Self-organization versus top-down planning in the evolution of a city – Supplementary Information

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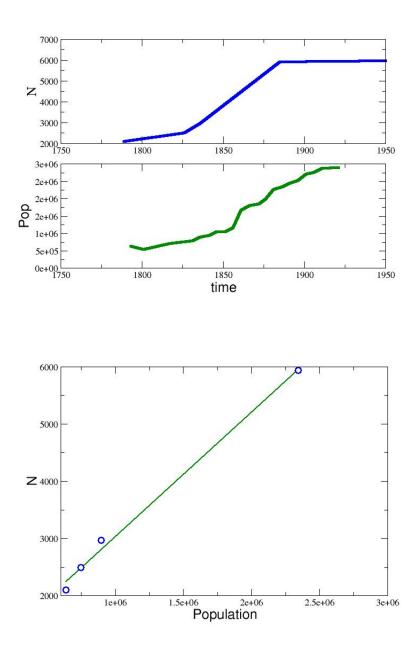
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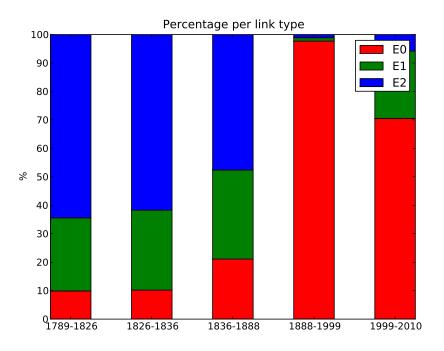
1 Population and nodes

In figure SI1, we show the evolution of the number of nodes and of the population of Paris (for the 12 districts delimited by the 'fermiers generaux' for the period 1789-1851 and for the 20th districts of Paris after). The area under consideration for the calculation of the population is not exactly the same, and only the order of magnitude can be trusted here. We can compute the number of nodes N versus the population P and we observe a linear dependence (with coefficient dN/dP = 0.0021) in agreement with previous studies on different areas (Groane region in Italy [refGroane], where the linear coefficient was found to be equal to dN/dP = 0.019). It is thus clear

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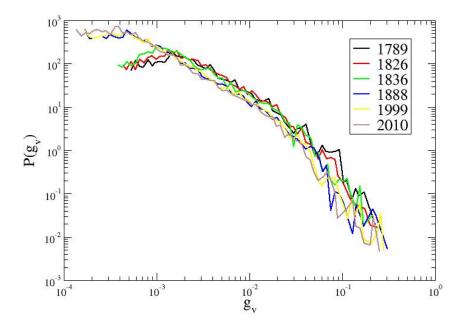
Supplementary Figure 1: Top panel: Evolution of the number of nodes versus time for Paris. Middle panel: evolution of the Paris population. Bottom panel: Number of nodes versus population. The line is a linear fit ($r^2 > 0.99$).



Supplementary Figure 2: Evolution of the percentage of different types of links. E_i corresponds to new links with creation of *i* new nodes. The denser the network, the smaller E_2 and in particular, we observe that the Haussmann period is not radically different in this respect from other periods. that the number of nodes follows the demographic population and that the large increase observed during the Haussmann period is largely due to the demographic pressure.

2 Type of new links

In figure SI2, we show the evolution of the proportion of the different types of new links. We see in this figure that the evolution is rather smooth and that from this point of view, the Haussmann period is not radically different from previous ones.



Supplementary Figure 3: Vertex BC distribution for all links and all periods. Vertex BC probability distribution for the different time snapshots considered in this study. We note that the average BC decreases (indicating a larger navigability in the system) and that the overall shape and tail remain the same across all times.

3 Stability of the BC distribution

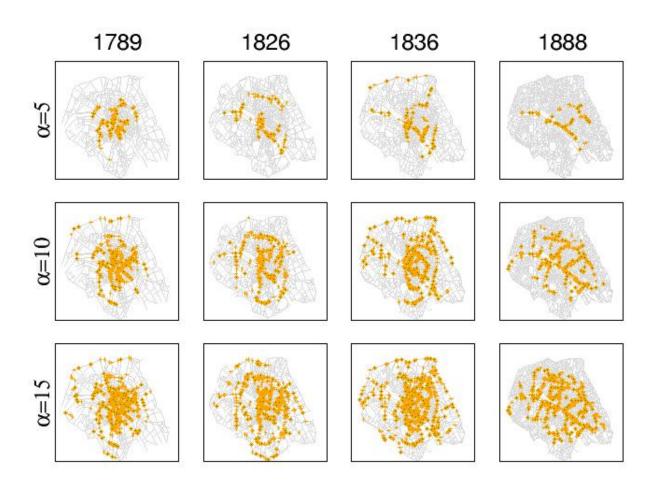
We consider here the evolution of the vertex BC with time. In figure 3 of this SI, we see that the average BC decreases slightly and that the overall probability distribution remains constant in time.

4 Most central nodes: stability of spatial patterns

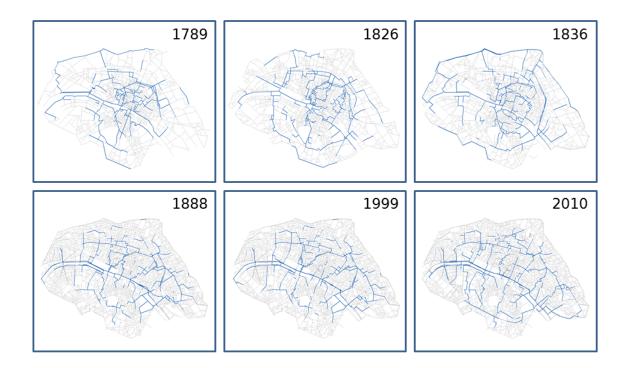
The most central nodes are such as their centrality is $g_v > \max g_v / \alpha$. In the article we consider $\alpha = 10$ and we show in figure SI4 the results for $\alpha = 5, 10, 15$. A visual inspection shows that the patterns are rather robust versus α and that $\alpha = 10$ corresponds to an intermediate situation displaying interesting patterns.

5 Spatial pattern of the most central edges

Instead of the most central nodes, we can also represent the most central edges such that their centrality is $g_e > \max g_e/\alpha$. If we consider here $\alpha = 20$ we obtain for the different dates the results presented in Fig. SI5. We can see that the pattern for the edges is naturally consistent with the one obtained with the node centrality.



Supplementary Figure 4: Spatial patterns of most central points defined by $g_v > \max g_v/\alpha$ for different values of α . We see that for the 4 time points considered here that the pattern is robust with respect to the value of α .



Supplementary Figure 5: Spatial patterns of most central edges defined by $g_e > \max g_e/\alpha$ for $\alpha = 20$.