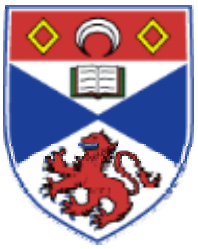
CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS
WORKING PAPER SERIES



CDMA05/12

Understanding financial derivatives during the South Sea Bubble: the case of the South Sea subscription shares

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DECEMBER 2005

ABSTRACT

South Sea Company subscription shares were compound call options on the firm's own original shares. From the description of shares found in 6 Geo. 1, c.4, a theory of their pricing is developed. A method for computing subscription share values is also developed. Calculated theoretical values for subscription shares are compared to the shares' historical values and a close correspondence between the two is demonstrated. The pricing of the subscriptions appears to have been quite rational and explainable using simple financial economic theory.

JEL Classification: N23, G13.
Keywords: South Sea Company, Financial Revolution, Bubble Act, compound options, partly-paid shares.

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Introduction

In this paper we demonstrate the existence of something whose existence has never been noted before. During the South Sea Bubble of 1720 there was not only a market for fully-paid original South Sea shares – a market that famously rose to dizzying heights before it came crashing down - there were also markets for financial derivatives that were based on South Sea shares. We do not refer here to the occasional option contract that was privately negotiated and drawn up between two individuals. We refer instead to financial derivative markets that attracted a trade amongst large numbers of people. These individuals traded contracts that were fixed in their form and were issued by the South Sea Company itself. The trading values for these contracts were reported everyday in what passed for the financial press of that time. We refer here to the markets for the South Sea subscription shares, for surely they were financial derivatives. In fact, they were call options. In point of fact, they were compound call options on South Sea fully-paid shares.

Why should anyone care that such contracts and such markets existed? The South Sea Bubble itself is of course a prominent landmark in the financial economic history of the western world. The more we can learn about the South Sea Bubble, the more we can learn about the early history of British corporate and public finance.[1] The South Sea Bubble is of interest to the non-economic historian as well. It was such a traumatic event for so many people that the historical evidence left in its wake has long been an important source of information about early Georgian society and politics. All persons concerned with such matters should be interested in the history that is about to be told in this paper. We hope, however, that this history will be especially interesting to the specialised financial economist. To this person historical market crashes and bubbles might appear to be mere curiosities or

[1] An organising principle in Dickson's (1967) history of the Financial Revolution is that the South Sea Bubble shook (literally) British financial institutions into the forms that they largely kept for the rest of the eighteenth century.

very interesting phenomena that are just too remote in time and are describable only with crude and unreliable data. This person also knows that a modern bubble or market crash is difficult enough to analyse. A powerful effort in data collection and theorising is the order of the day in coming to grips with a modern market crash. Absolutely essential to the task is the information that can be found only in markets for financial derivatives. In the markets for derivatives we know that investment professionals deal in the contracts that they use to construct schemes that are supposed to insure against loss of value in portfolios. Even when such schemes are not successful we see in the prices of financial derivatives the traces of what investment professionals hoped to achieve. There might be evidence of a change in the market price of risk or the first evidence of the arrival of essential information that has not yet reached other financial markets. Without data from the markets for financial derivatives the work of the modern financial economist is seriously impeded.

We cannot convince the financial economist that to be a student of the South Sea Bubble is as easy or even remotely like being a student, for example, of the stock market crash of 1987. We hope that the most optimistic of our readers might just convince themselves that the financial world of 1720 is not that remote and there might just be a chance that the South Sea Bubble, as a scientific problem in financial economics, will someday have a solution. To such a reader the demonstrated existence of derivatives markets might be the basis of such optimism.

This paper is divided into several sections. In the next section a short history of the subscription shares is presented. The problems in understanding South Sea subscription share values are presented; the relationship with modern day partly-paid shares is discussed. In the section after that a theory is developed that says the South Sea subscription shares were compound call options. In the third section a simple method is proposed for computing the values of subscription shares based upon the compound call option theory. In the fourth

section the data to be analysed are described and their limitations are discussed. Essential facts about returns' distributions of South Sea original shares in 1720 are developed. These facts are used to calibrate the computational model of subscription share values that is used in the following section. In that section we use the computational model to test the performance of our compound call option theory. Discussion of possible further research is found in the concluding section of this paper. There is a short bibliography, but the footnotes will guide the interested reader to a selection of works that contain much fuller historical bibliographies on the South Sea Bubble. Tables, figures and special supplementary appendices are at the end of the paper.

The subscription shares and the problem of their pricing

In this section are briefly described the South Sea subscription contracts and the theoretical problems we face in understanding their values. There were four issues of South Sea subscription shares in 1720. They were authorised under the authority of 6 Geo 1, c.4 (April 1720). Near the end of that very long Act is a passage devoted to defining the terms under which subscription finance could be raised and any special abilities or disabilities that would attach to subscription shares.[2] The Act also defined the uses to which subscription finance could be applied. 6 Geo. 1, c.4 thus fulfilled in part the role that would be played today by a share prospectus, as well as being an expression of corporate law. The Company would leave some of the other provisions of subscription shares, such as the instalment schedules, to later definition. The important features of South Sea subscription shares defined by this Act were as follows:

[2] The relevant passage from 6 Geo. 1, c.4 is reproduced in full in supplementary appendix 1.

1) If a subscriber failed to pay an instalment, the Company could deprive him of any 'Share, Dividend, Annuity, or Profits' that he might otherwise be entitled to;[3]
2) If a subscriber failed to pay an instalment, the Company could also 'stop the Transfers or Assignments' of the defaulter's interests in the firm;[4]
3) The defaulter's liability for a missed payment, plus 5 per cent per annum interest, would be met from the 'Shares and Stocks of such Defaulter'; this meant that if the subscriber persisted in default for a space of three months, the Company, or anyone whom the Company designated, could sell the defaulter's 'Stocks';[5]
4) If in the event of such a forfeiture to the Company, the forfeited shares would be sold by the Company for whatever they were worth. If the value of such a sale was in excess of the defaulter's liability to the Company, the defaulter would receive the excess value - the 'Overplus', as it was called.[6]

These four provisions in 6 Geo. 1, c.4, when taken together, allow us to build a theory of how the prices of subscription shares should have behaved. Items 3) and 4) are the important ones. Any subscriber considering default would be liable to the Company only to the extent of his holding of shares in the Company. In other words default would result in a simple forfeiture of the shares to the Company. Forfeiture, however, could not be finalised until three months had passed after which the defaulter could restore himself to ownership of the shares by paying the missed instalment plus three-months' interest. This could be thought of as an option to 'wait-and-see', but the cost of exercising this option was so low (only the interest cost of waiting), we may treat it as simply an extension of the instalment schedule itself. Indeed, it is no wonder that we have so much evidence that instalments for

[3] supplementary appendix 1, lines 27-9.
[4] supplementary appendix 1, lines 33-4.
[5] supplementary appendix. I, lines 35-44.

each of the four subscription issues were heavily in arrears.[7] It cost subscribers practically very little to delay the payments of their instalments for at least three months.

We find modern examples of partly-paid shares that impose the penalty of forfeiture if instalments are not paid and in which the issuer continues to incur an obligation to the owner of the partly-paid share as a result of forfeiture.[8] This is a feature of the South Sea subscription shares that plays actually a very important role in the theory that will be presented in the next section. The restriction of the subscriber's liability to his share holdings, however, is also very important. This is a feature of partly-paid shares not to be generally found today (see Section 144 of the Companies Act of 1985) except perhaps in the No Liability (NL) corporate structure allowed to Australian mining companies today.

The payment schedules of instalments were left to the Company to declare. In the cases of each of the four subscription issues the Company declared a fixed schedule of payments.[9] The first two issues' schedules were defined in April 1720 and the Company stayed with these schedules until December. See table 1.

*********table 1 here**********

One thing the Company did not do was to reserve to itself the option of suddenly and surprisingly making a call for an instalment. When it made any surprise announcement about

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[6] supplementary appendix 1, line 45.

[7] As early as the 2 June there was discussion in the Company as to how forfeitures of subscription shares would be managed (Minute book of the committee of the treasury, B64, House of Lords Record Office, p. 17). Then even though it was resolved in General Court that a £100 deposit was required upon subscription to the 3rd series of shares (BL Add. MS 25,499, 15 June 1720) there was discussion in July about the large numbers of subscribers who had failed to pay their deposits (Boyer, The political state of Great Britain, Vol. 20, p. 132).

[8] Multiplex, the builder of the new Wembley, faces some problems similar as those faced by the South Sea Company in late 1720. In 2004 Multiplex called for the final instalment to be paid on some of its partly-paid shares when, as is alleged in a potential class action against Multiplex, the Company did not reveal the true extent of its problems with the Wembley project. Many shareholders did default on the last instalment at that time and Multiplex duly informed them of their liability and that their shares were forfeit. Many of these shareholders, like South Sea subscribers in the autumn of 1720, felt that their liability to the Company to provide capital was compromised by the Company's failure to reveal fully its business problems. For the Multiplex controversy refer to see the webpages http://www.multiplex.biz/page.asp?partid=294&ID=125 and http://www.mauriceblackburncashman.com.au/areas/class_actions/multiplex.asp. The complaints of subscribers against the South Sea Company are studied in Section III of Shea, "Financial market analysis can go mad…."

the instalment schedules, it was always for a delay in a scheduled payment. Having a fixed instalment schedule is another feature of the South Sea subscription shares that makes the construction of a pricing theory for them relatively easy to achieve.

With a fixed instalment schedule it is possible to calculate the present value of instalments yet to be paid. Since nothing would have prevented an owner of a subscription share from paying all the instalments and obtaining a fully paid share, it straightforwardly follows that the value of a fully-paid, original share would have been a lower bound on the total value of a subscription contract. That is, the trading value of a subscription share plus the present value of the cost of converting it into a fully-paid share (the present value of the instalments yet to be paid) would have been no lower than the value of a fully-paid share. This lower bound – a kind of arbitrage bound – allows us to illustrate quite straightforwardly the challenge in understanding the values of subscription shares. In figures 1 and 2 are respectively illustrated the value of subscription shares Nos. 1 and 2 (plus the present values of their instalments) alongside the values of fully-paid shares.

********figures 1 and 2 here*******

Although we can remark that fully-paid share values do provide a lower bound on the subscription share values, more remarkable still is how far above original share values subscription values are. Can we understand why subscription shares had such value? The important clue is found in the levels of the original share values themselves. When original values were high, such as on 20 June 1720, the following array of facts is found. An original share was worth about £760 ($P_{original}$) and a subscription share (from the first series) was worth about £590 ($P_{subscription}$). The difference between the two values was £170. The present value (PV) of the remaining 7 instalments (calls) of £30 each would have to have been

[9] In the very short trading history of the 3rd and 4th subscriptions, however, the Company changed the instalment schedules frequently and in radical ways. Supplementary appendix 2 contains a full history of the Company's management of the instalment schedules.

smaller than £210, but a realistic value (at about a 4 or 5 per cent per annum discount rate) would also have been greater than £200. Thus we see that the value of a subscription contract was a bit more than £790 on 20 June. Contrast this with 25 October 1720 when fully paid shares were worth about £210. 1st series subscription shares were worth about £130 and at the same time the present value of their remaining 7 instalments was still a bit more than £200. Thus when South Sea share values had reached low levels in the autumn of 1720, the value of subscription contracts relative to fully-paid shares actually rose. The figures are strongly suggestive that subscription share values relative to fully-paid share values followed a nonlinear function of the level of fully-paid shares - the hallmark of an option value.

But what kind of option could be embodied in the subscription shares that would give them such high values when fully-paid share values were low? Like any call option on shares, the subscription shares certainly gave the right to their owners to own a fully-paid share at a low price in the contingency that share prices rose and continued to stay above the issue-price of the subscription shares. But the subscription shares required only that their owners make a sequence of decisions to pay instalments and each instalment that was paid only secured to the payer the right to make another instalment payment later. How would we value such rights? That question is answered in the next section.

This is not the first scholarly examination of the subscription shares and their prices. Dale, Johnson and Tang (2005) assumed that the subscription issues were packages of fractional shares and the subscriber was required to make instalment payments with no possible recourse to default, just as if the current Companies Act had been enacted in 1720. The natural conclusion they drew from their assumption was that subscription share values and original share values should have been locked together into <u>linear</u> pricing relationships. Of course, when no sensible linear relationships were found by these authors for data such as those in figures 1 and 2, they concluded that irrational pricing relationships were the only

possible explanation. There is however, strong evidence that these authors did not examine 6 Geo 1, c.4 and ignored as well much evidence in pamphlet literature and in Company minutes about how the subscription shares worked.[10] In the next section we shall use the facts we have so far to develop a pricing theory for the subscription shares that will make a little more sense of the South Sea subscription share values of 1720.

Theory of South Sea subscription share pricing

The purpose of this section is to present a theory of South Sea subscription share pricing. This is not difficult when subscription contracts have the very simple structure that the South Sea subscription contracts had. With a failure to pay an instalment there was to be an eventual forfeiture of the subscription share to the Company. The Company would take possession of the subscription share for the purpose only of extracting the value of the instalment that was due to it. When the Company disposed of the forfeited share, the defaulting subscriber would receive any excess value (the 'overplus'). These were the happy circumstances of the South Sea Company's subscription contracts, as defined in 6 Geo. 1, c. 4, that were discussed in the previous section. Even though a subscriber would have no reason to default if the present value of the subscription share (call it $P_{subscription}$) was greater than a due instalment (call it K), it is nevertheless important to remember that if he were to default for any reason, he would still be due to be paid whatever the Company was able to sell the subscription share for, minus his liability of K to the Company. Under these conditions, we shall see that a subscription share will amount to what can be called an n-tuple or n-fold compound call option on an original share. By a compound call option we mean a series of call options on call options.

[10] The special styles of financial/historical analyses followed by Dale, Johnson and Tang are the particular subject studied by Shea in "Financial market analysis can go mad…," and are discussed no further here.

Our argument is quite direct and requires no elaborate formal presentation. Imagine a subscription share for which the number of instalments due to be paid is n. Let us now consider the value of that share after n-1 instalments have been paid and only one instalment remains to be paid.

Proposition 1: After n-1 instalments of K each have been paid, an n-instalment subscription share becomes nothing more than a call option upon one original share.

The exercise price of this call option is the last instalment itself (K) and the exercise date is the date on which the last instalment is to be paid. If the last instalment is paid, the subscriber has (in net terms) a full original share less the instalment (K). If the subscriber instead defaults upon the last instalment, the Company receives the subscription share in forfeit. When the Company sells the subscription share for whatever it is worth, the defaulting subscriber will then possess the excess value of the share ($P_{subscription}$) over and above his liability (K) to the Company (the 'overplus'). In the case of the last instalment to be paid, $P_{subscription}$ equals $P_{original}$; that is, the subscriber gets $\max[P_{original} - K, 0]$ or, in other words, $P_{original} - K$ (the "overplus") or 0, whichever is greater. This is, of course, the classic expression of the terminal value of a call option upon an original share for which the exercise price is K.[11] Regardless of whether the subscriber defaults or not, he will end up with

[11] Another happy circumstance for our analysis is that the Company declared that all subscription shares carried rights to the 10 percent midsummer dividend that was declared in April for fully-paid shares. A holder of a subscription share therefore knew that his share had the same dividend rights that a fully-paid share had and that he would not have to pay any instalments earlier than the schedules required in order to obtain the dividend. As a consequence, the options that we study here can all be evaluated as straightforward European-style options. We should also note here that we are purposely ignoring the effects of dilution that would result when a subscription share becomes eventually a fully-paid share. The subscription shares were like bundles of warrants in that they would result in the creation of new fully-paid shares by the Company. Important as subscription share finance was in the events of 1720, however, the numbers of subscription shares were not very great relative to the numbers of fully-paid shares already in existence and the very large number of fully-paid shares that were being created by other means. We are confident that the dilution effects on subscription share values are minor and can be safely ignored.

$\max[P_{original} - K,0]$. Thus the terminal value of a subscription share is the terminal value of a call option upon an original share. It follows that the present value of a subscription share should, from the payment of the penultimate instalment to the last instalment, be the present of a call option (with a strike price of K) on an original share. We now extend this argument when we consider the value of a subscription share when the next-to-last instalment payment is to be made.

Proposition 2: After n-2 instalments of K each have been paid, but before the penultimate instalment is paid, the subscription share is a call option (with strike price K) on the call option described in Proposition 1.

Again, the subscriber will either default or not default. If he defaults, he gets either nothing or $P_{subscription} - K$, when the Company sells his forfeited subscription share for what it is worth. This is also what he has in terms of net asset value if he does not default and pays the due instalment of K. Regardless whether he defaults or not, the value of his subscription share (when the penultimate instalment is due) is therefore $\max[P_{subscription} - K,0]$. If we imagine a call option on the subscription share whose exercise date is the same date as that of the due instalment and whose exercise price is K as well, the terminal value of that call option will also be either $P_{subscription} - K$ or 0. We would thus conclude that the present value of the subscription share, before the penultimate instalment was due and after the previous instalment was paid, would have to be the value of a call option on the subscription share (with exercise price K). Since the subscription share itself will become a call option upon an original share after the penultimate instalment is paid, the subscription share must be viewed, before that time, as a call option upon a call option.

The argument of the last paragraph can be extended recursively to apply to the values of the subscription share for any number of instalments due upon it. For n instalment payments, the subscription share would be equivalent to a call option on a call option on a call option…(n-1 times) on a call option on an original share. Although we may note that a subscriber would most likely default whenever an instalment (K) was due and whenever $P_{subscription} < K$, it will nevertheless be true that, regardless of whether he defaults or not, the net value (at the time of an instalment payment) of a subscription share will always be $\max[P_{subscription} - K, 0]$.

Values of compound call options are not difficult to compute even though closed-form solutions for compound option values are difficult to derive. The first successful effort was Geske's (1979) solution for a call on a call. Good textbook treatments of compound options readily extend his solution to calls on puts, puts on puts and puts on calls.[12] To our knowledge, however, no one has presented a closed-form solution for a general n-fold compound call option of the type we are describing here. Examples of modern partly-paid shares that have a forfeiture sanction attached to them are not hard to find, but they usually have other features that make them difficult to value. After forfeiture, there may not always be a requirement that an 'overplus' be delivered to defaulters. Although it is unlikely an owner of a partly-paid share would ever default in a situation in which he could expect to get a positive 'overplus', it is true nevertheless that the promise of an 'overplus' is the only guarantee that the terminal value of a partly-paid share will be exactly the same as the terminal value of a call option on an original share. Modern partly-paid shares are more complex also because the schedule of instalments to be paid may not be known or fixed. The South Sea Company published fixed schedules for instalment payments. It did change them occasionally, but the subscription shares were not subject to sudden surprise calls for

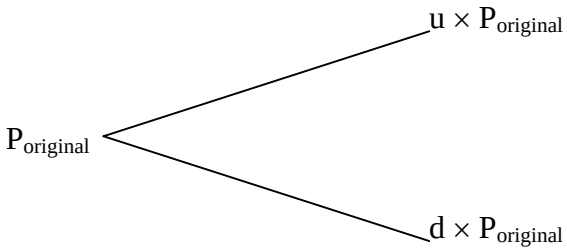
instalments to be paid. Additionally there may also attach different abilities and disabilities to original shares than attach to their partly-paid counterparts, such as rights to dividends. Matters were simpler in 1720 when it came to the definition of partly-paid shares and it took some time for partly-paid shares and related law to evolve into their modern forms. By the time we come to 1790's and beyond, the definition of partly-paid share finance became a regular feature of incorporating Acts. In any Act for an infrastructure project, such as a canal, railway, bridge, gasworks or waterworks, can be found passages that deal with how finance for the firm can be raised with partly-paid shares. Restrictions on instalment schedules came to be specified and many Acts came to contain a mixture of provisions as to how forfeiture of shares would be handled.[13] But all that was to come after 1720. In the next section of this paper we discuss briefly how we can turn our theory into a method for computing values of the South Sea subscription shares.

Subscription share value computations

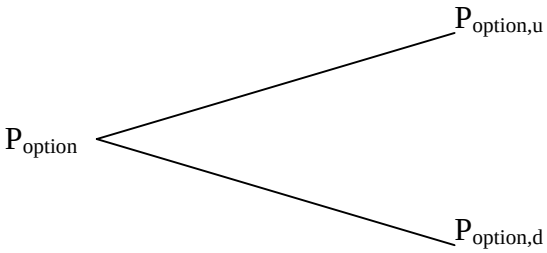
We use a simple algorithm to compute subscription share values as compound option values. Although the techniques we use can be found in most any intermediate textbook on derivatives pricing, we adapt the theory, terminology and notation presented in Cox and Rubinstein's Options Markets.[14] There are a number of frameworks in which to present the fundamentals of option pricing theory, but the framework is determined largely by the way in which the stochastic character of share returns is modelled. The range of choice in such stochastic models is immense, but in this paper we use the simplest of them all - the binomial

[12] See Geske (1979), "The valuation of compound options". A good textbook treatment, with computer programs, for valuing 2-fold compound options is found in McDonald, Derivatives Markets, pp. 441-4.
[13] Even then the principle of the subscriber's obligation to pay all instalments was far from completely established as it would eventually become in, say, the Companies Act of 1985. A typical early nineteenth century incorporation Act, such as 6 Geo 4, c. 67 for the Ashton under Lyne Gas and Waterworks (1825), gave the Company an option to pursue defaulting subscribers with Actions on Debt, but would also allow the

random walk. The binomial random walk is a simple, but is also a powerful means of describing the way in which prices, such as share and option prices, evolve through time. Using it can lead to a number of exact pricing formulae for options that serve as accurate approximations to formulae that can be obtained from other more complex stochastic models. Its construction starts in the following simple manner. Over some appropriately short interval of time, the price of an original share is assumed to take a random walk either "up" or "down". The potential size of these steps is denoted u (u>1) and d (0<d<1). This binomial random walk is illustrated thus:



If we now imagine an option contract that is written on the share, we can depict the option value's random walk, which corresponds to the random walk in the share price, thus:



We express the solution for P_{option} in a standard way that uses Δ, the number of original shares that are held in a portfolio whose value will replicate the value of the option. When Δ shares in this replicating portfolio are combined with an appropriate amount of borrowing

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Company to take subscription shares in forfeit. DuBois, The English Business Company, is essential reading for the general legal basis of corporate subscription finance in the eighteenth century.

[14] Options Markets, section 5-3, pp. 171-8 contains the theory used here.

(B), the possible future values of the portfolio will mimic the possible future values of the option and hence it must be the case that the present value of the portfolio will mimic the present value of the option. The exact expressions for Δ and B that define such a 'replicating' portfolio are standard. These expressions are respectively:

$$\Delta = \frac{P_{option,u} - P_{option,d}}{(u-d)P_{original}} \text{ and } B = \frac{u \times P_{option,d} - d \times P_{option,u}}{(u-d)r}.$$

The value of the replicating portfolio is $P_{original}\Delta + B$, which is also the value of the option, P_{option}. These formulae can be used recursively to compute the value of an option back to the present period so that between the present and the terminal date for the option a binomial random walk for option values can be constructed.

With a binomial random walk for option values defined, there is nothing to prevent us from valuing another option on the original option in the same manner. This computational algorithm can then be nested within itself to compute the value of an n-fold compound option. Further details of how we implement this algorithm and our resulting programs are described in Supplementary Appendix 3. There the interested reader can learn how to use the programs that generated all the results presented in this paper. In the next section we will describe the South Sea data that we analyse in this paper. We will also extract from this data a range of likely values for "u" and "d" with which our simulations of subscription share values can be calibrated.

Data and distributions of original share returns in 1720

In this section we provide some more details of the data that we will analyse – why we have selected the data we have and its special limitations. Secondly, we will also study the

distribution of share returns contained in this data. Such study will provide guidance for how we should calibrate the programs we use to calculate theoretical subscription share values.

If we distinguish between data and numbers, data being numbers whose sources we can verify and whose reliability we can understand, then data pertaining to the South Sea Bubble are very scarce. For example, if we look at the original share values depicted in figures 1 and 2, we see that there are two substantial periods for which there are no depicted values. These were the periods in which the Company's transfer ledgers were closed and trade in original shares could only be carried on in a forward market for delivery of shares. This does not mean that there were no share value numbers reported for these periods. We have such numbers in plenty, but it is not easy to understand what they represent actually. The primary sources for value data in 1720 are the two price courants, Freke's, *Prices of stocks* and Castaing's *Course of the exchange*.[15] During the midsummer period (23 June – 22 August) the company's ledgers were closed for making up the midsummer dividend. It was common practice amongst companies such as the South Sea Company to close their ledgers for such purposes over lengthy periods of time. In this case, both price courants indicate that reported share values were values for forward delivery "at the opening", i.e. on or about 22 August. No further information is reported about the forward contracts to which these values were attached. What were their size and what kind of securities was embedded into these contracts that would ensure performance by the two contracting parties? This kind of information, that is, everything that must be known in order to locate an underlying spot value for an original share, is simply not revealed by the publishers of the price courants. The second closing was a sudden closing of the ledgers on 26 August, when it was also announced that the closing would last until 22 September. The transfer ledgers were re-opened on 12 September. We know of this 'second' closing because discussion of it that can be found in Company records

and in a court case that was a consequence of the Company's sudden decision to re-open the ledgers.[16] But no mention of this can be found in the price courants and so we have no word from them about the nature of the value numbers they reported for the 26 August – 12 September period. They were very probably forward delivery values as well "for the opening", but that is only a reasonable speculation. The best evidence we have so far of forward premia in private financial contracting in 1720, is that such premia could be very large and positive. We conclude that, for the time being at least, it is very difficult to locate approximate spot original share values in those periods in which transfer ledgers were closed. That is why we exclude their depiction from figures 1 and 2.[17]

Both figures also depict subscription share values and these values can be found in either Freke's or Castaing's publication. We can depict values for these shares throughout the "closing" periods because it appears that spot trade in subscription shares was carried out by a trade in endorsable receipts. By late June it was certainly the case that such trade was possible and legal because it was ratified in the Bubble Act (6 Geo.1, c.18). But still that does not mean that we know how Freke and Castaing got their subscription value data. Although it is a reasonable presumption that they were conducting surveys of values in spot trade of subscription receipts, they do not say so anywhere in their publications. This is a concern because we know that only in the case of the 1st subscription were receipts delivered out to

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[15] The genesis of the British financial press and the relative standing of Freke's and Castaing's reportage are discussed in chapter 2 of Neal's The rise of financial capitalism.

[16] BL, Add. MS 25,499, Court minutes, 25 August 1720. The closing was thought opportune while the management of subscriptions was discussed. There was discussion of a possible rights issue for those persons who were subscribing government annuities into South Sea stock. This idea was abandoned and there was discussion about a possible further money subscription. In the end this idea too was also abandoned and the transfer books were ordered (11 September) to be re-opened on the next day. The forward delivery contract in dispute in Maber vs. Thornton, The cases of the defendant and plaintiff, was undertaken 7 September for 'the next opening'. On 12 September Maber failed to take delivery of the stock, which precipitated Thornton's successful suit.

[17] The evidence concerning size of forward premia in 1720 is summarised in "Financial market analysis can go mad…." In the Dale, Johnson and Tang paper, the authors implicitly assumed that the forward values reported in the price courants for the midsummer closing are what today would be called "zero-premium" forward values – forward values that differ from the underlying spot value by only the small interest-cost of carrying the stock

subscribers soon after the subscription was taken in (14 April). Even in the case of the 2nd subscription (taken on 20 April) receipts were apparently not ordered to be delivered until July and throughout July there is mention in the Company records that the receipts for the 2nd subscription had to be delivered out more quickly.[18] So we might well wonder just how transactions in subscriptions were actually effected, especially in the early trade of the 1st and 2nd subscriptions, and what precisely were the numbers that Freke and Castaing were reporting.[19] Some of the early subscription values reported by Freke and Castaing splice nicely with some special transactions records of sales of 1st and 2nd subscriptions by Company directors through brokers. There is thus the possibility that some of the early values reported by price courants were the results of sales by directors of places on the subscription lists. In other words some of the early reported values could have been the result of primary sales rather than secondary trade in subscription receipts.[20]

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forward. Under this assumption they calculated imputed "spot" values from the reported midsummer forward values.

[18] The Duke of Portland had to wait until the end of May before he received the receipts for his very substantial purchases of the first subscription (Portland (London) MS, Pw B 165, pp.13-4). According to An account of the subscriptions, pp. 15-6, when the third subscription lists were being made out (17-21 June), there were still people trying to obtain receipts for the earlier subscriptions. In the Minute book of the committee of the treasury, B64, House of Lords Record Office, p. 19, we find that it was not until 8 June that receipts for the 2nd subscription were ordered to be made and delivered. On 22 July (p. 24) the Committee of Treasury called for further expedition from the Cashier in the delivery of those receipts. This was corroborated by Pheasant Crisp in one of his many newsletters to the Duke of Portland; he wrote how the Company was making an effort to issue receipts for the second subscription as late as 25 July 1720 (Portland (London) MS, Pw B 8, 10). When an instalment was paid, the owner of a receipt was supposed to have the payment acknowledged by endorsement entered onto the receipt by one of Company's clerks or officials. Subscribers thus could not easily present evidence to buyers of their receipts that instalments had been paid without having had their receipts endorsed by a Company official.

[19] In the case of the short-lived trade in 3rd and 4th subscription shares, these problems were all the greater. After much vacillation, the Company never did decide to deliver the previously promised receipts to subscribers. One of the great controversies of late 1720 was the distress to which the Company had caused in financial markets because of these actions. One of the related questions pertaining to South Sea data is the meaning of the values reported by Freke and Castaing (and others) for trade in the 3rd and 4th subscriptions. Because of these complications, we do not model 3rd and 4th subscription values in this paper.

[20] The Committee of Secrecy made its reports to the House of Commons in 1721 and left behind a valuable, but little used collection of papers at the House of Lords Record Office. The Committee was keenly interested in Directors' trade in South Sea liabilities and thus we have a number of abstracts of ledgers belonging to brokers who had dealings with South Sea directors. From these ledger abstracts I have recorded as many trades and in as great detail as possible. These records themselves are far from perfect. They do not cover the entirety of Directors' dealings, only those dealings accomplished through brokers, and they most certainly exclude some accounts that the Committee of Secrecy would have liked to have seen, but were to remain missing. (Box 158 of the Parchment Collection, HLRO).

In the next section we study the pricing of 1st and 2nd subscriptions shares relative to original share values in two very different periods: 1) before the midsummer closing of the company ledgers when share values were high and rising quickly and 2) after the second closing when prices were much lower and declining quickly. The former period was dominated by positive price changes and the latter period was dominated by negative price changes, but daily price changes in both of these periods were strongly bimodal, as figures 3 and 4 attest. The difference in the two periods' distributions of returns might be easier to see over a longer returns' horizon. We therefore have run a Monte Carlo experiment to obtain an idea of the possible range of returns over a longer one-month horizon. With 10,000 independent 30-day random samplings from the discrete distributions represented in figures 3 and 4, we obtained the simulated 30-day cumulated returns distributions illustrated in figure 5 and 6.

**********figures 3 through 6 here**********

If we look at the distributions of the simulated 30-day returns, we can obtain a better idea of the likely range in returns' variation. For example, in the 14 April – 22 June period, the middle 90 percent of the distribution of monthly returns in figure 5 lies between +160 percent and +10 percent. Although negative monthly returns are possible in the simulated distributions, they are extremely unlikely, whereas very large monthly returns are highly likely. This suggests that very large values of "u" and only very small negative values for "d" might be good choices for calibrating our model in the early period. In figure 6 we see a very different distribution. The middle 90 percent of that distribution lies in the monthly return range of –85 percent to +55 percent, suggesting that a very different set of estimates for "u" and "d" are appropriate for the later period. In the next section we see how our choices for "u" and "d" affect the empirical performance of our pricing theory.

Empirical results

How well does the model of compound call option pricing fit the empirical data on subscription share values? To answer this question we do not fit an econometric model, as such. What are reported in this section are the calculated theoretical values of subscription shares that come from our model when the model is calibrated, or tuned we might say, to calculate these values for a given set of volatilities, or (u,d)-pairs. The questions concerning empirical performance can be framed basically as the following: 1) Can empirically reasonable volatilities result in theoretical subscription values that are close to empirical values? and 2) Are very unreasonable volatilities just as capable or more capable of producing good theoretical subscription share values? In this section we present our results in four tables, tables 2 through 5.

The reader can understand the results in the tables by keeping the following scheme in mind. Imagine three different volatilities: (u′,d), (u,d) and (u, d′) with $u' > u > 1$ and $1 > d > d' > 0$. For example, the two pairs (u′,d) and (u,d) can represent what we might a volatility range in that each pair defines a different random walk, but each random walk also has the same "downward" potential as the other one has. The two random walks differ only in their "upward" potential. Now imagine that we calculate two theoretical values for a subscription share using the two different volatilities that define a range. If an empirical subscription share value can be found that falls between the two theoretical values, then we can conclude that our model is capable of duplicating the empirical subscription share value precisely using some volatility that falls within the volatility range (u′,d):(u,d). Rather than solving for such volatilities precisely (and there is no reason that they have to be unique), the tables are a simple visual presentation of the model's capability is of "capturing" empirical subscription share values when using some reasonable (or unreasonable) volatility ranges. Each table shows a selection of 1720 dates, original share values on that day (these are taken from

Freke's, *Prices of stocks*), and the minimum and maximum subscription share value observed for that day. These latter values are taken from all available sources, but they come primarily come from either Freke or Castaing's *Course of the Exchange*. The reason we wish to use such extreme values is as follows. The interday and intraday variation in subscription share values, even from the same source, was quite large. The two price courants were more likely to disagree upon subscription share values than they were to disagree upon original share values. The publishers did not indicate any precise times of day when their reported data were collected, so we cannot be sure that subscription share values were even collected at the same time of day as were original share values. We strongly suspect on some days reported subscription share values were even "stale", left over values that were observed either earlier in the day or even on the day before when original share values were quite different. We are concerned only that our model can produce roughly correct magnitudes for subscription share values with reasonable volatilities. We ask the reader therefore to concentrate upon reading the tables along each row and across columns rather than comparing columns across time (rows). We also wish the reader to realise that there is a certain amount of conservatism built into the reported results. Sometimes volatility might have to be increased to an unreasonable level so that our model can capture an extreme empirical subscription value, when at that time there might have been actually a lot of trade in subscriptions at values that were not so extreme.

With these thoughts in mind, we hope the reader is prepared to follow our general conclusions about what the tables reveal. The results in Table 2 reflect quite well on the compound option model. Many empirical values are captured by our model when it uses volatilities with very small d's and large u's.[21] These would be the kinds of monthly share return volatilities that we would most likely observe in the early period before the share

transfer ledgers closed. (See figure 5). Volatilities with greater "downward" potential are much less capable of generating captures of empirical subscription values. Where the model fails most noticeably is in April and early May, in which it appears incapable of generating sufficiently low theoretical subscription values. We can only speculate, but one reason that the model "overprices" very early values could be that this was a period in which real trade in subscriptions was more costly and difficult because the trade in endorsable subscription receipts had not yet been made legal. Perhaps the small supply of subscription receipts themselves was putting the trade at a disadvantage.

In Table 3 are the results for the 1st subscription shares for the autumn of 1720. This table is successful in the sense that it generally shows that empirical subscription values are best captured by volatilities with very large "downward" potential. By reading across most rows carefully the eye will also confirm that the subscription values not only positively respond to volatility (as any option value should), but are more sensitive to downward volatility than upward volatility. The results in the table are perhaps not so successful in another sense. The model is most likely to capture empirical subscription values with positive and negative volatilities that are a bit unlikely if they were to be sampled from the discrete distribution pictured in figure 6. We have identified already that the monthly-return range +55 percent to -85 percent captures most of the distribution, but our model best generates "captures" with volatility ranges wider than +60 percent to -95 percent. Another way of stating this is that our model understates by a bit the subscription values per unit of volatility.

Before we turn to Tables 4 and 5, which contain the results for the 2nd subscription series, the reader should recall that the 2nd series differs from the 1st in two important respects. It had a higher issue price (£400 per share, as opposed to £300 per share) and thus should *ceteris paribus* have been less valuable than the first series. Its instalments were also

[21] In the tables 'captures' are indicated by outlined and boldfaced simulated values that straddle one of the

spread over a much longer period of time than were the instalments of the 1st series (refer again to Table 1), which *ceteris paribus* should have made the 2nd series more valuable than the 1st. There were brief times in 1720 when the 2nd series subscription shares appeared to be as valuable and perhaps even more valuable than were the 1st series shares. Because the 2nd series shares had approximately a year longer duration than did the first series, it would be surprising indeed if the volatilities that best explained their values were quite as extreme as were the volatilities that best explained the 1st series shares' values. After all, in April-June 1720 it might have been felt that negative returns on South Sea shares were highly unlikely in the medium term, but it probably was not case that people felt that such lopsided volatility was likely to persist for as long as two years. Likewise, in the dark days of autumn 1720 people could probably have been more optimistic about the return of upside volatility in the long term than they could feel optimistic about its reappearance in the short term. Our calculations of subscription values, however, are quite crude in the sense that a uniform set of volatilities is employed to calculate long-term as well as short-term option values. Because this is a constraint imposed on our computations, it would not be surprising to find that the volatility ranges that best result in empirical subscription value "captures" will not be as extreme as they were for the 1st series subscription shares. That is indeed the case, and we consider it one of the more notable successes of our theory and computations, when Table 4 is compared to Table 2 and Table 5 is compared to Table 3. In the April-June period, volatility ranges that allow for more frequent negative returns and smaller and less frequent positive returns better account for the 2nd series values than they do for 1st series values (comparing Table 4 to Table 2). Similarly in autumn 1720, volatility ranges that allow for less frequent and smaller negative returns better account for the 2nd series values than they do for 1st series values (comparing Table 4 to Table 2).

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extreme empirical subscription values.

We conclude this section by noting that there are very few instances in which no captures can be effected with any reasonable volatility range. Even amongst these failures the model is still capable usually of producing one simulated share value that comes close to one of the extreme empirical subscription values. 1st subscription values were as valuable as £540 and worth as little as £130 in 1720. Yet these extreme values and most all values between them can be explained by our model using only the concurrent value of an original share and a reasonable set of returns volatilities. Similarly, the even wider range of 2nd series values (£540 to £15) in 1720 can be explained when using the same original share values and nearly the same volatilities that were used to explain the 1st series values. This is a rather robust performance for a model that has to explain subscription values in both the heady days of early 1720 and in the gloomy days of autumn 1720. These results were obtained despite the use of a very crude model for share returns. The distributions of original share returns were assumed to be constant over the life of our compound options, regardless of whether they were short-term or long-term options. We now present our final conclusions and suggestions for further research.

Conclusions and suggested further research

The South Sea subscription shares of 1720 were compound call options. They were the creation of a law, which when examined closely enough, clearly suggests how the shares' option-like nature can be given precise expression in a theory. From the theory it is but a short step to defining a computational method for theoretical subscription share values that can be compared to their historical values. As crude as our method was, it was still capable of producing the approximate values for subscription shares that were quite close to their values in history. The model performed well for early 1720 and it performed equally well for periods after the bursting of the South Sea Bubble.

Despite our current inability to explain the South Sea Bubble itself – the movement of its fundamental value through the course of 1720 – it has been demonstrated here that the relative pricing of South Sea original shares and subscription shares throughout 1720 was just about right. It is a tradition in much South Sea scholarship to rely upon the "madness of crowds" to explain every phenomenon that does not have an immediately obvious rational explanation. The events of 1720 will not give up their secrets easily, but now that we know there was a functioning financial derivatives market that operated alongside the market for fully-paid South Sea shares there is more hope that some progress will be made in our understanding. We can hope that the South Sea Bubble will attract the interest of more financial economists, as well as financial economic historians, as not only an episode in financial economic history, but also as a scientific problem that someday might actually be solved.

There are several directions in which further research can go. We should next turn our attention to the small amounts of data we have that pertain to the 3rd and 4th subscriptions to see if they conform to the theory presented in this paper. Then there are some broader South Sea questions on which some progress might be made with the help of data from financial

derivatives markets. Finding the actual path of the South Sea Bubble itself should be a priority. Locating spot values for original shares in the crucial midsummer period will be a true feat in financial archaeology, but perhaps data on financial derivatives values might be of some help. Financial derivatives textbooks, for example, instruct their readers in how synthetic forward contracts on assets can be constructed in portfolios that contain positions in derivatives and other assets. It would be straightforward to modify the exercise to create or estimate synthetic spot values for shares from data on their forward and option contract values. Another thing we can do with financial derivatives is to examine their relative values - an exercise that can be of some use in estimating implied volatilities of share returns. A time series of time-changing volatilities in returns could be just as useful way of measuring the progress of the South Sea Bubble as would be looking at a time path of original share values. Now that we know finally what the South Sea subscription shares were actually, more refined applications of option pricing theory may help solve some of the outstanding puzzles and help create new puzzles concerning the South Sea Bubble.

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6 Geo. 1, c. 18
6 Geo 4, c. 67

Tables and figures

First subscription shares instalment schedule			
	14 April 1720	£60	
	14 June 1720	£30	
	15 August 1720	£30	
	14 October 1720	£30	
	14 December 1720	£30	
	14 February 1721	£30	
	14 April 1721	£30	
	14 June 1721	£30	
	14 August 1721	£30	

Second subscription shares instalment schedule			
	29 April 1720	£40	
	14 September 1720	£40	
	14 January 1721	£40	
	14 May 1721	£40	
	14 September 1721	£40	
	14 December 1721	£40	
	14 March 1722	£40	
	14 June 1722	£40	
	14 September 1722	£40	
	14 December 1722	£40	

Table 1: Instalment schedules for the 1st and 2nd South Sea subscription shares

Source: BL, Add. MS 25,499, Minutes of the General Court, 19/20 & 29 April 1720. See text in supplementary appendix 2.

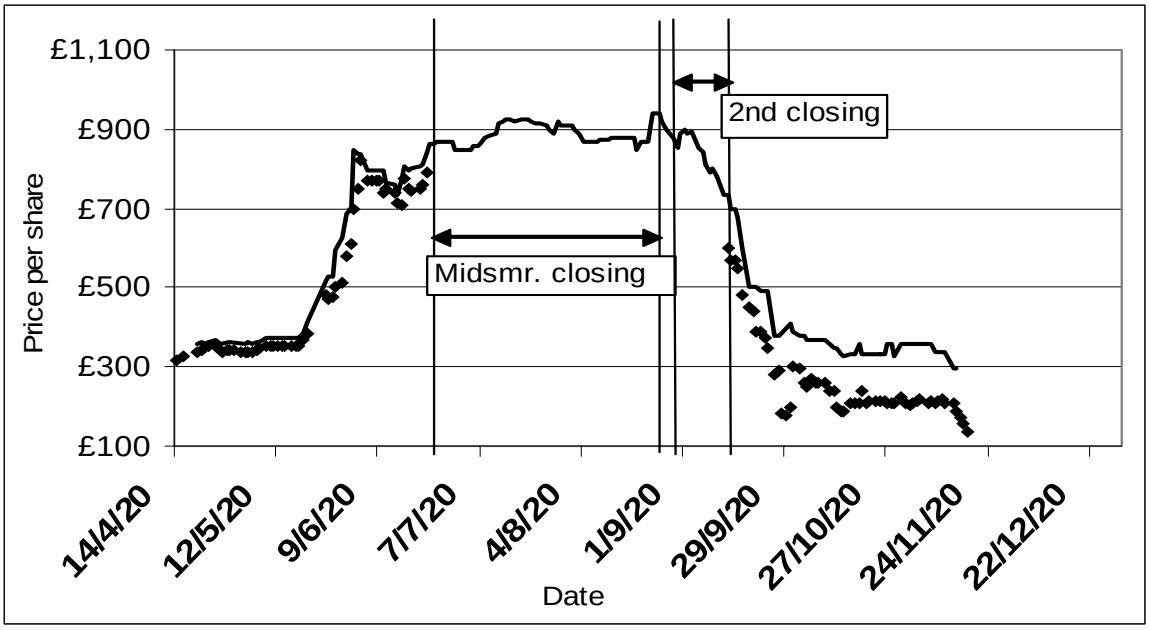

Figure 1. South Sea original and 1st. subscription share values, 1720

Source: Freke, *Prices of stocks* and HLRO Box 158 (see text)

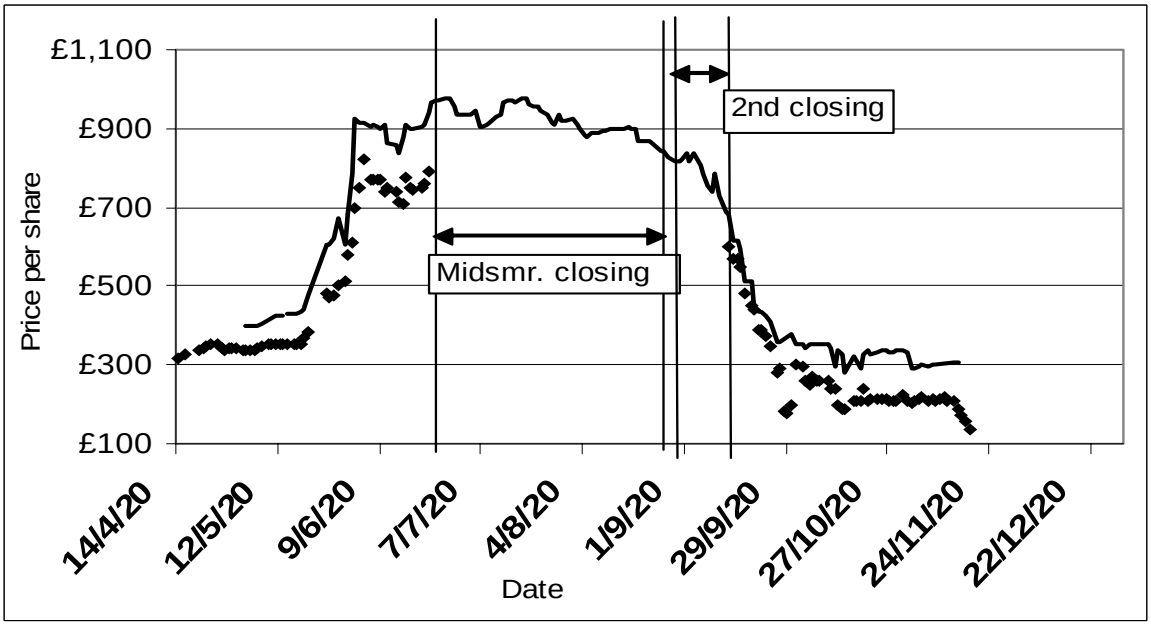

Figure 2. South Sea original and 2nd. subscription share values, 1720

Source: Freke, *Prices of stocks* and HLRO Box 158 (see text)

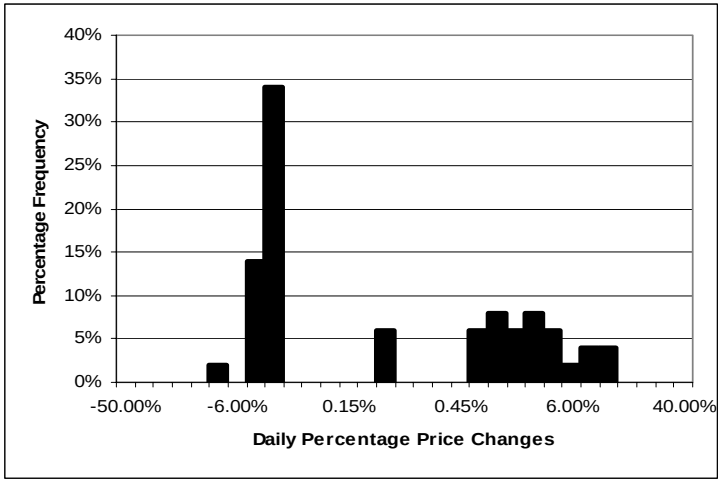

Figure 3: Discrete distribution of daily South Sea original share price changes, 14 Apr - 22 Jun 1720

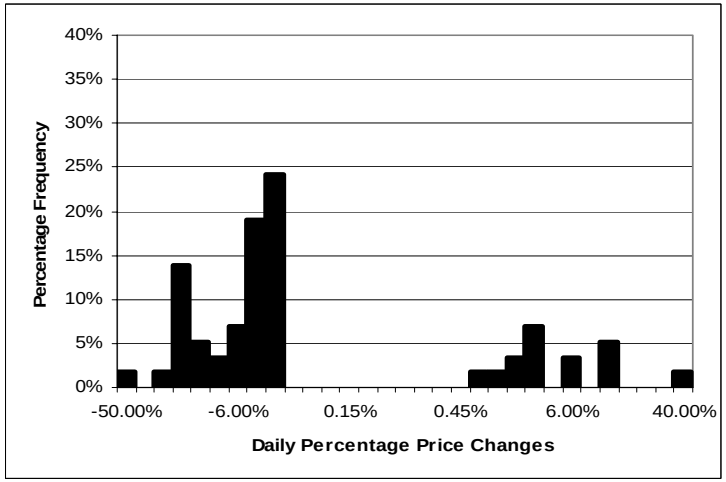

Figure 4: Discrete distribution of daily South Sea original share price changes, 13 Sep - 18 Nov 1720

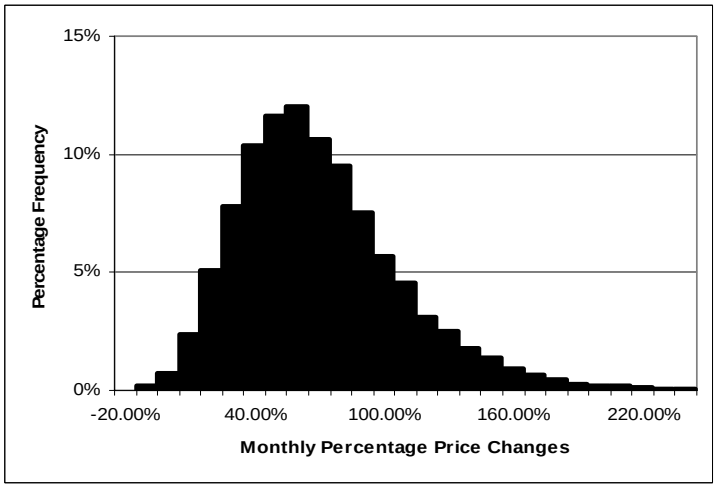

Figure 5: Discrete distribution of 30-day cumulated daily price changes, 10,000 Monte Carlo replications, 14 Apr - 22 Jun 1720

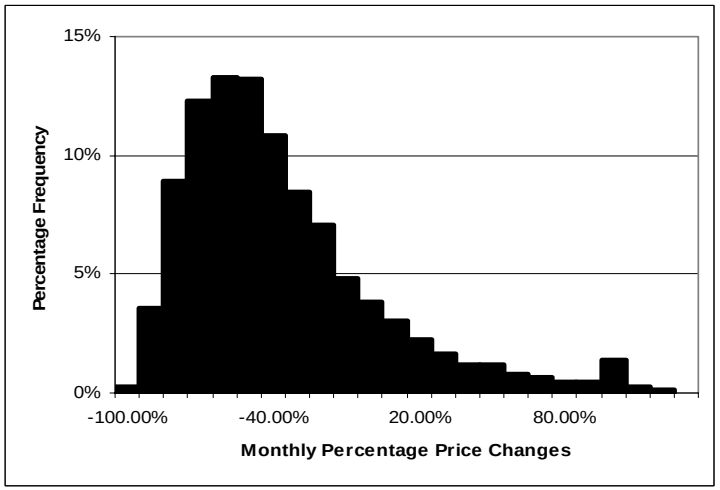

Figure 6: Discrete distribution of 30-day cumulated daily price changes, 10,000 Monte Carlo replications, 13 Sep - 18 Nov 1720

Table 2: Simulated 1st subscription share values, 20 Apr - 22 Jun 1720

Date	Original share value	Minimum observed 1st subscription value	Ranges of monthly volatility				Maximum observed 1st subscription value
			160 p.c. (u-1) to -1 p.c. (1-d)	160 p.c. (u-1) to -10 p.c. (1-d)	160 p.c. (u-1) to -20 p.c. (1-d)	20 p.c. (u-1) to -20 p.c. (1-d)	
20/04/20	£336	£60	£87	£123	£153	£114	£65
29/04/20	£337	£67	£88	£122	£151	£115	£75
07/05/20	£334	£70	£84	£118	£147	£111	£70
16/05/20	£352	£79	£102	£129	£157	£126	£80
23/05/20	£487	£220	£237	£254	£274	£258	£220
25/05/20	£480	£215	**£230**	£248	£267	£250	£230
26/05/20	£470	£230	**£220**	**£238**	£258	£241	£230
27/05/20	£474	£230	**£224**	**£242**	£261	£244	£230
28/05/20	£510	£270	**£260**	**£277**	£294	£280	£300
30/05/20	£555	£320	**£305**	**£322**	£336	£325	£340
31/05/20	£600	£376	£350	**£367**	**£379**	**£370**	£395
01/06/20	£675	£400	**£425**	**£442**	**£452**	**£445**	£525
02/06/20	£700	£540	£450	£467	£476	£470	£600
03/06/20	£750	£540	£500	£517	£525	£520	£550
04/06/20	£770	£510	£520	**£537**	**£544**	**£540**	£540
06/06/20	£770	£500	**£519**	**£537**	£543	£540	£520
07/06/20	£760	£500	**£509**	**£527**	£533	£530	£525
08/06/20	£760	£470	£509	£527	£533	£530	£500
09/06/20	£740	£480	**£489**	**£507**	**£513**	£510	£500
10/06/20	£740	£500	**£489**	**£507**	£513	£510	£500
11/06/20	£750	£470	**£499**	**£517**	**£523**	£520	£510
13/06/20	£740	£465	**£489**	£507	£513	£510	£490
14/06/20	£715	£440	**£464**	**£482**	**£488**	**£485**	£485
15/06/20	£710	£480	£459	£477	**£483**	**£480**	£500
16/06/20	£755	£450	**£504**	**£522**	**£527**	**£524**	£540
17/06/20	£750	£480	**£499**	**£517**	**£522**	**£519**	£520
18/06/20	£745	£505	**£494**	**£512**	£517	£514	£510
20/06/20	£750	£500	**£499**	**£517**	£522	£519	£510
21/06/20	£760	£500	**£509**	**£527**	£532	£529	£520
22/06/20	£790	£520	**£539**	**£557**	£562	£559	£550

Table 3: Simulated 1st subscription share values, 14 Sep - 15 Nov 1720

Date	Original share value	Minimum observed 1st subscription value	Ranges of monthly volatility				Maximum observed 1st subscription value
			70 p.c. (u-1) to -99 p.c. (1-d)	70 p.c. (u-1) to -95 p.c. (1-d)	60 p.c. (u-1) to -95 p.c. (1-d)	60 p.c. (u-1) to -90 p.c. (1-d)	
14/09/20	£570	£460	£492	**£460**	£453	£434	£460
15/09/20	£570	£460	£491	**£460**	£453	£434	£460
16/09/20	£550	£440	£472	**£441**	**£434**	£415	£440
17/09/20	£480	£370	£404	**£374**	**£369**	£349	£370
19/09/20	£450	£280	£375	£346	£340	£321	£280
20/09/20	£440	£280	£365	£336	£331	£311	£280
21/09/20	£390	£280	£318	£290	**£284**	**£265**	£280
22/09/20	£390	£270	£314	£287	**£281**	**£264**	£270
23/09/20	£375	£270	£314	£287	**£280**	**£264**	£270
24/09/20	£350	£270	£314	£287	**£280**	**£263**	£270
26/09/20	£280	£170	£209	£185	**£179**	**£166**	£170
27/09/20	£290	£170	£209	£185	**£179**	**£165**	£170
30/09/20	£200	£200	£134	£113	£108	£94	£200
01/10/20	£300	£180	£227	£202	£196	£182	£180
03/10/20	£295	£170	£221	£197	£191	£177	£170
04/10/20	£260	£170	**£188**	**£165**	£159	£146	£170
05/10/20	£250	£160	**£179**	**£156**	£149	£137	£160
06/10/20	£270	£160	£197	£174	**£167**	**£154**	£160
07/10/20	£260	£160	£187	**£164**	**£158**	£145	£160
08/10/20	£260	£160	£187	**£164**	**£158**	£145	£160
10/10/20	£260	£160	**£181**	**£158**	£152	£138	£160
11/10/20	£240	£150	**£163**	**£140**	£134	£120	£150
12/10/20	£240	£140	**£163**	**£140**	**£134**	£120	£140
13/10/20	£200	£140	£129	£106	£100	£85	£140
14/10/20	£190	£130	£120	£97	£92	£78	£130
15/10/20	£190	£125	£120	£97	£92	£77	£125
17/10/20	£210	£130	**£136**	**£113**	£107	£93	£130
18/10/20	£210	£130	**£136**	**£113**	£107	£92	£130
19/10/20	£210	£150	£136	£112	£107	£92	£150
20/10/20	£240	£130	£161	£137	**£132**	**£118**	£130
21/10/20	£210	£130	**£135**	**£112**	£106	£92	£153
22/10/20	£215	£130	**£136**	**£112**	£107	£92	£130
24/10/20	£215	£130	**£136**	**£112**	£107	£91	£130
25/10/20	£215	£130	**£136**	**£111**	£106	£91	£130
26/10/20	£212	£130	**£133**	**£109**	£104	£88	£130
27/10/20	£210	£120	**£131**	**£107**	£102	£87	£120
28/10/20	£210	£120	**£131**	**£107**	£102	£86	£120
29/10/20	£208	£125	**£129**	**£105**	£100	£85	£125
31/10/20	£222	£120	**£136**	**£115**	£109	£94	£120
01/11/20	£210	£120	**£125**	**£105**	£99	£85	£120
02/11/20	£205	£120	**£120**	£100	£94	£81	£120
03/11/20	£209	£120	**£124**	**£103**	£97	£84	£120
04/11/20	£212	£120	**£126**	**£106**	£100	£86	£130
05/11/20	£220	£120	**£133**	**£112**	£106	£91	£120
07/11/20	£210	£120	**£123**	**£103**	£97	£84	£120
08/11/20	£214	£120	**£127**	**£106**	£100	£86	£120
09/11/20	£209	£100	£122	**£102**	**£96**	£82	£100
10/11/20	£211	£120	**£124**	**£103**	£97	£84	£120
11/11/20	£216	£120	**£128**	**£107**	£101	£87	£120
12/11/20	£210	£100	£122	**£102**	**£96**	£82	£100
14/11/20	£210	£145	**£148**	**£129**	£123	£110	£145
15/11/20	£190	£145	£129	£112	£105	£94	£145

Table 4: Simulated 2nd subscription share values, 20 Apr - 22 Jun 1720

Date	Original share value	Minimum observed 2nd subscription value	Ranges of monthly volatility				Maximum observed 2nd subscription value
			160 p.c. (u-1) to -1 p.c. (1-d)	160 p.c. (u-1) to -10 p.c. (1-d)	160 p.c. (u-1) to -20 p.c. (1-d)	20 p.c. (u-1) to -20 p.c. (1-d)	
03/05/20	£337	£15	£21	£108	£168	£74	£20
04/05/20	£337	£18	£21	£108	£168	£74	£18
05/05/20	£336	£18	£21	£107	£167	£73	£19
06/05/20	£336	£16	£21	£107	£167	£73	£20
07/05/20	£334	£22	**£21**	**£106**	£165	£72	£22
09/05/20	£346	£30	**£22**	**£114**	£174	£79	£32
11/05/20	£351	£30	**£22**	**£117**	£177	£81	£40
18/05/20	£351	£50	**£22**	**£116**	£176	£81	£60
19/05/20	£370	£55	**£36**	**£129**	£190	£93	£75
20/05/20	£381	£90	**£47**	**£137**	£198	£102	£90
24/05/20	£476	£215	£142	**£205**	**£275**	**£177**	£220
25/05/20	£480	£220	£146	**£207**	**£279**	**£181**	£220
26/05/20	£470	£220	£136	**£200**	**£270**	**£172**	£220
27/05/20	£474	£235	£140	**£203**	**£273**	**£176**	£235
28/05/20	£510	£275	£176	**£223**	**£295**	**£196**	£290
30/05/20	£555	£220	£221	£231	**£303**	**£205**	£325
31/05/20	£600	£300	£266	**£290**	**£364**	**£268**	£420
01/06/20	£675	£400	£341	£315	£389	£296	£500
02/06/20	£700	£540	£366	£398	£468	£380	£600
03/06/20	£750	£530	£378	£445	£513	£427	£560
04/06/20	£770	£520	£436	**£464**	**£531**	**£446**	£530
06/06/20	£770	£520	£436	**£464**	**£531**	**£446**	£545
07/06/20	£760	£520	£426	**£464**	**£531**	**£446**	£540
08/06/20	£760	£490	£426	**£464**	**£531**	**£446**	£525
09/06/20	£740	£510	£406	**£464**	**£530**	**£446**	£515
10/06/20	£740	£525	£406	£435	£503	£417	£525
11/06/20	£750	£480	£416	**£445**	**£512**	**£426**	£525
13/06/20	£740	£475	£378	**£435**	**£502**	**£417**	£500
14/06/20	£715	£455	£381	**£411**	**£479**	**£393**	£510
15/06/20	£710	£495	£376	£406	£474	£388	£520
16/06/20	£755	£465	£421	**£468**	**£533**	**£450**	£525
17/06/20	£750	£490	£416	**£444**	**£510**	**£426**	£515
18/06/20	£745	£520	£411	£439	£506	£421	£525
20/06/20	£750	£520	£416	£443	£510	£426	£525
21/06/20	£760	£510	£426	**£453**	**£519**	**£435**	£535
22/06/20	£790	£530	£456	**£481**	**£546**	**£464**	£565

Table 5: Simulated 2nd subscription share values, 14 Sep - 18 Nov 1720

Date	Original share value	Minimum observed 2nd subscription value	Ranges of monthly volatility				Maximum observed 2nd subscription value
			60 p.c. (u-1) to -70 p.c. (1-d)	60 p.c. (u-1) to -50 p.c. (1-d)	20 p.c. (u-1) to -50 p.c. (1-d)	20 p.c. (u-1) to -20 p.c. (1-d)	
14/09/20	£570	£310	£455	£379	**£317**	**£252**	£310
16/09/20	£550	£290	£431	£378	**£297**	**£233**	£290
17/09/20	£480	£210	£365	£315	**£239**	**£171**	£210
19/09/20	£450	£210	£338	£288	**£215**	**£146**	£210
20/09/20	£440	£140	£329	£279	**£206**	**£137**	£200
21/09/20	£390	£130	£284	£234	**£166**	**£97**	£130
23/09/20	£375	£110	£270	£221	**£154**	**£86**	£120
24/09/20	£350	£110	£248	£199	**£134**	**£67**	£110
26/09/20	£280	£60	£185	£139	**£78**	**£29**	£60
27/09/20	£290	£60	£194	£147	**£86**	**£34**	£70
30/09/20	£200	£50	**£116**	**£83**	**£34**	£4	£100
01/10/20	£300	£60	£202	£155	**£93**	**£38**	£60
03/10/20	£295	£60	£197	£150	**£89**	**£36**	£60
04/10/20	£260	£50	£166	£122	**£62**	**£19**	£50
05/10/20	£250	£60	£156	£115	**£56**	**£13**	£60
06/10/20	£270	£60	£173	£129	**£66**	**£22**	£60
07/10/20	£260	£50	£164	£122	**£61**	**£17**	£60
08/10/20	£260	£60	£164	£121	**£61**	**£17**	£60
10/10/20	£260	£60	£157	**£119**	**£60**	£15	£60
11/10/20	£240	£50	£139	**£104**	**£49**	£9	£55
12/10/20	£240	£45	£139	£103	**£49**	**£9**	£45
13/10/20	£200	£50	£105	**£73**	**£29**	£0	£50
14/10/20	£190	£40	£96	**£65**	**£24**	£0	£50
19/10/20	£210	£46	£113	**£80**	**£33**	£2	£46
20/10/20	£240	£40	£138	£102	**£48**	**£9**	£40
21/10/20	£210	£50	£112	**£79**	**£33**	£2	£50
22/10/20	£215	£45	£116	**£83**	**£35**	£3	£45
26/10/20	£212	£50	£113	**£80**	**£34**	£2	£50
28/10/20	£210	£45	£111	**£78**	**£32**	£2	£55
29/10/20	£208	£50	£109	**£77**	**£31**	£1	£50
31/10/20	£222	£50	£121	**£87**	**£38**	£4	£50
01/11/20	£210	£45	£111	**£78**	**£32**	£2	£50
02/11/20	£205	£45	£106	**£74**	**£29**	£1	£45
03/11/20	£209	£45	£110	**£77**	**£31**	£1	£45
04/11/20	£212	£45	£108	**£73**	**£31**	£1	£50
05/11/20	£220	£50	£115	**£79**	**£35**	£3	£50
07/11/20	£210	£45	£106	**£72**	**£29**	£1	£45
08/11/20	£214	£40	£110	**£74**	**£31**	£1	£45
09/11/20	£209	£45	£105	**£71**	**£29**	£0	£45
10/11/20	£211	£40	£107	**£72**	**£30**	£1	£40
11/11/20	£216	£40	£111	**£75**	**£32**	£2	£40
14/11/20	£210	£40	£102	**£64**	**£23**	£0	£40
15/11/20	£190	£25	£87	**£54**	**£11**	£0	£40
18/11/20	£175	£15	£75	£52	**£15**	**£6**	£15

Supplementary Appendices

Supplementary Appendix 1: Concerning Additional South Sea Stock (Subscriptions) – the passage from 6 George 1, c. 4 (April 1720)

And for the better enabling the said Governor and Company of Merchants of Great Britain, and their Successors, to raise Money to be Paid, for or in part of the said Sum of Four million one hundred fifty six thousand three hundred and six Pounds, four Shillings, and Eleven Pence, or for or in part of the said Sums to be paid after the said Rates of Four Years and an half Purchase, and One Years Purchase respectively, or for Purchasing or Paying off all or any the Annuities and Debts to be taken in or paid off in pursuance of this Act, or for exchanging for Ready Money the New Exchequer Bills to be made forth, as hereafter in this Act is mentioned, or for defraying the Interest thereof, or for carrying on their Trade, and other necessary Occasions: Be it further Enacted by the Authority aforesaid, That it shall and may be Lawful to and for the said Governor and Company of Merchants of Great Britain, and their Successors, from time to time, as they shall see cause, to call in or direct to be paid from and by their respective Members for the time being, proportionably according to their respective Interests in the Capital stock or Stocks which do or shall belong to the said Governor and Company of Merchants of Great Britain, and which shall be increased, as aforesaid, or by Opening Books of Subscription, or by Granting Annuities redeemable by the same Governor and Company, and their Successors, or by any other Method, Ways, and Means as they shall think proper to raise any Sum or Sums of Money, as in a General Court of the same Governor and Company shall, from time to time, be judged necessary, and ordered to be called in, or raised; and that all Executors, Administrators, Guardians, Trustees, and Mortgagees, shall be indemnified in paying, and are hereby impowered to pay in their respective Proportions of the Money so called in or raised; and in case any such Member or Members shall refuse or Neglect to pay his, her or their Share of the said Money so called for that Purpose, by Notice inserted in the London Gazette, and fixed upon the Royal Exchange in London, It shall and may be Lawful to and for the said Governor and Company of Merchants of Great Britain, and their Successors, not only to stop the Share, Dividend, Annuity, and Profits, which shall, from time to time, become payable to such Member or Members so neglecting or refusing, of the Funds, Stocks, Annuities, or Profits of the said Governor and Company of Merchants of Great Britain, and to apply the same, from time to time, for or towards Payment of the Share of the

Money so called for, and which ought to have been paid by such member or Members, so neglecting or Refusing, until the same shall be satisfied, but also to stop the Transfers or Assignments of the Share and Shares of every such Defaulter and Defaulters with Interest, after the Rate of Five Pounds per Centum per Annum, for the Money so by him, her or them omitted to be paid, from the time the same was appointed to be paid until the Payment thereof; and that the Share and Stock, Shares and Stocks of such Defaulter and Defaulters shall be liable to make good and answer the said Monies so appointed to be paid, and the Interest thereof, as aforesaid; and in case the Principal and Interest, as aforesaid shall be unpaid by the space of Three Months, then the said Governor and Company of Merchants of Great Britain, or their Successors, or their Court of Directors for the time being, shall have Power to Authorize such Person or Persons as they shall think fit, to Sell, Assign, and Transfer so much of the said Stock or Stocks of such Defaulter or Defaulters as will satisfy and pay the same, rendring the Overplus (if any be) to the Proprietor; And the said Governor and Company of Merchants of Great Britain, or their Successors, in a General Court, from time to time, when they shall judge their Affairs will admit thereof, shall or may cause any sum or Sums of Money which shall be called in, or any Part thereof, to be divided and distributed to and amongst the then members of that Corporation according and in proportion to their respective Interests in the Capital Stock or Stocks of the same; And former Law or Stature, Restriction, or other Matter or Thing whatsoever to the contrary notwithstanding.

Supplementary Appendix 2: South Sea subscription shares and instalment payments

In this Appendix is described the mechanics of the South Sea Company's Subscription issues. There were four issues. The first two issues in April 1720 provide the most useable data. The third and fourth subscriptions priced shares initially at £1000 each and were quickly 'out-of the-money' as share prices declined. The fourth subscription indeed offers very little in the way of data. Each issue is described below along with its instalment schedules. At the end of the Appendix we explain how the present values of instalments are calculated.

FIRST and SECOND SUBSCRIPTION ISSUES

The first subscription issue was planned soon after 6 Geo. 1, c. 4 came into force on 7 April 1720. Formal issues began in the week of 14 April and in the Court Minutes of the 19 and 20 April the management of the first issue and the newly planned second issue were discussed. The subscription price was set to £300 to be paid in 9 instalments. The first issue's instalment schedule was thus:

1st Subscription Shares Instalment Schedule	
14 April 1720	£60
14 June 1720	£30
15 August 1720	£30
14 October 1720	£30
14 December 1720	£30
14 February 1721	£30
14 April 1721	£30
14 June 1721	£30
14 August 1721	£30

In the 29th April meetings of the Company's General Court a schedule for the 2nd Subscription was determined. The subscription price was £400 to be paid in 10 equal instalments.

2nd Subscription Shares Instalment Schedule	
29 April 1720	£40
14 September 1720	£40
14 January 1721	£40
14 May 1721	£40
14 September 1721	£40
14 December 1721	£40
14 March 1722	£40
14 June 1722	£40
14 September 1722	£40
14 December 1722	£40

These schedules were printed from time to time, and with more or less accuracy, in *The course* from mid-July 1720, but it was not until the 9 August that *The course* began to print accurate instalment schedules. The first mention of instalment schedules appears in Freke's *Prices of stocks* (No. 75, 24 June 1720) and a detailed set of schedules for Subscriptions No. 1 through 3 appears in the next issue and subsequent issues. On 13 October the Court of Directors thought it advisable (and in discussions with the Bank of England the latter did concur) to delay the payment of the fourth instalment on the First Subscription till 15 November with interest to be charged at 5 p.c. p.a.[22]

THIRD and FOURTH SUBSCRIPTION ISSUES

The last two series of subscription shares were issues of expensive shares. South Sea share values were at their peak and the subscription price was set at £1000 for both issues. The first accurate reporting of the instalment schedule for the 3rd Subscription does not

[22] Add. Ms. 25,499, 13 October 1720, p. 7. Freke's *Prices of stocks* reports this change in the instalment schedule the next day and thereafter from Issue No.107.

appear in *The course* until the 9 August issue, but the schedule printed there was adopted on 15 June.[23] That schedule was:

3rd Subscription Shares Instalment Schedule, 15 June 1720	
16 June 1720	£100
2 January 1721	£100
2 July 1721	£100
2 January 1722	£100
2 July 1722	£100
2 January 1723	£100
2 July 1723	£100
2 January 1724	£100
2 July 1724	£100
2 January 1725	£100

By mid-August 1720 it was clear that the firm was starved for cash. There was a noticeable panic in the measures the Directors proposed for raising more cash. The Court Minutes record a proposed unworkable scheme for getting subscribers to pre-pay their instalments and then at the same meeting the 4th and final cash subscription of shares was proposed, again at a subscription price of £1000 and with the following instalment schedule.[24]

4th Subscription Shares Instalment Schedule, 12 August 1720	
25 September 1720	£200
25 August 1721	£200
25 August 1722	£100
25 August 1723	£100
25 August 1724	£100
25 February 1725	£100
25 August 1725	£100
25 February 1726	£100

[23] Add. Ms. 25,499, 15 June 1720, p.136. Freke's *Prices of stocks* reports a rough instalment schedule until, in the 2 August issue (No.86), there appears a fairly accurate version of the schedule adopted on 15 June.
[24] Add. Ms. 25,499, 12 August 1720, pp. 158-164.

At the same meeting, however, they also decided to amend the instalment schedule for the 3rd Subscription (£100 supposedly already paid). One indication that the 3rd subscription issue was failing was that the Company allowed a 25 September deposit date for the first £100 to those individuals who had not yet paid their deposit:

3rd Subscription Shares Instalment Schedule, 12 August 1720	
25 September 1720	£100
15 August 1721	£100
15 August 1722	£100
25 August 1723	£100
25 August 1724	£100
25 February 1725	£100
25 August 1725	£100
25 February 1726	£100
25 August 1726	£100
25 February 1727	£100

Neither *The course* nor *Prices of stocks* ever displays this revised schedule. Both price courants continue to print the 15 June-published schedule until the Company radically alters the terms of the 3rd and 4th Subscriptions on 30 September.

The Company soon changed its mind yet again with respect to the 4th Subscription and put forward the following revised shortened instalment schedule[25], which would again be quickly relengthened:

4th Subscription Shares Instalment Schedule, 23 August 1720	
at subscription	£200
25 March 1721	£200
25 September 1721	£200
25 March 1722	£200
25 September 1722	£200

[25] This schedule appears briefly in the *Price of stocks* from 26 August (No. 93) through 13 September (Numb. 98).

On 25 August the Court resolved that the 4th Subscription would follow the same instalment schedule followed by the 3rd Subscription. It did not say what those shared schedules would be. In the September 16 issue (Numb. 99) of Freke's *Price of Stocks* a new schedule for the 4th Subscription is shown alongside Freke's long-running 15-June schedule for the 3rd Subscription. The two schedules are almost the same and may have accurately reflected the adjustments referred to in the 25 August *Court minutes*. The newly reported schedule for the 4th Subscriptions instalments was:

4th Subscription Shares Instalment Schedule, reported Freke, 16 September 1720	
August 1720	£200
February 1721	£100
August 1721	£100
February 1722	£100
August 1722	£100
February 1723	£100
August 1723	£100
February 1724	£100
August 1724	£100

The Court Minutes recorded (25 August) that something had to be done to make the payment schedules on the 3rd and 4th Subscriptions more equitable, but we do not see a resolution of this issue until after share values have largely collapsed. At the Court of Directors on 30 September the 3rd and 4th Subscriptions' subscription prices were announced to both be revised downwards to £400 p.s. Two new instalment schedules were devised as shown below.[26] The revised terms and schedules appeared in the respective 4 October and 7 October issues of *The course* and in the 4 October issue (No. 104) of *Prices of stocks*.

[26] Add. Ms. 25,499, 30 September 1720, pp. 201-2.

3rd Subscription Shares Instalment Schedule, 30 September 1720	
16 June 1720	£100
2 July 1721	£40
2 January 1722	£40
2 July 1722	£40
2 January 1723	£30
2 July 1723	£30
2 January 1724	£30
2 July 1724	£30
2 January 1725	£30
2 July 1725	£30

and the revised instalment schedule for the 4th Subscription became:

4th Subscription Shares Instalment Schedule, 30 September 1720	
August 1720	£200
26 September 1721	£25
26 March 1722	£25
26 September 1722	£25
26 March 1723	£25
26 September 1723	£25
26 March 1724	£25
26 September 1724	£25
26 March 1725	£25

Supplementary Appendix 3: Computational programs for South Sea subscription share values

In this appendix we describe the two programs that we use to generate our results. One program is for simulating the 1st subscription share values and is entitled: "Binomial Option pricing subscription shares 1st SS Sub.xls". The second is "Binomial Option pricing subscription shares 2nd SS Sub.xls" and is for simulating the values of subscription shares of the 2nd series. These notes are provided to supply enough information so that an interested reader can run the programs and understand the results presented in the main body of this paper.

The figure below is of what we might call the control panel of the spreadsheet programme that will compute 1st series subscription share values for us. The program statements and the very 'look and feel' of the program are very much like those of option pricing examples contained in any of the editions of Holden's textbooks on financial spreadsheet programming.[27]

[27] See for example Holden, Excel Modelling in Corporate Finance. A study of Holden's construction of the spreadsheet he entitles "BINOMULT.xls" will reveal the programming principles upon which the current spreadsheet is based.

Binomial Pricing for 1st series South Sea Subscription Shares

Date	6/22/20
Expiration (last instalment) date	8/14/21
South Sea Subscription Price	£520.00
South Sea Original Share Price	£790.00
Up Movement/Month (%)	160.00%
Down Movement/Month (%)	-10.00%
Risk-free @ 5 p.c. p.a.	0.20%
Subscription Share Issue Price	£300.00
Time to Expiration (Years)+ 3 months	1.39
Number of instalments + deposit	10
Modeling Periods	34

1st Sub Calls	Yearfrac + 3 months
6/14/20	0.23
8/15/20	0.40
10/14/20	0.56
12/14/20	0.73
2/14/21	0.89
4/14/21	1.06
6/14/21	1.23
8/14/21	1.39

Up Movement/Period (u-1)	60.04%
Down Movement/Period (1-d)	-5.05%
Theoretical-Actual Price Difference	£36.69
Instalment Size	£30.00
PVcalls	£293.53
Theoretcial Subscription Price	£556.69
Theoretical Subscription Price + PVcalls	£850.22

Figure A.1.1: Control panel for South Sea 1st subscription share value simulation program

The control panel illustrated in the figure contains the input data and control parameters for the simulation program. The time to each instalment payment (plus three months to account for the option that subscribers had to withhold payment for three-months before forfeiture could be enforced) is calculated in years. The time to the last instalment is then divided into 34 sections of what we might call modelling time. In the example above 1.39 years is divided into 34 equally size time intervals. In another worksheet in the program ("Stock Evolution") a binomial random walk in share values, starting with the current value (£790), is generated over the 34 modelling periods. Note that amongst the programme inputs are monthly percentage potential movements "up" or "down" for the share values. These are automatically converted into their modelling-period equivalents that are used to generate the "Stock Evolution" random walk. The real-time instalment schedule for the subscription share pictured in the control panel must also be converted into its modelling-period equivalent; this is also stored in the "Stock Evolution" worksheet as

well. The workbook also consists of another 35 worksheets; each of these have titles such as “Period n-34”, “Period n-33” and so forth.

In the first of these worksheets (“Period n-34”) the binomial random walk for call option values on an original share is calculated. This is the call option on an original share that is discussed in Proposition 1 in the main body of the paper. In each of the successive worksheets a binomial random walk is calculated for each of the program’s 34 successive modelling periods. If no instalment payment is due in a particular modelling period corresponding to such a worksheet, then the binomial random walk found in that worksheet is simply the continuation of the binomial random walk found in the previous sheet. If, however, an instalment payment is due, then the binomial random walk calculated will be for the values of the next call option in the compound call option sequence. In the example above there are only 8 instalments due to be paid and thus in only 8 of the entire 34 modelling periods will instalments be due.

The programs first appear to be complex, but the user must remember that most of their work is performed by one simple algorithm that is employed again and again in a highly recursive fashion. Care must be exercised in modifying inputs to value any particular subscription share, but guidance is provided in a number of cell comments that the user can open and read. Users may also correspond with the author to deal with any other problems they might encounter and of course to report any errors that they find. The author does not warrant that these programs are useful for any other purpose other than for producing the simulation results that are reported in the main body of this paper.

ABOUT THE CDMA

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