b) real GDP levels following the historical path during 1984–2004, and (c) trend real GDP growing at a constant rate of 3.1 percent. The initial value of GDP was assumed to be 100. The figure shows that, in case (a), strong growth of real GDP during initial periods was followed by slower growth, thus reducing the number of trigger events. In this scenario, GLWs would have paid a present value of 7.93 in domestic currencies, compared with our baseline prediction of 6.36. In case (b), overall growth patterns were less favorable to the investors, thereby reducing the number of trigger events and the amount of each payments, and paying a present value of only 4.54 in domestic currencies. In the light of this exercise, our model appears to have reasonable prediction capacities, as the predicted value of 6.36 falls between historical values of 7.93 and 4.54.

Figure 2. Hypothetical Historical Paths of Real GDP Levels



Sources: JPMorgan; IMF, *International Financial Statistics* ; author's estimation.

IV. Concluding Remarks

The objective of this paper was to numerically examine the quantitative properties of GLWs to evaluate the issuer government's capacity to service debts and to estimate the price of GLWs from the investor's point of view. The literature has claimed that GLWs would provide opportunities to share growth risks between the issuer and the holder; however, the literature has also suggested that there might be moral hazard issues in the context of debt overhang, where the issuer country may lose incentives for promoting growth policies. Despite these moral hazard problems, some have argued that, under asymmetric information, debt payments should be linked to the issuer's GDP in order to facilitate debt-service relief. Experience has shown that the poor design and low quality of statistics have been major problems in the history of GLWs. Although the initial experience may have led to negative connotations to GLWs, the successful issuance of growth-linked shares in Singapore suggests that GLWs can be managed successfully when accompanied by proper design.

On the basis of the discussion of the benefits of GLWs and the problems encountered in past issuances, this paper proposed how GLWs could be designed such that the trigger conditions are clearly identifiable and the payment amount easily calculated. The maturity was set to 20 years to smooth payments over business cycles, and the trigger conditions were designed as the function of GDP in real value and domestic currency terms. The design also ensured that a country growing above potential in its recovery phase would not pay on the warrants until the actual GDP level had exceeded its long-term potential trend. Reference dates were set to account for the significant amount of time required to calculate GDP value, and all payments on the GLWs were capped to ensure that, in the event of extreme growth surprises, the cash-flow payments would not exceed the country's capacity to service debt.

This paper developed a simple modeling method to evaluate cash flows from GLWs under different scenarios, providing a framework to analyze the risk profile of GLWs from both the issuer's and investor's point of view. Based on the specific design and the historical data for the main EMBI countries, GLWs were evaluated in terms of their ability to match an issuer's revenue flows with the payment flows from the GLWs. The WI ratio—the ratio of the warrant payments to incremental tax revenues—was devised to evaluate such a capacity, and the ratio was calculated under different growth scenarios. Estimated cash flows were used to assess the risk the government could face under different growth scenarios.

Subsequently, the price of GLWs was estimated both in the issuer country's currency and in U.S. dollar terms. In both currencies, the price was found to change significantly under different growth scenarios. In particular, estimated prices in both currencies were sensitive to changes in average growth rates and volatility, while other parameters had limited effects. In addition, the price in foreign currency terms changed significantly with respect to exchange rates. Thus, deviation of forward exchange rates from the expected path implied by the interest rate parity condition could significantly change the price of GLWs in foreign currencies, and this risk would be borne by the foreign investors. Finally, the CAPM was used to calculate the size of the indexation premium, which was found to be even lower than the results in the literature.

The model can be enriched by changing a number of assumptions. For instance, the ratio of tax revenues to GDP could be modeled to be state-dependent, and tax revenues intertemporally storable. Deterministic variables such as interest rates could be modeled as stochastic and in a way to reflect the state-dependent default risk. Finally, an issuer's repayment capacity could be assessed based on a criterion different from what was assumed in this paper.

Theoritical Model

I. The Model

This section derives equations (1), (3) and (4) presented in the main text.

A. Estimating Cash Flows

Starting from the initial value of v_0 in period 0, real GDP in period t in state i, v_t^i is assumed to follow a geometric Brownian motion, with drift equal to average growth rates of g and volatility of σ:

$$dv_t^i = v_t^i(g + \sigma\varepsilon_t^i), \tag{A1}$$

where ε_t^i ($i = 1, 2, \ldots, N$) are random draws from $\phi(0,1)$. Hence,

$$v_t^i = v_0 \prod (1 + g + \sigma\varepsilon_{t-N}^i). \tag{A2}$$

Actual growth rates of real GDP g_t^i can be written as

$$g_t^i = \frac{dv_t^i}{v_t^i}. \tag{A3}$$

Threshold real GDP, q_t, is assumed to start from the same initial value of v_0, growing at an annual rate of θ:

$$q_t = v_0(1+\theta)^t. \tag{A4}$$

Realized and threshold nominal GDP, V_t^i and Q_t, respectively, are given by adjusting v_t^i and q_t for the deterministic inflation rates, γ_t:

$$V_t^i = v_0 \prod_{N=0}^{t-1} (1 + g + \sigma\varepsilon_{t-N}^i)(1 + \gamma_{t-N}) \tag{A5}$$

$$Q_t = v_0 \prod_{N=0}^{t-1} (1+\theta)(1 + \gamma_{t-N}). \tag{A6}$$

Excess cash flows observed in period t and paid in period $t+1$, in state i, $ECF^i_{t,t+1}$, are calculated as the difference between realized and trend GDP in real terms, multiplied by realized nominal GDP levels:

$$\begin{aligned} ECF^i{}_{t,t+1} &= (g^i{}_t - g)V^i{}_t \\ &= \sigma\varepsilon^i{}_t v_0 \prod_{N=0}^{t-1}(1+g+\sigma\varepsilon^i{}_{t-N})(1+\pi^i{}_{t-N}). \end{aligned} \tag{A7}$$

The investor would receive excess cash flows as warrant payments, which are capped at C percent of trend nominal GDP in nominal terms in period $t-1$:

$$W^i_t = \min(ECF^i_{t-1,t},\, Q_{t-1} * C), \tag{A8}$$

when the two trigger conditions are satisfied:

$$W^i_t > 0, \text{ if } v^i_t > q_t \text{ and } g^i_t > 0$$

$$W^i_t = 0, \text{ otherwise.}$$

A cap is defined as a fraction of trend GDP levels as a way to smooth warrant payments.

B. Evaluating Debt-Servicing Capacity

The warrant payments are matched with the revenue available to the issuer for servicing debts to assess debt-servicing capacity. Total tax revenues, T^i_t, are given by the tax revenues-to-GDP ratio, κ:

$$\begin{aligned} T^i_t &= \kappa V^i_t \\ &= \kappa[v_0 \prod_{N=0}^{t-1}(1+g+\sigma\varepsilon^i_{t-N})(1+\gamma_{t-N})]. \end{aligned} \tag{A9}$$

Note that tax revenues include a risk factor, ε^i_t. GLWs are assumed to be serviced with incremental tax revenues associated with contemporaneous growth. Incremental tax revenues in period t in state i are the difference between total tax revenue in period $t-1$ and t:

$$\begin{aligned} \Delta T^i_t &= T^i_t - T^i_{t-1} \\ &= \kappa(V^i_t - V^i_{t-1}). \end{aligned} \tag{A10}$$

The issuer government's capacity to service GLWs is evaluated by the ratio of the warrant payments to incremental tax revenues (WI ratio). The WI ratio, WI_t^i, is warrant payments divided by incremental tax revenues:

$$WI_t^i = \frac{W_t^i}{\Delta T_t^i}. \tag{A11}$$

C. Estimating the Price – Domestic Currency

The price of GLWs is estimated as the expected present value of a sequence of WI_t^i. It is first estimated in domestic currency terms. Each coupon payment in nominal terms is first deflated by cumulative inflation rates, and is discounted by the domestic real interest rate, R_t:

$$PV[WI_t^i] = \frac{W_t^i}{\prod_{N=0}^{t-1}(1+R_{t-N})(1+\gamma_{t-N})}. \tag{A12}$$

The price of GLWs is the expected present value of each warrant summed over the periods:

Domestic currency price: $\sum_t \underset{i}{E}\{PV(W_t^i)\}$. (A13)

D. Estimating the Price – Foreign Currency

The price of GLWs is also estimated in foreign currency terms. Forward exchange rates, E_t, are estimated from the domestic and foreign country's nominal interest rate term structures, on the basis of the interest rate parity conditions:

$$E_t = \frac{\prod_{N=0}^{t-1}(1+R_{t-N})(1+\gamma_{t-N})}{\prod_{N=0}^{t-1}(1+R_{t-N})(1+\lambda_{t-N})}, \tag{A14}$$

where r_t is the foreign real interest rate and λ_t is foreign inflation. By rewriting equation (A14),

$$E_t = E_0\prod_{N=0}^{t-1}(1+\eta_{t-N}), \tag{A15}$$

where E_0 is the initial value of exchange rates and η_t is an annual rate of change in exchange rates. The warrant payments in domestic currency terms are converted into U.S. dollars with E_t and are discounted by the issuer country's nominal risky foreign rate, χ_t:

$$PV[W^i_{\$,t}] = \frac{W^i_t}{E_t \prod_{N=0}^{t-1} (1+\chi_{t-N})}, \tag{A16}$$

The price of GLWs is the expected present value of each warrant summed over the periods:

Foreign currency price: $\sum_t E_i\{PV(W^i_{\$,t})\}$. (A17)

E. Disturbances

Disturbances to the inflation rates and rates of change in exchange rates are introduced into the model to check the robustness of the model's estimations to changes in parameter values. Disturbances are applied in each period to such parameters, through random shocks and parallel shifts. Price levels and levels of exchange rates in period t in state i, P^i_t and E^i_t, are assumed to follow a geometric Brownian motion, with drift equal to deterministic growth rates of γ_t and η_t,[27] and volatility equal to υ and μ:

$$dP^i_t = P^i_t(\gamma_t + \upsilon\zeta^i_t) \tag{A18}$$

$$dE^i_t = E^i_t(\eta_t + \mu\omega^i_t), \tag{A19}$$

where ζ^i_t and ω^i_t $(i = 1,2,...,N)$ are random draws from $\phi(0,1)$.

Realized inflation rates include two more parameters: (a) the value of the pass-through of growth shocks to inflation from growth rates of real GDP to the inflation rates, δ, and (b) parallel shifts in deterministic inflation rates, s. The inflation rates are assumed to be affected by innovations, namely, the difference between realized and average growth rates of real GDP:

[27] η_t is deterministic in that stochastic shocks to exchange rates are separately defined. However, note that the value of η_t is affected by stochastic shocks to inflation.

$$\sigma\varepsilon_t^i = g_t^i - g. \tag{A20}$$

The degree of the impact of such innovation on inflation is controlled by δ. Parameter s is introduced to simulate the impact of parallel movements in the path of inflation. In summary, realized inflation rates in period t in state i, π_t^i, are the sum of deterministic inflation rates, γ_t, stochastic shocks to the inflation rates, $\upsilon\zeta_t^i$, shocks to growth rates of real GDP scaled by δ, $\delta\sigma\varepsilon_t^i$, and parallel shifts, s:

$$\pi_t^i = \gamma_t + \upsilon\zeta_t^i + \delta\sigma\varepsilon_t^i + s. \tag{A21}$$

A parallel shift parameter, ψ, is also added to realized rates of change in exchange rates for simulation purposes. Realized rates of change in exchange rates, h_t^i, are

$$h_t^i = \eta_t + \mu\omega_t^i + \psi. \tag{A22}$$

Equations (A7), (A12), and (A15) are modified to account for additional parameters in order to derive equation (1), (3), and (4) presented in the main text:

$$ECF^i{}_{t,t+1} = (g_t^i - g)V_t^i$$
$$= \sigma\varepsilon_t^i v_0 \prod_{N=0}^{t-1}(1 + g + \sigma\varepsilon_{t-N}^i)(1 + \pi_{t-N}^i)$$

Equation (1) in the main text

$$PV[W_t^i] = \frac{W_t^i}{\prod_{N=0}^{t-1}(1 + R_{t-N})(1 + \pi_{t-N}^i)}$$

Equation (3) in the main text

$$E_t^i = \prod_{N=0}^{t-1}(1 + h_{t-N}^i).$$

Equation (4) in the main text

The parameters employed in this model are summarized as follows:

g: average growth rates of real GDP
g_t^i: realized growth rates of real GDP
σ: volatility of growth rates of real GDP
v_0: initial value of real GDP
v_t^i: realized real GDP

V_t^i : realized nominal GDP
θ : growth rates of real threshold GDP
q_t : real threshold GDP
Q_t : nominal threshold GDP
ρ : trigger growth rate
κ : ratio of total tax revenues to GDP
T_t^i : total tax revenues
E_0 : initial value of exchange rates, domestic currency per foreign currency
m : fraction of excess cash flows payable
ε_t^i : random shocks to growth rates of real GDP
ζ_t^i : random shocks to domestic inflation rates
ω_t^i : random shocks to rates of change in exchange rates
υ : volatility of random shocks to domestic inflation rates
μ : volatility of random shocks to rates of change in exchange rates
s : parallel shifts, domestic inflation rates
ψ : parallel shifts, rates of change in exchange rates
δ : value of the pass-through of growth shocks to inflation
r_t : foreign real interest rates
R_t : domestic real interest rates
χ_t : issuer's risky foreign nominal interest rates
λ_t : foreign inflation rates
γ_t : domestic inflation rates

II. INDEXATION PREMIUM

The insurance value of GLWs is high if their return is negatively correlated with the return on the entire portfolio; a low return on the portfolio during bad times can be at least partially counterbalanced by a high return on GLWs. The return on GLWs can be evaluated by comparing it with what the theoretical insurance value of GLWs implies on the basis of the capital asset pricing model (CAPM).[28] While other factors would affect the theoretically required return, such as an illiquid market, only the insurance premium is considered here.

The CAPM divides the risk of holding risky assets into systematic and specific risk. Systematic risk is the risk of holding the market portfolio. As the market moves, each individual asset is more or less affected. To the extent that any asset participates in such general market moves, that asset entails systematic risk. Specific risk is the risk that is unique to an individual asset. It represents the component of an asset's return that is uncorrelated with general market moves.

According to the CAPM, the marketplace compensates investors for taking systematic risk, but not for taking specific risk. This is because specific risk can be diversified away. When an investor holds the market portfolio, each individual asset in that portfolio entails specific risk. Through diversification, however, the investor's net exposure is just the systematic risk of the market portfolio.

The risk premium the investor would theoretically require for holding GLWs is estimated using two closely related methods: the mean-variance CAPM and the consumption CAPM (CCAPM).

A. Mean-Variance CAPM

This section estimates the indexation premium GLWs should pay to compensate investors on the basis of a standard CAPM. Developed by Sharpe (1964) and Lintner (1965), the CAPM estimates the relationship between an asset’s risk and expected return:

$$E[r_i] = r_f + \beta(E[r_m] - r_f), \text{ and } \beta : \frac{Cov(r_i, r_m)}{Var(r_m)},$$

where r_i is the return on risky assets, r_f is risk-free interest rates, and r_m is the market return. The approach based on the mean-variance frontier was first presented in and applied to U.S. data, where systematic risk can be measured using the β. According to the CAPM, the expected return on a bond equals the risk-free rate plus the portfolio's β multiplied by the expected excess return of the market portfolio.

[28] See, for instance, Cochrane (2001) for further detail.

The application of the same logic in emerging markets is not straightforward, and the figures provided are to be interpreted as suggestive. Empirical studies suggest that the β of emerging market returns with respect to the world portfolio is zero or negative, implying a zero or negative risk premium. However, in reality, the required returns on emerging market products are never equal to or lower than the U.S. risk-free rate. Harvey (2001), for instance, explains how investment banks make ad hoc modifications to adjust up the low rates of return that the CAPM would predict, and that it is difficult to explain most of these modifications.

The parameter β is estimated only for the foreign investor because there are no well-defined domestic asset prices for our hypothetical issuer country. This section estimates the value of β, but not the indexation premium itself. Given that the relevance of the CAPM in the context of emerging markets is uncertain, its implications are at best suggestive.

The value of r_i is approximated by the real return on GLWs, while the value of r_m is approximated by two items: real growth rates of U.S. GDP and the real total return on the S&P 500, thus assuming that the return to the foreign investor's portfolio is highly correlated with them. Table A1 shows the estimated value of β for the foreign investor, for two different periods on the basis of the combinations of two (r_i, r_m). Estimated results imply that r_i is equal to or less than r_f, and that the indexation premium should take a small positive or negative value.

Table A1. Estimated β for Foreign Investors

Periods	U.S. Market Return	
	U.S. Real GDP Growth	S&P Real Return
1974-94	-0.06	-0.03
1984-2004	-0.02	0

B. CCAPM

This section considers another model of CAPM, CCAPM, in order to check the robustness of the estimated β. From utility maximization, the Euler equation is given as follows:

$$p_t = E_t\left\{\rho \frac{u'(c_{t+1})}{u'(c_t)}\right\} b_{t+1},$$

where ρ is the subjective discount factor. $p_t u'(c_t)$ is the loss of utility from the purchase of b_{t+1} at its price, p_t, and $E_t[\rho u'(c_{t+1}) b_{t+1}]$ is the increase in expected value of the discounted utility from the payoff of b_{t+1}. By rearranging,

$$E(r_i) - r_f = -r_f \operatorname{cov}[m_{t+1}, r_i]$$

where $m_{t+1} = \rho u'(c_{t+1})/u'(c_t)$. Assuming a utility function of the form $u = c^{1-\theta}/(1-\theta)$,

$$m_{t+1} = \rho(1+\gamma_{t+1})^{-\theta},$$

where θ is the degree of risk aversion, and γ_{t+1} is growth rates of consumption between period t and $t+1$. The standard parameter values for the subjective discount factor and concavity of preference are as follows: [29]

$$(\rho,\ \theta) = (0.95,\ 0.5),$$

and consumption growth is approximated by the growth rates of GDP. The estimated value of the covariance term is found to be very low for the foreign investor.

[29] The value of $1-\theta$ usually range from -2 to .5. With respect to the subjective discount factor ρ, its standard value may be inferred from the steady state relationship between ρ and the risk-free rate r, $\rho = (1+r)^{-1}$.

Sensitivity Analysis of WI Ratio

Figure A1. WI Ratio - Choice Parameter: Average Growth Rates of Real GDP (In percent)

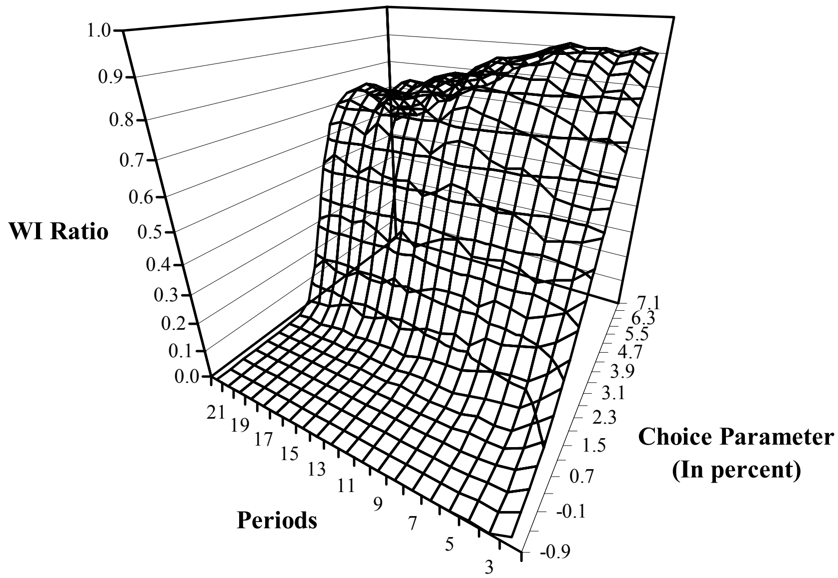
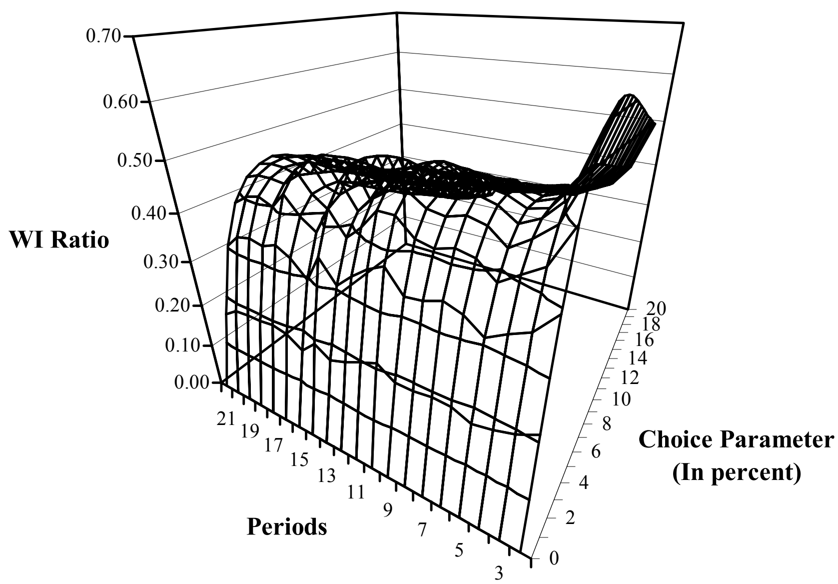


Figure A2. WI Ratio - Choice Parameter: Volatility of GDP Growth (In percent) Baseline Scenario (g=3.1 percent)



Source: Author's estimation.

Sensitivity Analysis of WI Ratio

Figure A3. WI Ratio - Choice Parameter: Volatility of GDP Growth (In percent)
High-Growth Scenario (g=5 percent)

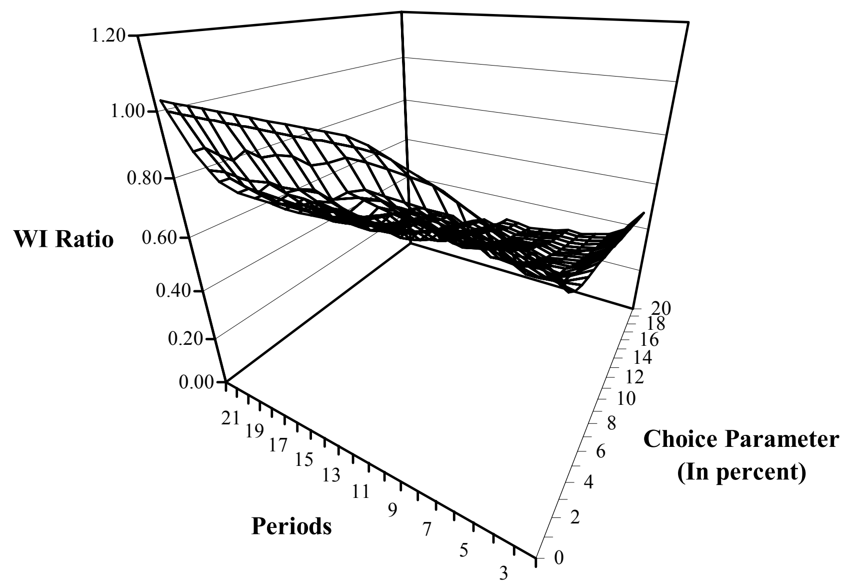
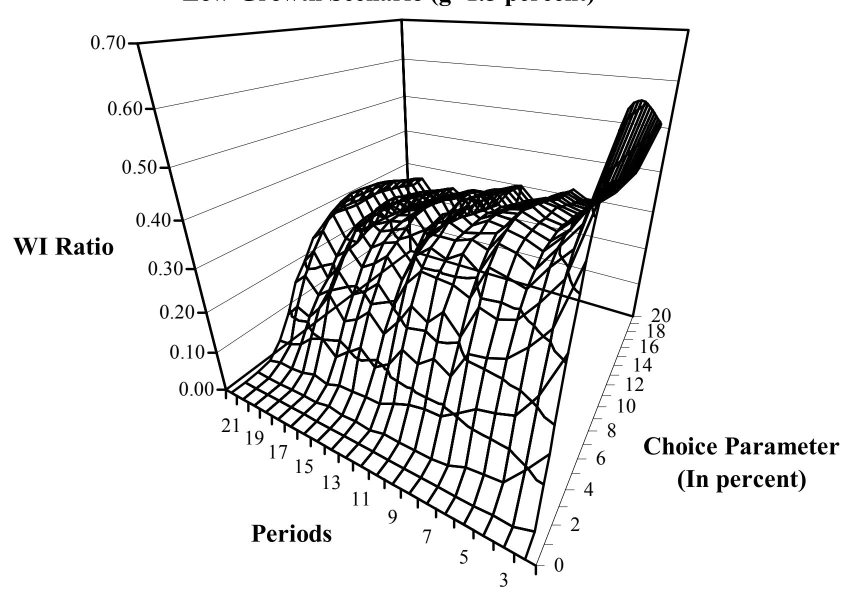


Figure A4. WI Ratio - Choice Parameter: Volatility of GDP Growth (In percent)
Low-Growth Scenario (g=1.5 percent)



Source: Authour's estimation.

Sensitivity Analysis of WI Ratio

Figure A5. WI Ratio - Choice Parameter: Pass-Through of Growth Shocks to Inflation
(In percent)
Baseline Scenario (g=3.1 percent)

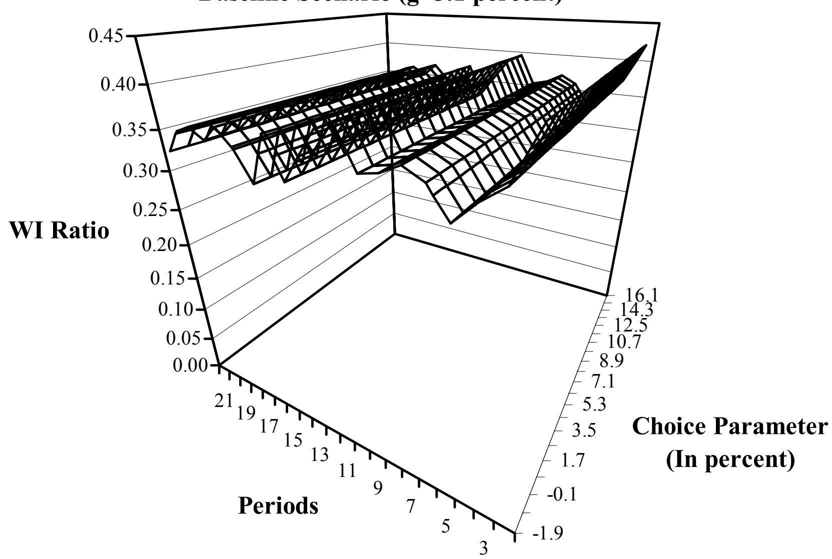
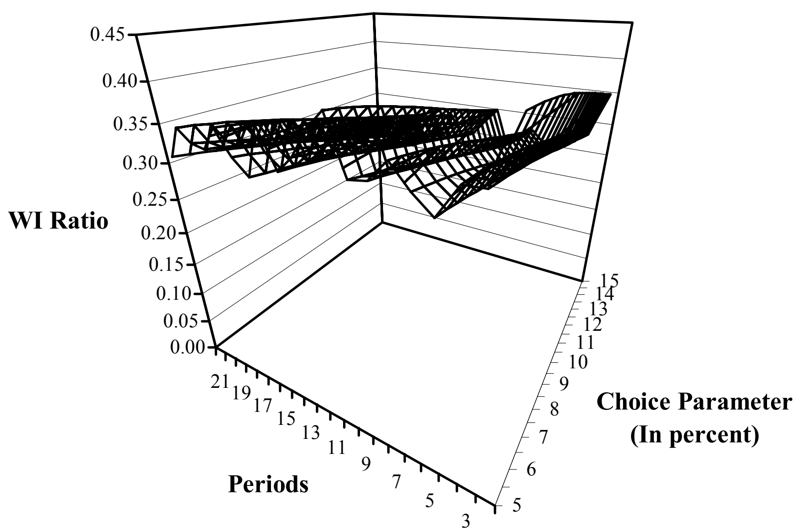


Figure A6. WI Ratio - Choice Parameter: Random Shocks to Inflation
Baseline Scenario (g=3.1 percent)



Source: Author's estimation.

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