Numerical Implementation of a Susceptible - Infected - Recovered (SIR) Mathematical Model of Covid-19 Disease in Nigeria

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Abstract: - In this study, we examine the dynamics of the Susceptible Infected Recovered (SIR) model in the context of the COVID-19 outbreak in Nigeria during the year 2020. The model is validated by fitting it to data on the prevalence and active cases of COVID-19, sourced from a government agency responsible for disease control. Utilizing the parameters associated with the disease prevalence, we calculate the basic reproduction number R_{cr}, revealing its approximate value as 10.84. This suggests an average infection rate of around 10 human individuals, indicating the endemic nature of the disease in Nigeria. The impact of variation of recovery rate via treatment is examined, demonstrating its effectiveness in reducing disease prevalence when R_{cr} is below or above unity. To numerically implement the model, we employ the Sumudu Decomposition Method (SDM) and compare its results with the widely used Runge–Kutta fourth-order (RK4) method, implemented through the Maple software. Our findings indicate a mutual efficiency and convergence between the two methods, providing a comprehensive understanding of the COVID-19 dynamics in Nigeria.

Key-Words: - COVID – 19, Basic reproduction number, Runge Kutta Method, Sumundu Decomposition Method, Susceptible-Infected-Recovered (SIR) model, Non-Pharmaceutical Strategies (NPIs).

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1 Introduction

Deterministic models use the concept of mathematical techniques to develop an accurate depiction of a system. Epidemiologically, models enable the description of the evolution and transmission of infection, future-term behavior, and possible control strategies to eradicate disease spread. Several works on mathematical modeling of

diseases have been developed together with determinant factors like incidence, spread, and persistence, [1]. The SIR model was first studied by [2] and [3], where the impact of demographic factors like births, deaths, and migration are studied. Later, SIR models with vital dynamics, [4] and other forms of extensions with vaccination, treatment, relapse, susceptibility, etc., are studied by different authors [3], [5], [6], [7], [8], [9].

A novel disease named coronavirus (COVID - 19) disease evolved in Wuhan, China, December 2019. This disease became the most devastating health challenge experienced in the world after the 1918/1919 pandemic of influenza. The World Health Organization (WHO) announced the disease as a pandemic on March 11, 2020, and by the end of the year 2020, over 90 million cases have been recorded and more than two million lives lost, as a result of the COVID – 19 menace. Nigeria is one of the most affected countries in Africa with COVID-19 cases. By the end of year 2020, 87607 and 1361 cases of COVID – 19 infection and casualties were recorded, [10], [11], [12], [13], while efforts by the WHO are ongoing to circulate vaccines and possible drugs across the world to treat and minimize the high rate of the infection spread.

Several deterministic and stochastic models have been derived to explain and predict the transmission of COVID – 19 in Nigeria. A study, [14], formulated a model with Non-Pharmaceutical Strategies (NPIs) fitted to the prevalence date as of March 30, 2020. Their results show that COVID – 19 can be effectively mitigated using a moderate level of compliance with NPIs to avoid a second wave of the pandemic. In [15], derivation of a model was done to forecast COVID – 19 dynamics using the prevalence data as of March 16, 2020. Their results reveal that if at least 55 percent of humans can adhere to social distancing and face mask usage, the disease will be eradicated. Also if the case findings for humans with symptoms are increased to 0.8 per day associated with social distancing will lead to a reduction of COVID-19 disease incidence. The studies in [16] and [17], considered the effect of optimal management in minimizing COVID – 19 infection in Nigeria. Other works on the formulation of COVID – 19 using qualitative and quantitative techniques include the works of [18], [19], [20], [21]. The SIR model is the basic framework for describing disease spread in population dynamics. The recent coronavirus (COVID-19) disease across the world has majorly been described using the SIR model, [22], as well as other diseases in [23], [24] and [25]. The idea of SDM was first conceived in [26]. Also, the studies [2] and [27], employed hybrid methods of SDM and Laplace to compute the system of ordinary differential equations, other works on the application of SDM can be seen in the works of [27], [28] and [29], while works on the modification of SDM, using the other semi-analytical approaches can be seen in [30], [31], [32] and [33].

Inspired by the cited works on the mathematical modeling approach to COVID – 19 disease spread in Nigeria together with different applications of numerical methods to obtain approximate solutions of models, in this work we consider fitting a SIR model to the COVID – 19 prevalent and active cases in Nigeria in relation to year 2020 utilizing the non-linear least square method by the use of MAPLE computational software, such that the estimated and fitted values were used to analyze and obtain the value of R_{cr} [25]. Also, the numerical solution of the model using the SDM in comparison with the RK4 method is obtained. It is to the best understanding of the authors that this has not been done by the aforementioned authors. The subsequent parts of the article are sectionalized. Section 2 involves the model formulation and analysis and data fitting analysis. Section 3 involves the numerical implementation of the model equations by the use of SDM and RK4 methods, while Section 4 discusses the results and conclusion. See also a study in [34] as a case study of Lagos State, Nigeria.

2 Model Formulation

The model is divided into the Susceptible $S_c(t)$; Infected $I_c(t)$; and Recovered $R_c(t)$, where the whole human population $N(t)$ yields $N(t) = S_c(t) + I_c(t) + R_c(t)$. $\prod$ denotes the crude birth rate, β represents the transmission rate per COVID – 19 infective, ϕ is the recovery rate for COVID – 19 infection, κ is the mortality related to COVID – 19 infection and μ is the natural death rate. Using these descriptions, the model is expressed as:

$$\begin{aligned} \frac{dS_c}{dt} &= \prod - \beta S_c I_c - \mu S_c \\ \frac{dI_c}{dt} &= \beta S_c I_c - (\mu + \phi + \kappa + \sigma) I_c \\ \frac{dR_c}{dt} &= \phi I_c - \mu R_c \end{aligned} \qquad (1)$$

Analytically, Eq. (1) is positively invariant and well posed in the region:

$$\Delta_* = \left\{(S_c, I_c, R_c) \in \mathbb{R}_+^3 : S_c + I_c + R_c \leq \frac{\prod}{\mu}\right\} \quad (2)$$

The average time of COVID - 19 infection is $\frac{1}{(\mu+\phi+\kappa+\sigma)}$ and since the infectious individual transmits COVID - 19 disease at the rate β, then R_{cr} is computed to be $\frac{\prod\beta}{(\mu+\phi+\kappa+\sigma)}$. R_{cr} measures the number of secondary COVID-19 infectious humans per COVID-19 index case in a naive population of vulnerable human population. Solving for equilibrium solutions in Eq.(1) when independent of time, yields

$$e^1 = (S_c, I_c, R_c) = \left(\frac{\prod}{\mu}, 0, 0\right) \quad (3a)$$

$$e^2 = (S_c^*, I_c^*, R_c^*) = \left(\frac{\mu}{R_0}, \frac{\mu}{\beta}(R_{cr} - 1), \frac{\phi}{\beta}(R_{cr} - 1)\right) \quad (3b)$$

Where e^1 and e^2 represent the COVID – 19 free and endemic equilibrium points respectively. If $R_{cr} < 1$, then the disease vanishes, but if $R_{cr} > 1$, then the infection becomes persistent in the human and environment host population.

2.1 Model Fitting

To validate the model system Eq. (1), Data on cumulative and active cases of COVID-19 in Nigeria, reported by NCDC [12] for the year 2020 is applied. The parameters estimated are the transmission rate β and progression rate. The parameters were obtained from the literature. For instance, $\prod$ is the crude birth rate of Nigeria , which is estimated to be 37.269 per 1000 people, and the death rate in Nigeria is taken to be 11.577, so that $\frac{\prod}{\mu}$ = 3.2192 year^{-1} is the restricted human population in a COVID-19 free community and $\frac{1}{\mu}$= 0.0863 year^{-1}. Also, the total COVID-19 induced death rate κ in Nigeria for year 2020 is 1,361, where $\frac{1}{\kappa} = 0.00073/\text{day}$ [12]. To apply the non-linear least square method for model validation using the available data, at time t, vector z of Eq. (1) and vector θ of the unknown parameter, Eq. (1) follows the form:

$$z^1 = f(t, z, \theta), \quad z(t_0) = z_0. \quad (4)$$

Also, the residual form of Eq.(4) is expressed as:

$$\text{Residual } (\theta) = f(t_i, z(i), \theta) - z_{real}(i) \quad (5)$$

and the error is given by:

$$\text{Error } (\theta) = \sum_{i=1}\left(z(i) - z_{real}(i)\right)^2, \quad (6)$$

where $z_{real}(i)$ is denoted by the actual data and $y(i) = y(t_i, z)$ is the solution to Eq. (4) for θ. In addition, the minimization function is given by:

$$\min \text{ error } (\theta) \text{ based on Eq. (4)} \quad (7)$$

is applied to compute the optimal parameters.

2.1.1 A Non Linear Algorithm For Estimation of the Parameter

The non-linear algorithm governing the parameter estimation applied to obtaining the fitted values is given below.

1. Take up state and parameter values.
2. Compute Eq.(1) by the use of RK4 method together with step 1.
3. Check the error.
4. Minimize to derive a new set of parameter values of Eq.(1) to agree with actual data.
5. Investigate convergence. If it doesn't converge, return to 2.
6. Continue iteration, till the convergence for the acquired parameters are achieved.

The model fit simulation using the data in [25], via the non-linear least square algorithm is displayed in Figure 1 and Figure 2.

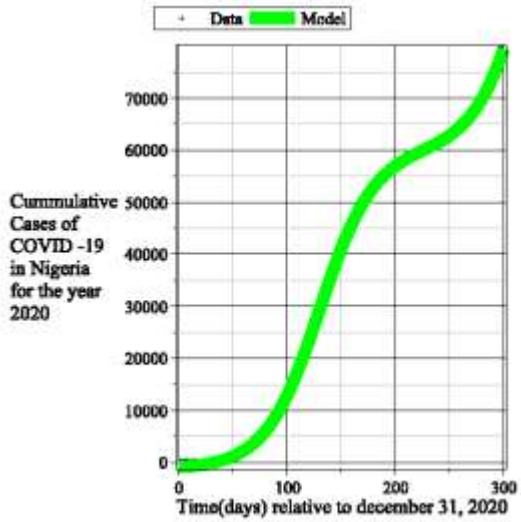

Fig. 1: Model fit of cumulative cases of COVID – 19 in Nigeria relative to the year 2020

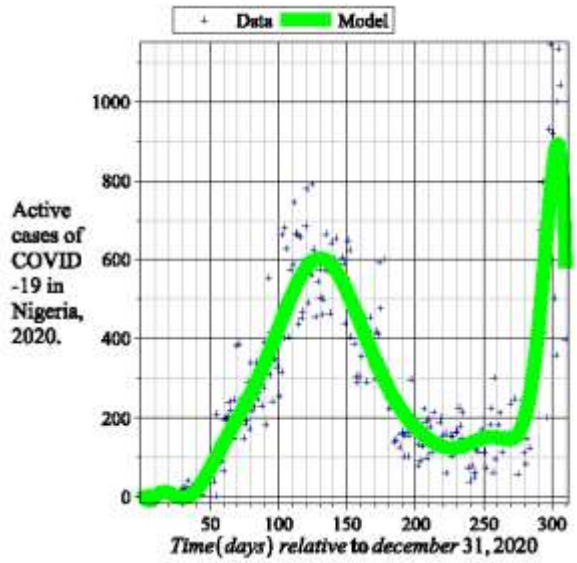

Fig. 2: Model fit of active cases of COVID – 19 in Nigeria in relation to the year 2020.

3 Numerical Implementation

3.1 The SDM and RK4 Method

To understand the essentials of SDM, we consider the denotation of in-homogenous non-linear ordinary differential equation with its initial data as:

$$H_c + K_c + M_c = g(t), \quad l(t) = a. \tag{8}$$

Where H_c represents the derivative of first order, K_c represent a differential operator, M_c denotes the non-linear term, while $u(t)$ is the source term, [24].

On the application of SDM to Eq. (8) yields:

$$S^*[H_c] + S^*[K_c] + S^*[M_c] = S^*[l(t)] \tag{9}$$

Also, following the same procedure, Eq. (9) yields:

$$S^*[H_c] = a + u\big[S^*[l(t)] - S^*[K_c] - S^*[M_c]\big]. \tag{10}$$

The inverse of Eq. (10), yields:

$$c(t) = S^{*-1}\Big[u\big[S^*[l(t)] - S^*[K_c] - S^*[M_c]\big]\Big]. \tag{11}$$

So that Eq. (11) is denoted as an infinite series given by:

$$c(t) = \sum_{n=0}^{\infty} c_n(t). \tag{12}$$

Also, the non-linear term in Eq.(11) can be expressed as:

$$M_c(t) = \sum_{n=0}^{\infty} {A_n}^*. \tag{13}$$

Where ${A_n}^*$ are Adomian polynomials of $c_0, c_1, c_2, \ldots$ such that:

$${A_n}^* = \frac{1}{n!}\frac{d^n}{d\lambda^n}\Big[M\Big(\sum_{i=0}^{\infty}\lambda^i c_i\Big)\Big]_{p=0}, n = 0,1,2\ldots. \tag{14}$$

Putting Eqs. (13) and (14) into Eq. (12), we have:

$$\sum_{n=0}^{\infty} c_n(t) = P(t) - S^{*-1}\Big[cS^*\big[K\sum_{n=0}^{\infty} c_n(t) + \sum_{n=0}^{\infty} {A_n}^*(t)\big]\Big]. \tag{15}$$

Where $P(t)$ is the expression arising from the source term and the initial data.

Making use of the Adomian Decomposition Method (ADM) to Eq.(15), we obtain:

$$c_0(t) = P(t), \tag{16}$$

$$c_1(t) = -S^{*-1}\big[cS^*[Kc_0(t) + {A_0}^*]\big], \tag{17}$$

$$c_2(t) = -S^{*-1}\big[cS^*[Kc_1(t) + {A_1}^*]\big], \tag{18}$$

$$c_{n+1}(t) = -S^{*-1}\big[cS^*[Kc_n(t) + {A_n}^*]\big]. \tag{19}$$

From Eqs. (16-19), we obtain the values of c_0, c_1, c_2. We implement the SDM on Eq. (1) by using the values; $\prod = 3.2192$, $\beta = 0.830$, $\mu = 0.0863$, $\phi = 0.1429, \kappa = 0.001137$ and $\sigma = 0.016$. For the purpose of illustration, let $S_c(0) = 0.87607$, $I_c(0) = 0.13610$, $R_c(0) = 0.67507$.

The SDM of Eq. (1) yields:

$$\begin{aligned} (\dot{S_c}) &= \frac{S^*(S_c(t)-S_c(0))}{S_c} = S^*[\prod - \beta S_c I_c - \mu S_c] \\ S^*(\dot{I_c}) &= \frac{S^*(I_c(t)-I_c(0))}{I_c} = S^*[\beta S_c I_c - (\mu + \phi + \kappa + \sigma)I_c], \\ S^*(\dot{R_c}) &= \frac{S^*(R_c(t)-R_c(0))}{R_c} = S^*[\phi I_c - \mu R_c]. \end{aligned} \tag{20}$$

Further, the inverse SDM of Eq. (20) yields

$$\begin{aligned} S_c(t) &= S^{*-1}[S_c S^*(\prod - \beta S_c I_c - \mu S_c)] \\ I_c(t) &= S^{*-1}[I_c S^*(\beta S_c I_c - (\mu + \phi + \kappa + \sigma)I_c)], \\ R_c(t) &= S^{*-1}[R_c S^*(\phi I_c - \mu R_c)]. \end{aligned} \tag{21}$$

Assuming the solution in Eq. (21) as infinite series of unknown function then:

$$\begin{aligned}
&\sum_{n=0}^{\infty} S_{c_n}(t) = S^{*-1}\left[S_c S^*\left[\prod - \beta \sum_{n=0}^{\infty} A_n^*(t) - \mu \sum_{n=0}^{\infty} S_{c_n}(t)\right]\right] \\
&\sum_{n=0}^{\infty} I_{c_n}(t) = \\
&S^{*-1}\left[I_c S^*\left[\beta \sum_{n=0}^{\infty} A_n^*(t) - (\mu+\phi+\kappa+\sigma) \sum_{n=0}^{\infty} I_{c_n}(t)\right]\right] \\
&\sum_{n=0}^{\infty} R_{c_n}(t) = S^{*-1}\left[R_c S^*\left[\phi I_c - \mu R_c\right]\right].
\end{aligned} \tag{22}$$

$$\begin{aligned}
A_0^* &= S_{c_0} I_{c_0}, \\
A_1^* &= S_{c_0} I_{c_1} + S_{c_1} I_{c_0}, \\
A_2^* &= S_{c_0} I_{c_2} + S_{c_1} I_{c_1} + S_{c_2} I_{c_2}.
\end{aligned} \tag{23}$$

So that:

$$\begin{aligned}
S_{c_0} &= 0.87607 \\
S_{c_1} &= S^{-1}\left[S_c S\left(\prod - \beta A_0^* - \mu S_{c_0}\right)\right] \\
S_{c_2} &= S^{-1}\left[S_c S\left(\prod - \beta A_1^* - \mu S_{c_1}\right)\right] \\
I_{c_0} &= 0.13610 \\
I_{c_1} &\ S^{-1}\left[I_c S\left(\beta A_0^* - (\mu+\phi+\kappa+\sigma) I_{c_0}\right)\right] \\
I_{c_2} &= S^{-1}\left[I_c S\left(\beta A_1^* - (\mu+\phi+\kappa+\sigma) I_{c_1}\right)\right] \\
R_{c_0} &= 0.67507 \\
R_{c_1} &= S^{-1}\left[R_c S\left(\phi I_{c_0} - \mu R_{c_0}\right)\right] \\
R_{c_2} &= S^{-1}\left[R_c S\left(\phi I_{c_1} - \mu R_{c_1}\right)\right]
\end{aligned} \tag{24}$$

Further computation of Eq. (23) together with Eq. (24) yields the following series solution as:

$$S_c(t) = 0.87607 - 3.0447t - 2.5649t^2 + 2.2699t^3 + \ldots \tag{25}$$

$$I_c(t) = 0.13610 + 0.0654t + 0.3754t^2 + 0.6355t^3 + \ldots \tag{26}$$

$$R_c(t) = 0.67507 - 0.0388t + 0.0127t^2 + 0.0525t^3 + \ldots \tag{27}$$

Moreover, the RK4 scheme is applied to Eq. (1). The RK4 scheme is given by:

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2\,k_2 + 2\,k_3 + k_4). \tag{28}$$

Where

$$\begin{aligned}
k_1 &= hf\,(x_n, y_n), \\
k_2 &= hf(x_r + {}^1/_2 h,\ y_r + {}^1/_2 k_1), \\
k_3 &= hf(x_r + {}^1/_2 h,\ y_r + {}^1/_2 k_2), \\
k_4 &= hf(x_r + h,\ y_r + k_3).
\end{aligned} \tag{28}$$

and

$$\begin{aligned}
S_{c_{n+1}} &= S_{c_n} + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\ h, \\
I_{c_{n+1}} &= I_{c_n} + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)\ h, \\
R_{c_{n+1}} &= R_{c_n} + \frac{1}{6}(m_1 + 2m_2 + 2m_3 + m_4)\ h.
\end{aligned} \tag{29}$$

and

$$\begin{aligned}
k_1 &= \prod - \beta S_c I_c - \mu S_c, \\
L_1 &= \beta S_c I_c - (\mu + \phi + k + r)\ I_c\ , \\
M_1 &= \phi I_c - \mu R_c.
\end{aligned} \tag{30}$$

and

$$\begin{aligned}
k_2 &= \prod - \beta\left(S_{c_n} + k_1 \frac{h}{2}\right) \\
&\left(I_{c_n} + k_1 \frac{h}{2}\right) - \mu\left(S_{c_n} + k_1 \frac{h}{2}\right), \\
L_2 &= \beta\left(S_{c_n} + k_1 \frac{h}{2}\right)\left(I_{c_n} + k_1 \frac{h}{2}\right) \\
&- (\mu + \phi + k + r)\left(I_{c_n} + k_1 \frac{h}{2}\right), \\
M_2 &= \phi\left(I_{c_n} + k_1 \frac{h}{2}\right) - \mu\left(R_{c_n} + k_1 \frac{h}{2}\right).
\end{aligned} \tag{31}$$

and

$$k_3 = \prod - \beta(S_{c_n} + k_2\frac{h}{2})(I_{c_n} + k_2\frac{h}{2}) - \mu(S_{c_n} + k_2\frac{h}{2}),$$

$$L_3 = \beta(S_{c_n} + k_2\frac{h}{2})(I_{c_n} + k_2\frac{h}{2}) - (\mu + \phi + k + r)(I_{c_n} + k_2\frac{h}{2}), \quad (32)$$

$$M_3 = \phi(I_{c_n} + k_2\frac{h}{2}) - \mu(R_{c_n} + k_2\frac{h}{2})$$

and

$$K_4 = \prod - \beta(S_{c_n} + k_3\frac{h}{2})(I_{c_n} + k_3\frac{h}{2})$$

If n = 0

$$S_{c_{n_1}} = S_{c_0} + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h,$$

$$L_4 = \beta(S_{c_n} + k_3\frac{h}{2})(I_{c_n} + k_3\frac{h}{2}) - (\mu + \phi + k + r)(I_{c_n} + k_3\frac{h}{2}), \quad (33)$$

$$M_4 = \phi(I_{c_n} + k_3\frac{h}{2}) - \mu(R_{c_n} + k_3\frac{h}{2}).$$

$$I_{c_{n_1}} = I_{c_0} + \frac{1}{6}(L_1 + 2L_2 + 2L_3 + L_4)h,$$

$$R_{c_{n_1}} = R_{c_0} + \frac{1}{6}(m_1 + 2m_2 + 2m_3 + m_4)h,$$

$$S_{c_{n_2}} = S_{c_1} + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h, \quad (34)$$

$$I_{c_{n_2}} = I_{c_1} + \frac{1}{6}(L_1 + 2L_2 + 2L_3 + L_4)h,$$

$$R_{c_{n_2}} = R_{c_1} + \frac{1}{6}(m_1 + 2m_2 + 2m_3 + m_4)h.$$

4 Discussion of Results and Conclusion

4.1 Discussion of Results

In the course of simulation, the parameter and variable values given in Section 2 are adopted to obtain the numerical results in Table 1, by comparing SDM and RK4 method, while, the errors between the two methods are given in Table 2.

Table 1. Comparison between SDM and RK4 for Approximate Solutions of Model Eq. (1).

Time(Months)	S_c(SDM)	S_c(RK4)	I_c(SDM)	I_c(RK4)	R_c(SDM)	R_c(RK4)
1	2.4636	2.4640	1.2124	1.2315	0.7015	0.7020
2	2.6863	2.6875	6.8525	6.8555	1.0683	1.0693
3	29.9452	29.9463	20.8694	20.8703	2.0905	2.1001
4	92.9325	92.9341	47.0761	47.0779	4.0831	4.0850
5	205.2676	205.2685	89.2856	89.2871	7.3611	7.3630
6	308.5699	308.5710	151.3109	151.3121	12.2395	12.2405
7	632.4588	632.4599	236.9650	236.9668	19.0333	19.0372
8	974.5537	974.5570	350.0609	350.0615	28.0575	28.0620
9	1420.4740	1420.4770	494.4116	494.4120	39.6271	39.6352
10	1983.8391	1983.8402	673.8301	673.8312	54.0571	54.0582
11	2678.2684	2678.2715	892.1294	892.1301	71.6625	71.6637
12	3517.3813	3517.3831	1153.1225	1153.1225	97.7583	97.7590

Table 2. Errors between RK4 and SDM approximate solutions for model Eq. (1).

Time(Months)	S_c(RK4 – SDM)	I_c(RK4 – SDM)	R_c(RK4 – SDM)
1	0.0004	0.0011	0.0005
2	0.0012	0.0030	0.0010
3	0.0011	0.0009	0.0016
4	0.0006	0.0018	0.0019
5	0.0009	0.0011	0.0019
6	0.0011	0.0012	0.0010
7	0.0011	0.0018	0.0039
8	0.0033	0.0006	0.0045
9	0.0030	0.0004	0.0081
10	0.0011	0.0011	0.0011
11	0.0031	0.0007	0.0012
12	0.0018	0.0075	0.0007

The graphical representations in Figure 1 and Figure 2 depict the model fit for cumulative and active COVID-19 cases in Nigeria throughout the year 2020. Notably, a consistent upward trend is evident over time, attributed to a significant lack of adherence to COVID-19 protocols. This underscores the imperative for stringent enforcement of non-pharmaceutical interventions to curb the rapid spread of the disease. Additionally, the examination of Table 1 and Table 2 reveals a harmonious agreement between the two numerical methods, displaying minimal errors. Furthermore, it is noteworthy that the Sumudu Decomposition Method (SDM) exhibits better performance in both efficiency and convergence when compared to the Runge–Kutta fourth-order (RK4) method, while Figure 3 and Figure 4 describes the effect of recovery rate ϕ on R_{cr} in 12 months the host community. It is observed that the curve converges to the disease – free and endemic equilibrium when $R_{cr} < 1 \; and \; R_{cr} \approx 10.84$. When the recovery rate through treatment is increased, that is, $\phi = 0.8429$, R_{cr}reduces but not below unity. This highlights the challenge of completely eradicating the virus through treatment alone. The relevance of umerical modeling techniques and recovery rate through treatment for the containment of the spread of COVID-19 has been shown in this work. Though effective treatment is needed to ameliorate the impact of the virus, preventive measures is essential to reduce the spread due to the endemic nature of the disease.

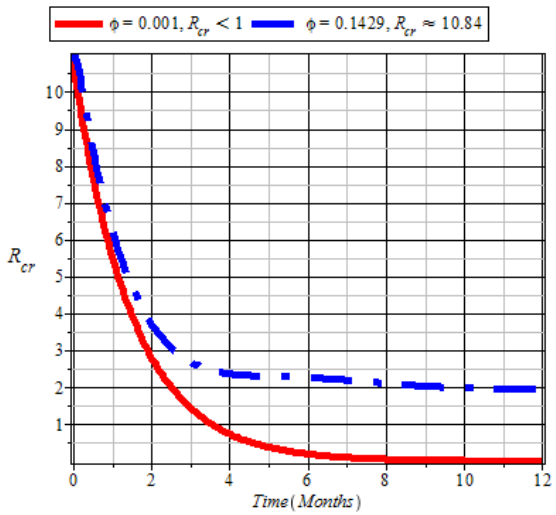

Fig. 3: Behavior of recovery rate $\phi = 0.1429$ on R_{cr} when R_{cr}<1 and $R_{cr} > 1$

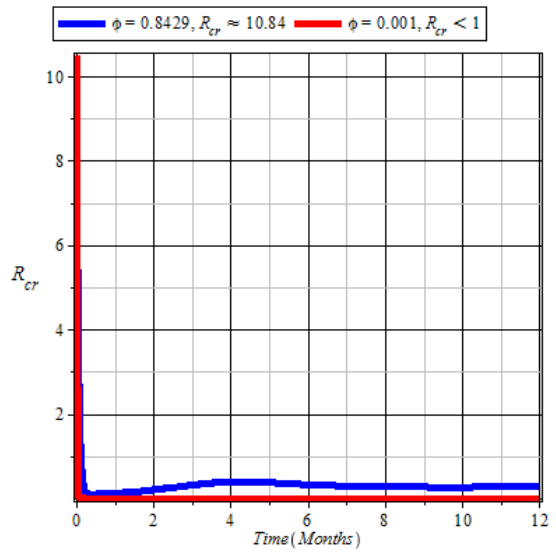

Fig. 4: Behavior of recovery rate $\phi = 0.8429$ on R_{cr} when R_{cr}<1 and $R_{cr} > 1$

5 Conclusion

The compartmental model has been used in this study to analyze COVID-19 cumulative and active cases in Nigeria throughout the year 2020. The simulations, based on fitted and estimated parameters from existing literature, showed a basic reproduction number R_{cr} of approximately 10.84. This finding reveals the endemic nature of COVID-19 in Nigeria, with an average infection rate of at least 10 individuals. Furthermore, the investigation into the impact of the recovery rate on R_{cr} showed that an increase in the recovery rate through treatment can reduceR_{cr}, although it remains above unity. This suggests that treatment alone may not be sufficient to effectively combat the disease. The numerical implementation of the model equations using the Sumudu Decomposition Method (SDM) and the Runge–Kutta fourth-order (RK4) method demonstrated their efficiency, with SDM exhibiting better convergence. Consequently, health policy-makers in Nigeria are advised to intensify the implementation of Non-Pharmaceutical Interventions (NPIs) recommended by the World Health Organization (WHO). This strategic scaling up of NPIs is crucial to mitigate the spread of COVID-19 and reduce R_{cr} below unity, ultimately aiming to eliminate the disease. Mathematically, this study suggests potential extensions into spatial, fractional order, stochastic, and optimal control problems. These avenues of research could further enhance our understanding of the dynamics of COVID-19 and contribute to the development of more effective strategies for disease control and prevention.

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