Notices

OF THE

AMERICAN MATHEMATICAL SOCIETY

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November 1973
Issue No. 149

Calendar

This Calendar lists all of the meetings which have been approved by the Council up to the date this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change; this is particularly true of meetings to which no numbers have yet been assigned.

Abstracts should be submitted on special forms which are available in most departments of mathematics; forms can also be obtained by writing to the headquarters of the Society. Abstracts to be presented at the meeting *in person* must be received at the headquarters of the Society in Providence, Rhode Island, on or before the deadline for the meeting.

Meeting Number	Date	Place	Deadline for Abstracts* and News Items
711	January 15–19, 1974 (80th Annual Meeting)	San Francisco, California	Oct. 31, 1973
712	March 7–8, 1974	Gainesville, Florida	Jan. 15, 1974
713	April 10–13, 1974	New York, New York	Feb. 21, 1974
714	April 27, 1974	Santa Barbara, California	Feb. 21, 1974
715	May 13–18, 1974	De Kalb, Illinois	March 27, 1974
716	August 1974	No summer meeting; International Congress (see below)	June 15, 1974 (News Items only)
	November 8–9, 1974	Nashville, Tennessee	
	November 22–23, 1974	Los Angeles, California	
	November 23, 1974	Houston, Texas	
	January 23–27, 1975 (81st Annual Meeting)	Washington, D.C.	
	April 18–19, 1975	Monterey, California	
	January 22–26, 1976 (82nd Annual Meeting)	San Antonio, Texas	

*Deadline for abstracts not presented at a meeting (by title).

February 1974 issue: January 8
April 1974 issue: February 14
June 1974 issue: March 20

OTHER EVENTS

January 13–14, 1974	Preceptorial Introduction to Computer Science for Mathematicians San Francisco, California	
February 25–26, 1974	Symposium on Some Mathematical Questions in Biology San Francisco, California	Dec. 1, 1973
August 21–29, 1974	International Congress of Mathematicians Vancouver, B.C., Canada	April 15, 1974

The zip code of the Post Office Box of the Society has been changed from 02904 to 02940. Correspondents are requested to note this change in their records.

Please affix the peel-off label on these *Notices* to correspondence with the Society concerning fiscal matters, changes of address, promotions, or when placing orders for books and journals.

The *Notices* of the American Mathematical Society is published by the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940, in January, February, April, June, August, October, November, and December. Price per annual volume is $10. Price per copy $3. Special price for copies sold at registration desks of meetings of the Society, $1 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available), and inquiries should be addressed to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940. Second class postage paid at Providence, Rhode Island, and additional mailing offices.

Notices

OF THE

AMERICAN MATHEMATICAL SOCIETY

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RESERVATION FORMS A-691–A-692

The Seven Hundred Ninth Meeting
Georgia Institute of Technology
Atlanta, Georgia
November 16 – 17, 1973

The seven hundred ninth meeting of the American Mathematical Society will be held at Georgia Institute of Technology in Atlanta, Georgia, on Friday and Saturday, November 16–17, 1973.

By invitation of the Committee to Select Hour Speakers for the Southeastern Sectional Meetings, there will be three one-hour addresses, all of which will be presented in the Space Science and Technology Building. Professor Ben Fitzpatrick, Jr., of Auburn University will give an address entitled "Moore spaces." An address entitled "Commutative groups and the cardinal connection" will be given by Professor Paul Hill of Florida State University, and Professor M. Zuhair Nashed of Georgia Tech will present an address entitled "Regularization and numerical analysis of ill-posed operator equations."

There will be two Special Sessions in addition to the regular sessions. Professor John W. Heidel of the University of Tennessee, Knoxville, is arranging a Special Session on Applications of Ordinary Differential Equations. The speakers will include John V. Baxley, Stephen R. Bernfeld, Jagdish Chandra, T. G. Hallam, Christopher Hunter, M. Z. Nashed, Paul Nelson, and M. B. Sledd. Professor M. D. Plummer of Vanderbilt University is arranging a Special Session on Graph Theory. The slate of speakers will include Richard Duke, Donald L. Greenwell, R. L. Heminger, Renu Laskar, Roy B. Levow, Peter V. O'Neill, K. B. Reid, Richard Schelp, David Sumner, and William T. Trotter, Jr. There may be an added session for late papers Saturday afternoon if needed.

The registration desk will be located in the lower lobby of the Space Science and Technology Building. Registration hours will be from noon to 5:00 p.m., on Friday, November 16, and from 9:00 a.m. to noon, on Saturday, November 17. The sessions will be held in Skiles Classroom Building and the Space Science and Technology Building.

Atlanta is accessible from Interstates 85, 75, and 20, and is served by all major airlines. Atlanta is also served by Greyhound and Continental Trailways Bus Lines, and Southern Railroad from many cities as well as the Georgia Railroad from Augusta, Georgia. Limousine service is available from the Hartsfield International Airport to motels near the Georgia Tech campus. The limousine fare is $2.08. There are five automobile rental companies which maintain offices at the Atlanta airport (American International, Avis, Budget, Hertz, and National).

The cafeteria in the Student Center will be available for all three meals, and meals will also be available at commercial establishments.

Three motels near the campus were holding blocks of rooms for reservations with a deadline of October 26. Reservations should have been made directly with them, with mention of this meeting included in that correspondence.

TECH MOTEL
120 North Avenue (Three blocks from Space Science Building)
Phone: (404) 873-3721
Single $13 up
Double 16 up

ATLANTA TOWNEHOUSE MOTOR INN
100 10th Street (Six blocks from Skiles Classroom Building)
Phone: (404) 892-6800
Single $18 up
Double 26

LANDMARK MOTOR INN
1152 Spring Street (Eight blocks from Skiles Classroom Building)
Phone: (404) 873-4361
Single $16 up
Double 23

Additional accommodations are:

STOUFFER'S ATLANTA INN
590 West Peachtree (Five blocks from Space Science and Technology Building)
Phone: (404) 873-1551
Single $25–$31
Double 27 up

ATLANTA CABANA MOTOR HOTEL
870 Peachtree (Eight blocks from Skiles Classroom Building)
Phone: (404) 875-5018
Single $19
Double 21

SHERATON-BILTMORE
817 West Peachtree (Six blocks from Skiles Classroom Building)
Phone: (404) 875-3461
Single $21 up
Double 27 up

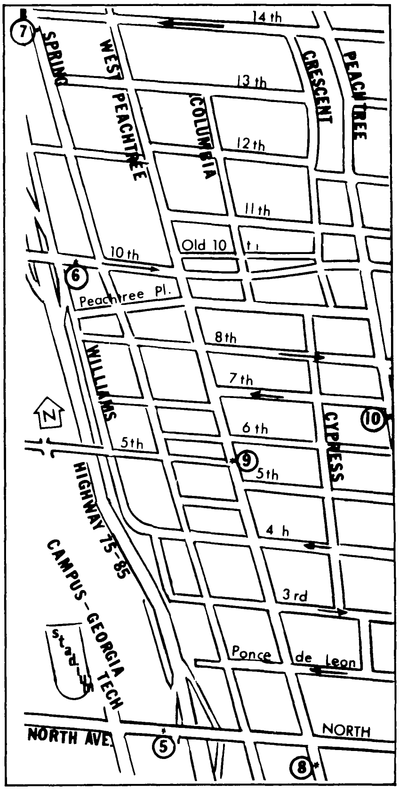
There will be a social at the Atlanta Townehouse Motor Inn with a Dutch treat bar on Friday, November 16.

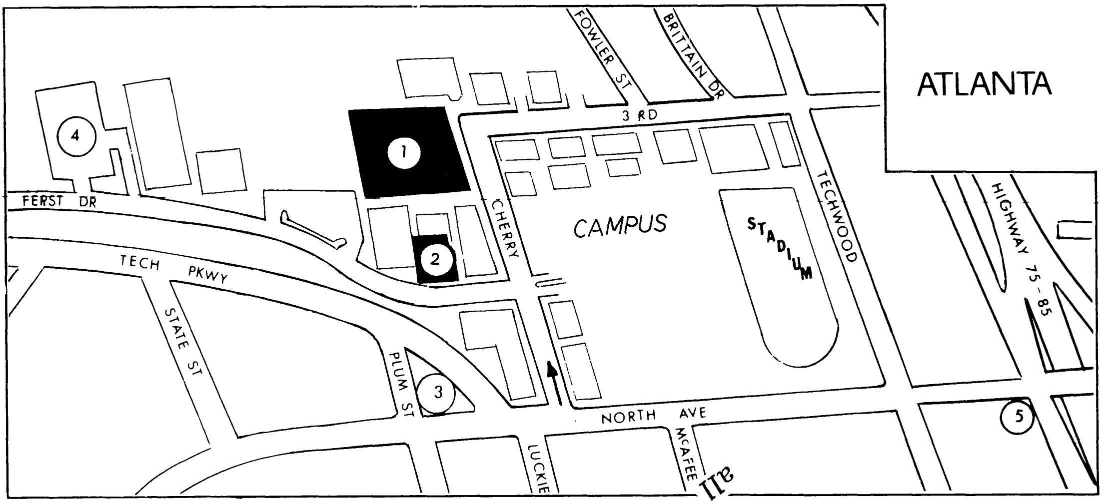
Emergency messages may be left for delivery at 894-2701.

1. Skiles Classroom Building
2. Space Science & Technology Building, #3
3. Plum Lot, parking
4. Student Center Lot West, parking

Hotel and Motel Directory:

5. Tech Motel
6. Atlanta Townehouse Motor Inn
7. Landmark Motor Inn
8. Stouffer's Atlanta Inn
9. Sheraton-Biltmore
10. Atlanta Cabana Motor Hotel





PROGRAM OF THE SESSIONS

The time limit for each contributed paper in the general sessions is ten minutes and in the special sessions is twenty minutes. To maintain this schedule, the time limits will be strictly enforced.

FRIDAY, 1:00 P. M.

Invited Address, Lecture Room 4, Space Science and Technology Building

(1) Moore spaces. Professor BEN FITZPATRICK Jr., Auburn University (709-G15)

FRIDAY, 1:00 P. M.

Special Session on Graph Theory, Room 108, Skiles Classroom Building

2:15- 2:35	(2)	Some recent results on paths in tournaments. Professor KENNETH B. REID, Louisiana State University (709-A14)
2:45- 3:05	(3)	Nearly planar graphs and the reconstruction problem. Preliminary report. Dr. PETER V. O'NEIL, College of William & Mary (709-A3)
3:15- 3:35	(4)	Periodic line digraphs. Professor ROBERT L. HEMMINGER, Vanderbilt University (709-A9)
3:45- 4:05	(5)	Graphs with 1-factors. Preliminary report. Dr. DAVID P. SUMNER, University of South Carolina (709-A10)
4:15- 4:35	(6)	Path connected graphs. Professor RALPH J. FAUDREE and Professor RICHARD H. SCHELP*, Memphis State University (709-A13)

FRIDAY, 2:15 P. M.

Special Session on the Applications of Ordinary Differential Equations I, Room 202, Skiles Classroom Building

2:15- 2:35	(7)	Some minimax problems arising from systems. Professor M. ZUHAIR NASHED, Georgia Institute of Technology (709-B20)
2:45- 3:05	(8)	A monotone method for quasi-linear boundary value problems. Dr. JAGDISH CHANDRA*, U.S. Army Research Office and Professor PAUL WILLIAM DAVIS, Worcester Polytechnic Institute (709-B8)
3:15- 3:35	(9)	An analysis of some third order initial value problems arising in magneto hydrodynamics. Professor THOMAS G. HALLAM, Florida State University (709-B19)
3:45- 4:05	(10)	Synthesis problems for self-consistent stellar systems. Dr. CHRISTOPHER HUNTER, Florida State University (709-C2) (Introduced by Professor John W. Heidel)

FRIDAY, 2:30 P. M.

Session on Algebraic Topology and Lie Groups, Room 218, Skiles Classroom Building

2:30- 2:40	(11)	A unipotent, but not contractible group. Mr. PETER R. MUELLER-ROEMER, East Carolina University (709-G5)
2:45- 2:55	(12)	The semilattice of left translations of a compact semilattice. Dr. THOMAS T. BOWMAN, University of Florida (709-G16)
3:00- 3:10	(13)	Invariants for Stiefel manifolds in Hilbert algebras. Professor KENNETH I. GROSS, University of North Carolina (709-G19)
3:15- 3:25	(14)	On dense subgroups. Preliminary report. Professor HARIHARAIER SUBRAMANIAN, State University of New York at Buffalo and Professor M. RAJAGOPALAN*, Memphis State University (709-G20)
3:30- 3:40	(15)	Differentiable semigroups. Dr. JOHN P. HOLMES, Auburn University (709-G13)
3:45- 3:55	(16)	A condition equivalent to covering dimension for normal spaces. Professor JAMES AUSTIN FRENCH, David Lipscomb College (709-G7)
4:00- 4:10	(17)	Automorphisms of handlebodies. Preliminary report. Dr. RICHARD DEE BARNHART, Bryan College (709-G8)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

4:15- 4:25 (18) A simple proof of the Künneth theorem for Alexander cohomology. Professor KERMIT N. SIGMON, University of Florida (709-G29)

FRIDAY, 2:30 P. M.

Session on Group Theory and Generalizations, Room 208, Skiles Classroom Building

2:30- 2:40 (19) Elementary types of some groups related to infinite symmetric groups. Preliminary report. Professor RICHARD A. SANERIB, Jr., Emory University (709-E1)

2:45- 2:55 (20) The structure of the semigroup of Boolean circulant matrices. Preliminary report. Professor KIM KI-HANG BUTLER and Professor JAMES RICHARD KRABILL*, Pembroke State University (709-A42)

3:00- 3:10 (21) The semigroup of Hall relations. Professor KIM KI-HANG BUTLER, Pembroke State University (709-A37)

3:15- 3:25 (22) On Moufang and extra loops. Dr. PALANIAPPAN KANNAPPAN, University of Waterloo (709-A31)

3:30- 3:40 (23) Universal compact inverse semigroups. Mr. DORIAN P. YEAGER, University of Tennessee (709-A26)

3:45- 3:55 (24) Fully invariant subgroups of totally projective abelian groups. Preliminary report. Mr. RONALD C. LINTON, University of South Alabama (709-A16) (Introduced by Professor Richard Vinson)

4:00- 4:10 (25) An elemental divisibility property of groups. Professor HAROLD S. FINKELSTEIN, Emory University (709-A15)

FRIDAY, 2:30 P. M.

Session on Functional Analysis, Room 211, Skiles Classroom Building

2:30- 2:40 (26) An order topology in ordered topological vector spaces. Preliminary report. Mr. LYNE HYNER CARTER, Florida State University (709-B2)

2:45- 2:55 (27) Approximation of compact homogeneous maps. Professor JOHN R. HUBBARD, Columbus College (709-B3)

3:00- 3:10 (28) Results on Auerbach bases for finite-dimensional normed spaces. Professor ROBERT J. KNOWLES*, University of Connecticut at Waterbury and Professor THURLOW A. COOK, University of Massachusetts (709-B13)

3:15- 3:25 (29) Approximation numbers of diagonal maps from ℓ^p to ℓ^q, $1 \leqq p < q \leqq \infty$. Preliminary report. Mr. PETER D. JOHNSON, Jr., Emory University (709-B22)

3:30- 3:40 (30) Distributions of exponential growth and their Fourier transforms. Dr. RICHARD D. CARMICHAEL, Wake Forest University (709-B23)

3:45- 3:55 (31) Linear isometries of subspaces of spaces of continuous functions. Professor WILLIAM P. NOVINGER, Florida State University (709-B28)

4:00- 4:10 (32) Absolute Schauder bases in C(X). Professor THURLOW A. COOK, University of Massachusetts (709-B36)

FRIDAY, 2:30 P. M.

Session on Special Functions, Fourier Analysis and Measures, Room 214, Skiles Classroom Building

2:30- 2:40 (33) Distributions for orthogonal polynomials whose recurrence is almost uniform. Preliminary report. Dr. WILLIAM P. McKIBBEN, Georgia Institute of Technology (709-B30)

2:45- 2:55 (34) Zeros of some recursively generated polynomials. Ms. GLADYS HAYES CRATES and Mr. JOHN W. JAYNE*, University of Tennessee, Chattanooga (709-B34)

3:00- 3:10 (35) Weighted Franklin series. Preliminary report. Dr. COKE S. REED, Auburn University (709-B41) (Introduced by Professor J. W. Rogers, Jr.)

3:15- 3:25 (36) The existence of realizations of hereditary systems. Professor JAMES A. RENEKE, Clemson University (709-B39)

3:30- 3:40 (37) A generalization of the Steinhaus-Kemperman theorem. Professor SOO BONG CHAE*, New College and Mr. VINCENT C. PECK, University of Rochester (709-B24)

3:45- 3:55 (38) Convergence sets and value regions for continued fractions. Preliminary report. Professor FRANCIS A. ROACH, University of Houston (709-B33)

FRIDAY, 2:30 P. M.

Session on Functions of a Complex Variable, Room 217, Skiles Classroom Building

2:30- 2:40 (39) Rational approximation of extremal length for doubly connected domains. Dr. C. WAYNE MASTIN, Mississippi State University (709-B6)

2:45- 2:55 (40) Toeplitz matrices generated by the Laurent expansion of an arbitrary rational function. Mr. K. MICHAEL DAY, University of Michigan (709-B11)

3:00- 3:10 (41) Concerning some subsets of functions analytic on the unit disc. Preliminary report. Professor DONALD E. RYAN, Northwestern State University (709-B18)

3:15- 3:25 (42) A characterization of completely convex functions. Dr. JAMES D. BUCKHOLTZ, University of Kentucky and Dr. J. K. SHAW*, Virginia Polytechnic Institute and State University (709-B26)

3:30- 3:40 (43) On univalent polynomials. Professor JOHN R. QUINE, Jr., Florida State University (709-B38)

FRIDAY, 2:30 P. M.

Session on Number Theory, Room 269, Skiles Classroom Building

2:30- 2:40 (44) On a problem of Ore: Harmonic numbers. Professor CARL POMERANCE, University of Georgia (709-A5)

2:45- 2:55 (45) Genera in abelian extensions. Professor ROBERT GOLD, Ohio State University (709-A12)

3:00- 3:10 (46) An intermediate theory for a purely inseparable Galois theory. Mr. JAMES K. DEVENEY, Florida State University (709-A19)

3:15- 3:25 (47) A software package for factoring in GF[q,x]. Preliminary report. Professor JACOB T. B. BEARD, Jr.* and Mrs. KAREN I. WEST, University of Texas at Arlington (709-A20)

3:30- 3:40 (48) Some analogs of arithmetic functions. Professor JULIANA DOWELL, East Carolina University (709-A22)

3:45- 3:55 (49) Class numbers of real quadratic number fields. Professor EZRA BROWN, Virginia Polytechnic Institute and State University (709-A23)

4:00- 4:10 (50) Factorization of certain decimal integers. Professor ANDREW SOBCZYK, Clemson University (709-A40)

FRIDAY, 2:30 P. M.

Session on Rings and Algebras, Room 270, Skiles Classroom Building

2:30- 2:40 (51) Finite basis theorem for rings and algebras satisfying a central condition. Professor CHANG MO BANG* and Professor KENNETH I. MANDELBERG, Emory University (709-A24)

2:45- 2:55 (52) Some finitely based varieties of rings. Professor TREVOR EVANS, Emory University (709-A32)

3:00- 3:10 (53) On subdirectly irreducible rings. Preliminary report. Mr. S. H. BROWN, Auburn University (709-A34) (Introduced by Professor C. C. Lindner)

3:15- 3:25 (54) Geometric rings. Preliminary report. Dr. JAPHETH HALL, Jr., Institute for Services to Education, Incorporated, Washington, D. C. (709-A39)

3:30- 3:40 (55) A characterization of finitely generated modules whose exterior rank equals the minimal number of generators. Preliminary report. Professor ROBERT B. GARDNER, University of North Carolina (709-A2)

3:45- 3:55 (56) Dimension theory of commutative rings without identity. Preliminary report. Professor JIMMY T. ARNOLD, Virginia Polytechnic Institute and State University, and Professor ROBERT GILMER*, Florida State University (709-A4)

4:00- 4:10 (57) Some countability conditions in a commutative ring. Preliminary report. Professor JIMMY T. ARNOLD, Virginia Polytechnic Institute and State University, Professor ROBERT GILMER, Florida State University and Professor WILLIAM J. HEINZER*, Purdue University (709-A17)

4:15- 4:25 (58) Some necessary conditions for pre-Prüfer domains. Preliminary report. Professor MONTE B. BOISEN, Jr. and Professor PHILIP B. SHELDON*, Virginia Polytechnic Institute and State University (709-A41)

FRIDAY, 2:30 P. M.

Session on General Topology I, Room 271, Skiles Classroom Building

2:30- 2:40 (59) Countable dense homogeneity of products of universal curves and manifolds. Mrs. NORMA F. LAUER, Auburn University (709-G1)

2:45- 2:55 (60) Generalizations of γ-spaces. Dr. PETER FLETCHER*, Virginia Polytechnic Institute and State University and Dr. WILLIAM F. LINDGREN, Slippery Rock State College (709-G2)

3:00- 3:10 (61) An example in fixed point theory. Professor GORDON G. JOHNSON, University of Houston (709-G3)

3:15- 3:25 (62) Characterizations of real functions by continua. Preliminary report. Mr. MAURICE HUGH MILLER, Jr., University of Alabama (Tuscaloosa) (709-G4) (Introduced by Professor Harvey Rosen)

3:30- 3:40 (63) Pseudo interiors of hyperspaces. Ms. NELLY KROONENBERG, Louisiana State University (709-G6)

3:45- 3:55 (64) On continuous images of Moore spaces. Professor GEORGE M. REED, Ohio University (709-G9)

4:00- 4:10 (65) Almost continuous retracts. Professor KENNETH R. KELLUM, Miles College (709-G10)

FRIDAY, 4:15 P. M.

Invited Address, Room 4, Space Science and Technology Building

(66) Regularization and numerical analysis of ill-posed operator equations. Professor M. ZUHAIR NASHED, Georgia Institute of Technology (709-C4)

SATURDAY, 9:00 A. M.

Invited Address, Lecture Room 4, Space Science and Technology Building

(67) Commutative groups and the cardinal connection. Professor PAUL D. HILL, Florida State University (709-A30)

SATURDAY, 9:00 A. M.

Session on Ordinary Differential Equations, Room 108, Skiles Classroom Building

9:00- 9:10 (68) Explicit solutions of a certain homogeneous autonomous system. Preliminary report. Professor RAYMOND H. ROLWING, University of Cincinnati (709-B40)

9:15- 9:25 (69) Oscillation of second order nonhomogeneous linear differential equations. Dr. STEVEN C. TEFTELLER, University of Alabama in Birmingham (709-B37)

9:30- 9:40 (70) Oscillation criteria for third order differential equations. Professor GARY D. JONES, Murray State University (709-B32)

9:45- 9:55 (71) Bounds for solutions of perturbed differential equations. Professor THOMAS G. PROCTOR, Clemson University (709-B27)

10:00-10:10 (72) Existence of solutions for ordinary differential equations in Banach spaces. Preliminary report. Mr. TIEN-YIEN LI, University of Maryland (709-B25)

10:15-10:25 (73) Continuous spectra of an even order equation. Professor DON B. HINTON, University of Tennessee (709-B12)

10:30-10:40 (74) A continuation theorem for a damped and forced nonlinear differential equation. Preliminary report. Professor JOHN W. BAKER, La Salle College (709-B10)

10:45-10:55 (75) Delay-feedback using derivatives for minimal time linear control problems. Dr. BERNARD A. ASNER*, University of Dallas and Dr. ARISTIDE HALANAY, University of Bucharest, Romania (709-B7)

11:00-11:10 (76) The discreteness of the spectrum of selfadjoint, even order, one-term, differential operators. Dr. ROGER T. LEWIS, Slippery Rock State College (709-B5)

SATURDAY, 10:15 A. M.

Special Session on Graph Theory, Lecture Room 4, Space Science and Technology Building

10:15-10:35	(77)	On r-partite graphs. Preliminary report. Dr. RENU LASKAR* and Mr. BRUCE AUERBACH, Clemson University (709-A11)
10:45-11:05	(78)	Realizable sets of boundary colorations. Preliminary report. Professor ROY B. LEVOW, Florida Atlantic University (709-A21)
11:15-11:35	(79)	Some Ramsey-type theorems for two-complexes. Preliminary report. Dr. RICHARD A. DUKE, Georgia Institute of Technology (709-A6)
11:45-12:05	(80)	Odd cycles and perfect graphs. Preliminary report. Professor DONALD L. GREENWELL, Emory University (709-A8)
12:15-12:35	(81)	Graph coloring and dimension theory. Preliminary report. Dr. WILLIAM T. TROTTER, Jr., University of South Carolina (709-A18)

SATURDAY, 10:15 A. M.

Special Session on the Application of Ordinary Differential Equations II, Room 202, Skiles Classroom Building

10:15-10:35	(82)	Initial-value problems for some infinite systems of ordinary differential equations. Preliminary report. Professor MARVIN B. SLEDD, Georgia Institute of Technology (709-B35)
10:45-11:05	(83)	Modeling the efficiency of the kidney. Preliminary report. Professor STEPHEN R. BERNFELD*, Memphis State University and Professor R. B. BECKMAN and Professor M. ZATZMAN, University of Missouri (709-C7)
11:15-11:35	(84)	On singular perturbation of nonlinear two-point boundary value problems. Professor JOHN V. BAXLEY, Wake Forest University (709-B9)
11:45-12:05	(85)	Ordinary differential equations in neutron transport theory. Professor W. ROBERT BOLAND, Clemson University and Professor PAUL NELSON, Jr.*, Texas Tech University (709-B29)

SATURDAY, 10:15 A. M.

Session on Combinatorics and General Systems, Room 218, Skiles Classroom Building

10:15-10:25	(86)	Primitive elements and one relation algebras. Dr. CATHERINE C. AUST, Georgia Institute of Technology (709-A28)
10:30-10:40	(87)	The chromatic polynomial of a complete r-partite graph. Professor RENU LASKAR and Professor WILLIAM R. HARE, Jr.*, Clemson University (709-A36)
10:45-10:55	(88)	Separable quasigroups. Dr. DWIGHT STEEDLEY, Auburn University (709-A35) (Introduced by Professor Charles C. Lindner)
11:00-11:10	(89)	The chromatic number of a graph with specified skewness. Professor PAUL C. KAINEN, Case Western Reserve University (709-A27)
11:15-11:25	(90)	Some remarks on the Steiner triple systems associated with Steiner quadruple systems. Dr. CHARLES C. LINDNER, Auburn University (709-A7)
11:30-11:40	(91)	A formula for the partition function. Preliminary report. Dr. ANDY N. C. KANG*, Virginia Commonwealth University, and Dr. C. K. KANG, Princeton University (709-A1) (Introduced by Dr. Robert J. Schwabauer)

SATURDAY, 10:15 A. M.

Session on Linear and Multilinear Algebras, Room 208, Skiles Classroom Building

10:15-10:25	(92)	On the classification of quadratic forms over semilocal rings. Professor KENNETH L MANDELBERG, Emory University (709-A25)
10:30-10:40	(93)	A relationship between characteristic values and vectors. Mr. EUGENE THOMAS BEASLEY, Jr.*, Ohio State University, and Professor PETER M. GIBSON, University of Alabama in Huntsville (709-A29)
10:45-10:55	(94)	Power sums of matrices over a finite field. Professor JOEL V. BRAWLEY*, Clemson University, Professor LEONARD CARLITZ, Duke University, and Professor JACK LEVINE, North Carolina State University (709-A33)
11:00-11:10	(95)	Unitary and orthogonal transformations on matrices. Professor PETER M. GIBSON, University of Alabama in Huntsville (709-A38)

SATURDAY, 10:15 A. M.

Session on Operator Theory, Room 211, Skiles Classroom Building

10:15-10:25	(96)	Nonlinear perturbation of m-accretive operators. Professor WILLIAM E. FITZGIBBON, University of Houston (709-B1)
10:30-10:40	(97)	Nonlinear eigenvalues. Professor E. LEE MAY, Jr., Salisbury State College (709-B4)
10:45-10:55	(98)	Generalized multi-parameter resolvents. Dr. RONALD SHONKWILER, Georgia Institute of Technology (709-B15)
11:00-11:10	(99)	Evolution system approximations of solutions to closed linear operator equations. Mr. SEATON D. PURDOM, Georgia Institute of Technology (709-B31)

SATURDAY, 10:15 A. M.

Session on Partial Differential Equations, Room 214, Skiles Classroom Building

10:15-10:25	(100)	Oscillation of hyperbolic equations. Professor CURTIS CLYDE TRAVIS, University of Tennessee (709-B14)
10:30-10:40	(101)	A characterization of hypoelliptic differential operators with variable coefficients. Preliminary report. Professor R. E. WHITE, North Carolina State University (709-B16) (Introduced by Professor Y. W. Chen)
10:45-10:55	(102)	Uniqueness theorems for a singular ultrahyperbolic equation. Preliminary report. Professor EUTIQUIO C. YOUNG, Florida State University (709-B17)
11:00-11:10	(103)	Existence and stability for partial functional differential equations. Professor CURTIS CLYDE TRAVIS, University of Tennessee and Professor GLENN F. WEBB*, University of Kentucky (709-B21)

SATURDAY, 10:15 A. M.

Session on Applied Mathematics, Room 217, Skiles Classroom Building

10:15-10:25	(104)	Potential operators for perturbed Markov processes. Preliminary report. Professor DHANDAPANI KANNAN, University of Georgia (709-F1)
10:30-10:40	(105)	A Markov inequality in several dimensions. Professor DON R. WILHELMSEN, University of Georgia (709-C8)
10:45-10:55	(106)	Projection methods and singular two-point boundary value problems. Professor G. W. REDDIEN, Jr., Vanderbilt University (709-C6)
11:00-11:10	(107)	Subspaces of transition probability spaces. Preliminary report. Professor JOHAN G. F. BELINFANTE, Georgia Institute of Technology (709-C5)
11:15-11:25	(108)	Support maximizing operators in quantum field theory. Dr. ALAN DAVID SLOAN, Georgia Institute of Technology (709-C3)
11:30-11:40	(109)	Complementarity problems over cones with monotone and pseudo-monotone maps. Preliminary report. Dr. STEPAN KARAMARDIAN, University of California, Irvine and Clemson University (709-C1)

SATURDAY, 10:15 A. M.

Session on General Topology II, Room 270, Skiles Classroom Building

10:15-10:25	(110)	Uniformly primitively complete mappings. Professor HOWARD H. WICKE* and Professor JOHN M. WORRELL, Jr., Ohio University (709-G23)
10:30-10:40	(111)	Weakly chainable circle-like continua. Preliminary report. Mr. GARY A. FEUERBACHER, University of Houston (709-G24) (Introduced by Dr. W. T. Ingram)
10:45-10:55	(112)	Baire's and Reed's convergence criteria in totally nonmeagre spaces. Professor JACK B. BROWN, Auburn University (709-G25)
11:00-11:10	(113)	A note on [a,b]-compactness. Professor RICHARD E. HODEL, Duke University and Professor JERRY E. VAUGHAN*, University of North Carolina, Greensboro (709-G26)
11:15-11:25	(114)	Semigroups of continuous relations. Preliminary report. Dr. EUGENE M. NORRIS, University of South Carolina (709-G27)
11:30-11:40	(115)	Extensions of homomorphisms in C(X,G). Preliminary report. Professor JEONG SHENG YANG, University of South Carolina (709-G28)

SATURDAY, 10:15 A. M.

Session on General Topology III, Room 271, Skiles Classroom Building

10:15-10:25	(116)	A sum theorem for confluent mappings. Professor ANDRZEJ LELEK, University of Houston (709-G11)
10:30-10:40	(117)	Jones' space. Dr. C. WAYNE PROCTOR, Stephen F. Austin State University (709-G12)
10:45-10:55	(118)	The index of periodicity of a transitive flow. Preliminary report. Professor ETHAN M. COVEN, Wesleyan University, and Professor BENJAMIN G. KLEIN*, Davidson College (709-G14)
11:00-11:10	(119)	Connectedness of noninvertible elements in semigroups. Preliminary report. Dr. JAMES C. KROPA, Judson College (709-G17)
11:15-11:25	(120)	Mapping arcwise connected continua onto cyclic continua. Professor W. KUPERBERG, University of Houston (709-G18)
11:30-11:40	(121)	Metric and symmetric spaces. Dr. PETER W. HARLEY III, University of South Carolina (709-G21)
11:45-11:55	(122)	Fixed and periodic points of local contraction mappings on probabilistic metric spaces. Professor GEORGE L. CAIN, Jr. and Professor ROBERT H. KASRIEL*, Georgia Institute of Technology (709-G22)

Tallahassee, Florida

O. G. Harrold, Jr.
Associate Secretary

Presentors of Papers

Following each name is the number corresponding to the speaker's position on the program

• Invited one-hour lectures

Asner, Bernard A. #75
Aust, Catherine C. #86
Baker, John W. #74
Bang, Chang Mo #51
Barnhart, Richard D. #17
Baxley, John V. #84
Beard, Jacob T.B., Jr. #47
Beasley, Eugene T., Jr. #93
Belinfante, Johan G. F. #107
Bernfeld, Stephen R. #83
Bowman, Thomas T. #12
Brawley, Joel V. #94
Brown, Ezra #49
Brown, Jack B. #112
Brown, S. H. #53
Butler, Kim Ki-Hang #21
Carmichael, Richard D. #30
Carter, Lyne H. #26
Chae, Soo Bong #37
Chandra, Jagdish #8
Cook, Thurlow A. #32
Day, K. Michael #40
Deveney, James K. #46
Dowell, Juliana #48
Duke, Richard A. #79
Evans, Trevor #52
Feuerbacher, Gary A. #111
Finkelstein, Harold S. #25
Fitzgibbon, William E. #96
•Fitzpatrick, Ben, Jr. #1
Fletcher, Peter #60
French, James A. #16
Gardner, Robert B. #55
Gibson, Peter M. #95
Gilmer, Robert #56
Gold, Robert #45
Greenwell, Donald L.
Gross, Kenneth I. #13
Hall, Japheth, Jr. #54
Hallam, Thomas G. #9
Hare, William R., Jr. #87
Harley, Peter W. III #121
Heinzer, William J. #57
Hemminger, Robert L. #4
•Hill, Paul D. #67
Hinton, Don B. #73
Holmes, John P. #15
Hubbard, John R. #27
Hunter, Christopher #10
Jayne, John W. #34
Johnson, Gordon G. #61
Johnson, Peter D., Jr. #29
Jones, Gary D. #70
Kainen, Paul C. #89
Kang, Andy N.C. #91
Kannan, Dhandapani #104
Kannappan, Palaniappan #22
Karamardian, Stepan #109
Kasriel, Robert H. #122
Kellum, Kenneth R. #65
Klein, Benjamin G. #118
Knowles, Robert J. #28
Krabill, James R. #20
Kroonenberg, Nelly #63
Kropa, James C. #119
Kuperberg, W. #120
Laskar, Renu #77
Lauer, Norma F. #59
Lelek, Andrzej #116
Levow, Roy B. #78
Lewis, Roger T. #76
Li, Tien-Yien #72
Lindner, Charles C. #90
Linton, Ronald C. #24
Mandelberg, Kenneth I. #92
Mastin, C. Wayne #39
May, E. Lee, Jr. #97
McKibben, William P. #33
Miller, Maurice H., Jr. #62
Mueller-Roemer, Peter R. #11
•Nashed, M. Zuhair #7 & #66
Nelson, Paul, Jr. #85
Norris, Eugene M. #114
Novinger, William P. #31
O'Neil, Peter V. #3
Pomerance, Carl #44
Proctor, C. Wayne #117
Proctor, Thomas G. #71
Purdom, Seaton D. #99
Quine, John R., Jr. #43
Rajagopalan, M. #14
Reddien, G. W., Jr. #106
Reed, Coke S. #35
Reed, George M. #64
Reid, Kenneth B. #2
Reneke, James A. #36
Roach, Francis A. #38
Rolwing, Raymond H. #68
Ryan, Donald E. #41
Sanerib, Richard A., Jr. #19
Schelp, Richard H. #6
Shaw, J. K. #42
Sheldon, Philip B. #58
Shonkwiler, Ronald #98
Sigmon, Kermit N. #18
Sledd, Marvin B. #82
Sloan, Alan D. #108
Sobczyk, Andrew #50
Steedley, Dwight #88
Sumner, David P. #5
Tefteller, Steven C. #69
Travis, Curtis C. #100
Trotter, William T., Jr. #81
Vaughan, Jerry E. #113
Webb, Glenn F. #103
White, R. E. #101
Wicke, Howard H. #110
Wilhelmsen, Don R. #105
Yang, Jeong Sheng #115
Yeager, Dorian P. #23
Young, Eutiquio C. #102

The Seven Hundred Tenth Meeting
University of Arizona
Tucson, Arizona
November 23 – 24, 1973

The seven hundred tenth meeting of the American Mathematical Society will be held at the University of Arizona in Tucson, Arizona, on Friday and Saturday, November 23–24, 1973. The invited addresses and regular sessions for contributed papers will be scheduled on Saturday. Special Sessions on Singular Perturbations will be scheduled on Friday afternoon and Saturday morning.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two invited addresses. Professor Andrew P. Ogg of the University of California, Berkeley, will lecture at 11:00 a.m. on Saturday; the title of his lecture will be "Diophantine equations and modular forms." Professor Robert R. Phelps of the University of Washington will lecture at 2:00 p.m. on Saturday on "The Choquet representation in the complex case." Both of these lectures will be given in Room 201 of the Physics, Mathematics, and Meteorology (PMM) Building, which is adjacent to the Mathematics Building.

There will be sessions for contributed papers on Saturday. All sessions will be held in the PMM Building. Late papers will be accepted for presentation at the meeting, but they will not be listed in the printed program.

Professors Paul C. Fife and Wilfred M. Greenlee of the University of Arizona are arranging two Special Sessions on Singular Perturbations consisting of thirty-minute talks. The first session will be held on Friday afternoon, and the second session will be held on Saturday morning. The speakers will be Donald S. Cohen, Julian D. Cole, Frank C. Hoppensteadt, Joseph B. Keller, J. Kevorkian, Norman R. Lebovitz, and Robert E. O'Malley, Jr.

The registration desk will be located in Room 402 of the Mathematics Building. Registration hours will be from 2:00 p.m. to 4:00 p.m. on Friday, November 23, and from 8:30 a.m. to 2:00 p.m. on Saturday, November 24.

The following hotels and motels are located in Tucson; only the Plaza International is within reasonable walking distance of the Mathematics Building. Reservations should be made directly with the hotel or motel. Advance deposits may be required.

EXECUTIVE INN
333 West Drachman (85705)
Phone: (602) 623-5781

Single	$14 up
Twin	17 up

FLAMINGO MOTOR HOTEL
1300 North Stone Avenue (85705)
Phone: (602) 624-5571

Single	$ 9 up
Double	13 up

PLAZA INTERNATIONAL HOTEL
1900 East Speedway at Campbell (85719)
Phone: (602) 327-7341

Single	$16 up
Double	18 up

ROYAL INN
1015 North Stone Avenue (85705)
Phone: (602) 624-8771

Single	$15 up
Double	18 up

Noon meals will be available at the Student Union. A list of off-campus eating establishments will be available at the registration desk.

Tucson is served by seven airlines and two bus lines. Limousine service is available from the airport. Persons driving to the meeting on Interstate 10 should take the Speedway exit, drive 1 1/4 miles east on Speedway to Euclid, turn right and drive 1/2 mile south to Sixth, turn left and drive 1/2 mile east to Highland, turn left and drive two blocks to the Science Library parking lot. The Mathematics Building may be reached by walking one block west and one block south from the entrance to the parking lot.

Emergency messages may be delivered to participants by telephoning the Campus Police (602) 884-1485.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper in the general sessions is ten minutes and in the special sessions is thirty minutes. To maintain this schedule, the time limits will be strictly enforced.

FRIDAY, 2:00 P. M.

Special Session on Singular Perturbations I, Room 486, PMM Building

2:00- 2:30 (1) Instabilities and relaxation oscillations in the spatial and temporal organizations of chemical systems. Professor DONALD S. COHEN, University of Arizona and California Institute of Technology (710-B5)

2:40- 3:10 (2) Some singular perturbation problems of cell physiology. Preliminary report. Professor JULIAN D. COLE, University of California, Los Angeles (710-B6). (Introduced by Professor Paul C. Fife)

3:20- 3:50 (3) Asymptotic stability of singularity perturbed systems. Professor FRANK C. HOPPENSTEADT, Courant Institute, New York University (710-B7)

4:00- 4:30 (4) Forced nonlinear vibrations and perturbed bifurcation theory. Professor JOSEPH B. KELLER, Courant Institute, New York University, and California Institute of Technology (710-C3)

SATURDAY, 9:00 A. M.

Special Session on Singular Perturbations II, Room 486, PMM Building

9:00- 9:30 (5) Resonance in systems with slowly varying coefficients and small nonlinearities. Professor J. KEVORKIAN, University of Washington (710-B12) (Introduced by Professor W. M. Greenlee)

9:40-10:10 (6) The singularly perturbed initial-value problem when the reduced path encounters a point of bifurcation. Preliminary report. Professor NORMAN R. LEBOVITZ*, and Mr. RICHARD J. SCHAAR, University of Chicago (710-B3)

10:20-10:50 (7) The singular perturbation solution to problems of cheap control. Professor ROBERT E. O'MALLEY, JR., University of Arizona (710-C1)

SATURDAY, 9:45 A. M.

General Session, Room 274, PMM Building

9:45- 9:55 (8) Polytope pairs and their relationship to linear programming. Professor VICTOR L. KLEE, JR., University of Washington (710-D1)

10:00-10:10 (9) The dilatation group of a generalized affine plane. Preliminary report. Dr. JAMES R. CLAY, University of Arizona (710-A4)

10:15-10:25 (10) A decomposition theorem for multi-sorted algebras. Dr. R. ARTHUR KNOEBEL, New Mexico State University (710-A2)

10:30-10:40 (11) The Weyl group of SU(n) on zero-weight spaces. Dr. DAVID A. GAY, New College (710-A3)

SATURDAY, 9:45 A. M.

Session on Analysis I, Room 276, PMM Building

9:45- 9:55 (12) A classification theorem for the Hilbert transform over a local field. Mr. CHARLES DOWNEY, New Mexico State University (710-B9) (Introduced by Dr. Keith Phillips)

10:00-10:10 (13) On the mathematical structure of a model converging in a space of semidefinite metric. Dr. KETILL INGÓLFSSON, University of Iceland, Reykjavík (710-C2) (Introduced by Professor Henry B. Mann)

10:15-10:25 (14) On algebras generated by composition operators. Professor JOSEPH A. CIMA* and Mr. WARREN R. WOGEN, University of North Carolina (710-B10)

10:30-10:40 (15) Differential recurrence theorems associated with a class of polynomials. Professor MOSES E. COHEN, California State University, Fresno (710-B11)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

SATURDAY, 11:00 A. M.

Invited Address, Room 201, PMM Building

(16) Diophantine equations and modular forms. Professor ANDREW P. OGG, University of California, Berkeley (710-A1)

SATURDAY, 2:00 P. M.

Invited Address, Room 201, PMM Building

(17) The Choquet representation theorem in the complex case. Professor ROBERT R. PHELPS, University of Washington (710-B8)

SATURDAY, 3:15 P. M.

Session on Analysis II, Room 276, PMM Building

3:15- 3:25 (18) Integrodifferential equations of real and complex Markov chains. Professor ALI KRYALA, Arizona State University (710-F1)

3:30- 3:40 (19) Variations on a theme of Coddington and Levinson. Professor JOHN V. BAXLEY, Wake Forest University (710-B2)

3:45- 3:55 (20) Diagonalization method in singular perturbations. Professor K. W. CHANG, University of Calgary (710-B4)

4:00- 4:10 (21) An application of Nagumo's lemma to some singularly perturbed systems. Dr. FREDERICK A. HOWES, University of Southern California (710-B13)

Kenneth A. Ross
Associate Secretary

Eugene, Oregon

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

The Eightieth Annual Meeting and Preceptorial Introduction to Computer Science for Mathematicians San Francisco Hilton Hotel San Francisco, California January 13 – 18, 1974

PRECEPTORIAL INTRODUCTION TO COMPUTER SCIENCE FOR MATHEMATICIANS

The American Mathematical Society will present a two-day Preceptorial Introduction to Computer Science for Mathematicians on Sunday and Monday, January 13 and 14, in the California Room of the San Francisco Hilton. This short course, which is open to all who wish to participate, is a repetition of the short course given originally at the summer meeting in Missoula on the recommendation of the AMS Committee on Employment and Educational Policy. The success of the program in Missoula led the committee to recommend that the course be repeated in order to make it available to a larger audience.

The program is under the direction of Professor Jacob T. Schwartz, Courant Institute of Mathematical Sciences, New York University. The members of the AMS Committee on Employment and Educational Policy are Richard D. Anderson (chairman), Michael Artin, John W. Jewett, Calvin C. Moore, Richard S. Palais, and Martha Kathleen Smith.

The program will consist of six lectures on various aspects of computer science intended to provide a concentrated introduction to the field, thus making it possible for the participants to judge if computer science is a subject which they would be interested in pursuing further. The speakers and their topics are Jacob T. Schwartz, Courant Institute of Mathematical Sciences, New York University, "Pragmatic and theoretical considerations concerning programming"; Richard M. Karp, University of California, Berkeley, "Lower and upper bounds on the computational complexity of combinatorial problems"; and Albert R. Meyer, Massachusetts Institute of Technology, "Discrete computation: Theory and open problems".

EIGHTIETH ANNUAL MEETING

The eightieth annual meeting of the American Mathematical Society will be held at the San Francisco Hilton Hotel in San Francisco, California, from Tuesday, January 15, through Friday, January 18, 1974. The meeting will be held in conjunction with the annual meeting of the Mathematical Association of America (January 17–19). The Society and the Association will cosponsor a panel discussion on Thursday, January 17, at 9:00 a.m. Professor H. L. Alder of the University of California, Davis, will serve as moderator; the topic to be discussed is "The problem of learning to teach." The members of the panel will be Professor Paul R. Halmos, Indiana University; Professor Edwin E. Moise, Queens College, City University of New York; and Professor George Piranian, University of Michigan. The Conference Board of the Mathematical Sciences will present a panel discussion on "Mathematics and Society" at 2:30 p.m. on Thursday, January 17.

The AMS Committee on Employment and Educational Policy is planning two panel discussions. The first is scheduled for Wednesday, January 16, 1974, at 8:30 p.m. Dr. Henry O. Pollak of the Bell Telephone Laboratories will serve as moderator; the topic to be discussed is "Nonacademic employment of Ph.D.'s." Additional members of the panel are Dr. Edward E. David, Jr., Executive Vice-president of Gould, Inc. and former Presidential Science Advisor; Mr. John M. McQuown, Vice-president and Director of Management Sciences, Wells Fargo Bank, San Francisco; and Dr. Carroll V. Newsom, former President, New York University. The second panel discussion is scheduled for Thursday, January 17, 1974, at 8:30 p.m., and will be moderated by Professor P. Emery Thomas, University of California, Berkeley. Members of the panel include Professor William Browder, Princeton University; Professor Karel deLeeuw, Stanford University; Professors Israel N. Herstein and Saunders Mac Lane, University of Chicago. The topic to be discussed is "The role of the dissertation in the Ph.D. program." Both panels will be held in the Continental Ballroom.

There will be two sets of Colloquium Lectures, each consisting of four one-hour talks. Professor Louis Nirenberg of the Courant Institute of Mathematical Sciences, New York University, will give one of the sets of Colloquium Lectures entitled "Selected topics in partial differential equations." The other set of lectures will be given by Professor John G. Thompson of the University of Cambridge; he will lecture on "Finite simple groups." All major addresses will be given in the Continental Ballroom.

The Retiring Presidential Address will be given by Professor Nathan Jacobson of Yale University at 2:45 p.m. on Wednesday, January 16, 1974. The title of his lecture will be "Some groups and Lie algebras defined by Jordan algebras."

The Josiah Willard Gibbs Lecture will be presented by Professor Paul A. Samuelson of the Massachusetts Institute of Technology at 8:30 p.m. on Tuesday, January 15, 1974. The title of his lecture will be "Economics and mathematical analysis."

In the normal course of events, the Bôcher Memorial Prize in analysis will be awarded at a session at 2:15 p.m. on Wednesday, January 16.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, there will be eight invited hour addresses. They will be given by Professor Elwyn R. Berlekamp of the University of California, Berkeley; Professor Richard J. Duffin of Carnegie-Mellon University; Professor Adriano M. Garsia of the University of California, San Diego; Professor Shoshichi Kobayashi of the University of California, Berkeley; Professor Barry M. Mitchell of Rutgers University; Professor Dijen K. Ray-Chaudhuri of Ohio State University; Professor Louis Solomon of the University of Wisconsin, Madison; and Professor Alan D. Weinstein of the University of California, Berkeley. The titles and schedules of these addresses are listed in the Summary of Activities, which follows this announcement.

There will be no limit on the number of contributed ten-minute papers. No provision will be made for late papers.

SPECIAL SESSIONS

There will be several special sessions of selected twenty-minute papers. The probable schedules of these sessions are given in the Summary of Activities which follows this announcement.

Professor Frank W. Anderson of the University of Oregon is organizing one or two special sessions on Ring Theory. Among the speakers will be Victor P. Camillo, Robert S. Cunningham, Carl Faith, Bruno J. W. Mueller, Richard S. Pierce, and Claudio Procesi.

Professor Lipman Bers of Columbia University is organizing two special sessions entitled Crash Course on Kleinian Groups. There will be eight interrelated talks which together will attempt to give an introduction to the subject, and a survey of the present state of the field. The speakers will aim their talks at nonspecialists assuming only the general knowledge of complex variables which every analyst has. It is hoped that mimeographed notes will be available at the time of the meeting. Among the speakers will be William Abikoff, Lipman Bers, Clifford J. Earle, Jr., Frederick P. Gardiner, Irwin Kra, Albert Marden, Bernard Maskit, and Halsey L. Royden.

Professor Lamberto Cesari of the University of Michigan and Professor Jack K. Hale of Brown University are organizing special sessions on Functional Analysis Methods in Nonlinear Differential Equations, Ordinary and Partial. Among the speakers will be Henry A. Antosiewicz, Lamberto Cesari, Jack K. Hale, William S. Hall, Rangachary Kannan, Alan C. Lazer, David A. Sanchez, Duane P. Sather, and Daniel Sweet.

Professor Laszlo Fuchs of Tulane University is organizing two special sessions on Abelian Groups. Invited expository talks on the most recent developments in certain areas will be given by Paul D. Hill, John M. Irwin, Everett L. Lady, Charles K. Megibben, Fred Richman, Carol Lee Walker, Elbert A. Walker, and Robert B. Warfield, Jr. The remainder of the talks will deal with various questions, primarily on the structure of abelian groups.

Professor Leslie C. Glaser of the University of Utah is organizing two special sessions on Geometrical Topology. A tentative list of speakers includes B. J. Ball, Marshall M. Cohen, Robert J. Daverman, Paul F. Duvall, Jr., William T. Eaton, Robert D. Edwards, Kenneth C. Millett, T. Benny Rushing, and Richard M. Schori.

Professor Ray A. Kunze of the University of California, Irvine, is organizing two or three special sessions on Non-Abelian Harmonic Analysis. The list of speakers will include Roe W. Goodman, Kenneth I. Gross, K. Johnson, Adam Kleppner, Bertram Kostant, Ronald L. Lipsman, A. Edward Nussbaum, L. Preiss Rothschild, Paul J. Sally, Jr., Elias M. Stein, Peter C. Trombi, Nolan R. Wallach, Guido L. Weiss, Norman J. Weiss, and Joseph A. Wolf.

Professor William J. LeVeque of Claremont Graduate School is organizing two special sessions on Distribution Modulo 1 and Random Number Generation. Both research and expository talks will be given at these sessions. Among the speakers are John H. Halton, Harald G. Niederreiter, Wolfgang M. Schmidt, and Stanislaw Zaremba.

Professor P. S. Mostert of the University of Kansas and Professor Alfred H. Clifford of Tulane University are organizing special sessions on Topological Semigroups and Algebraic Semigroups, respectively. The following will participate in the Topological Semigroups session: James H. Carruth, Charles F. Dunkl, J. G. Horne, Jr., Larry King, Jimmy D. Lawson, Michael W. Mislove, Donald E. Ramirez, and Albert R. Stralka. Speakers at the Algebraic Semigroups sessions will include Pierre A. Grillet, J. Leech, Donald B. McAlister, Kenneth D. Magill, Jr., John Rhodes, and Takayuki Tamura.

Professor Jean E. Rubin of Purdue University is organizing two special sessions on Set Theory and the Axiom of Choice. Two expository hour-talks are planned; one of them will be given by J. R. Buchi. The following will give twenty-minute talks: James D. Halpern, Paul E. Howard, Thomas J. Jech, Arthur Kruse, Hidegoro Nakano, David F. Pincus, Arthur L. Rubin, Gaisi Takeuti, and Martin M. Zuckerman.

Professor Gary M. Seitz of the University of Oregon is organizing two or three special sessions on Representations of Finite Groups. Among the speakers will be Thomas R. Berger, Burton I. Fein, Walter Feit, J. Sutherland Frame, I. M. Isaacs, Gerald J. Janusz, Robert W. Kilmoyer, Leonard L. Scott, Jr., Stephen D. Smith, Bhama Srinivasan, David Wales, and Warren J. Wong.

Professor Lawrence J. Wallen of the University of Hawaii is organizing a special session on Special Operators. Speakers include Joseph J. Bastian, Gerhard K. Kalisch, Paul S. Muhly, and Robert E. Waterman. A discussion period to follow the talks is planned.

COUNCIL AND BUSINESS MEETING

The Council will meet on Monday, January 14, at 2:00 p.m. in Continental Parlors 1 and 2 of the Hilton Hotel. Most of the meeting is open to members of the Society as observers. The agenda will be posted.

The Business Meeting will be held on Wednesday, January 16, at 4:00 p.m. in the Continental Ballroom of the Hilton Hotel. Both the Continental Ballroom and Parlors are located on the ballroom level of the hotel. The Secretary notes the following resolution of the Council: Each person who attends a Business Meeting of the Society shall be willing and able to identify himself as a member of the Society. In further explanation, it is noted that "each person who is to vote at a meeting is thereby identifying himself as and claiming to be a member of the American Mathematical Society."

STATED BUSINESS

In accord with Article X, Section 1, of the Bylaws, it is announced that three resolutions have been placed on the agenda of the Business Meeting for possible final action. Members should refer to pages 257–270 of the October issue of these *Notices* for a detailed discussion of these resolutions. The text of the resolutions is as follows:

Resolution A: In cases of alleged discrimination against mathematicians in matters of hiring, promotion or tenure for reasons of racial, sex or political bias, AMS shall pay legal expenses for the complainant in those cases where the complainant, in the opinion of the Council, has made out a prima facie case of such discrimination.

Resolution B: The AMS declares itself in favor of the massive transfer of funds from the federal military budget to the support of education, including higher education, and calls upon the Council and staff to engage the Society in lobbying and publicity efforts directed to this end.

Resolution C: The AMS views with alarm the present practice of increasing class sizes and teaching loads and other measures designed to save money by decreasing faculties at the expense of educational quality. The Council and staff of the Society are called upon to seek avenues for inducing colleges and universities to reverse this unhealthy trend.

MEETING PREREGISTRATION AND REGISTRATION

Computer short course participants may register at the desk in the West Lounge which is located outside of the California Room. The desk will be open from 8:30 a.m. to 2:00 p.m. on Sunday, January 13, and from 8:30 a.m. to 1:00 p.m. on Monday, January 14.

The registration desk for the joint meeting will be located in the Tower Lobby on the lobby level of the Hilton Hotel. The desk will be open from 2:00 p.m. to 8:00 p.m. on Monday, January 14; from 8:00 a.m. to 5:00 p.m. on Tuesday, January 15; from 8:30 a.m. to 4:30 p.m. on Wednesday through Friday, January 16–18; and from 8:30 a.m. to 2:30 p.m. on Saturday, January 19.

Participants who wish to preregister should complete the Meeting Preregistration Form found on the last page of these *Notices*. Those who preregister will pay a lower registration fee than those who register at the meetings, as indicated in the schedule below. Preregistrants will be able to pick up their badges and programs when they arrive at the meeting. Complete instructions on the procedure for making hotel reservations is given in the section entitled ACCOMMODATIONS.

Please note that separate registration is required for each of the two meetings. Registration fees for the meetings are as follows:

	Preregistration (by mail prior to 12/15)	At meeting
Computer Short Course		
All participants	$ 7	$ 10
AMS-MAA Annual Meeting		
Member	7	10
Student or unemployed member	1	1
Nonmember	12	15

There will be no extra charge for members of the families of registered participants.

The unemployed status refers to any member currently unemployed and actively seeking employment. It is not intended to include members who have voluntarily resigned or retired from their latest position.

Students are considered to be only those currently working toward a degree who do not receive an annual compensation totaling more than $7,000 from employment, fellowships, and scholarships.

Checks for the preregistration fee should be mailed to arrive not later than December 15, 1973. Participants may make their own reservations directly with any hotel in the area if they wish. It is essential, however, to complete the Meeting Preregistration Form on the last page of these *Notices* to take advantage of the lower meeting registration fee.

A fifty percent refund of the preregistration fee will be reimbursed for all cancellations received prior to January 14. There will be no refunds granted for cancellations received after that date nor to persons who do not attend the meetings.

EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register will be maintained from 9:00 a.m. to 4:00 p.m. on Wednesday, January 16, and from 9:00 a.m. to 5:40 p.m. on Thursday through Saturday, January 17–19. The addition of an extra day is a departure from past procedure. It was recommended by the Joint Committee on Employment Opportunities in an attempt to expand the interview schedule and to eliminate the necessity for evening interviews. The Register will be located in the Imperial Ballroom which is located on the ballroom level of the Hilton Hotel.

EXHIBITS

The book and educational media exhibits will be displayed in the Hilton Plaza of the Hilton Hotel, from Tuesday through Friday, January 15–18. The exhibits will be displayed from noon to 5:00 p.m. on Tuesday; from 9:00 a.m. to 5:00 p.m. on Wednesday and Thursday; and from 9:00 a.m. to noon on Friday. All participants are encouraged to plan a visit to the exhibits sometime during the meeting. The Hilton Plaza is located adjacent to the registration area.

AUDIO TAPES AND BOOK SALE

Audio tapes of invited addresses and books published by the Society and the Association will be sold for cash prices somewhat below the usual prices when these same books and tapes are sold by mail.

ACCOMMODATIONS

Forms for requesting accommodations will be found on the last two pages of these *Notices*. Please note that there are two separate and distinct forms: one for students and unemployed members who would like to reserve a triple room at the Hilton Hotel and a second form for regular preregistration and reservations.

The use of the housing services requires preregistration for the meeting. Persons desiring accommodations should complete the appropriate form (or a reasonable facsimile) and send it to the Mathematics Meetings Housing Bureau, P.O. Box 6887, Providence, Rhode Island 02940. The AMS will forward the reservation forms to the San Francisco Convention Bureau which will handle accommodations. Reservations will be made in accordance with preferences indicated on the reservation form, insofar as this is possible, and all reservations will be confirmed. Deposit requirements vary from hotel to hotel, and participants will be informed of any such requirements at the time of confirmation. REQUESTS FOR RESERVATIONS SHOULD BE MAILED TO ARRIVE IN PROVIDENCE NO LATER THAN DECEMBER 15, 1973.

BELLEVUE	
Doubles	$20–22
Twins	22–24
BERESFORD	
Singles	$12
Doubles	16
Twins	18
CALIFORNIAN	
Singles	$19
Doubles	23
Twins	24
CARLTON	
Singles	$ 8.50–$10.00
Twins	12.50– 15.50
Rollaways	2.50
EL CORTEZ	
Singles	$14
Doubles	16–$18
Twins	18– 22
Suites	38– 40
GAYLORD	
Singles	$17–$18
Doubles and Twins	21– 23
HILTON	
Singles	$19, 21, 22, 23, 24, 25, 27, 29
Doubles or Twins	27, 29, 30, 31, 32, 33, 35, 37
*Triples	24
Suites	74 and up in Main Building 95 and up in Tower
RAMONA	
Singles	$12
Doubles	13.50
Twins	15.50
Suites	30
ST. FRANCIS	
Singles	$19, 21, 22, 23, 24, 25, 27, 29, 31
Doubles or Twins	27, 29, 30, 31, 32, 33, 35, 37, 39
Suites	60–$200 in Main Building 90– 500 in Tower
SAN FRANCISCAN	
Singles	$18–$22
Doubles	22– 26
Twins	18– 22
Triples	22– 26
Quadruples	26– 30
Suites	45– 60
SENATOR	
Singles	$12
Doubles	14
Twins	16
SHAW	
Singles	$11.50
Doubles	13.50
Twins	15.50
SIR FRANCIS DRAKE	
Singles	$22–$28
Doubles and Twins	29– 35
Suites	97
STEWART	
Singles	$16–$20
Doubles and Twins	20– 28
STRATFORD	
Singles	$14
Doubles	17
Twins	18

*The triples are reserved for students and unemployed members; see special reservation form which may be found on penultimate page of these *Notices* for criteria governing eligibility for these rooms.

Y. M. C. A.	
Singles	$ 9.40
Doubles	11.50
Twins	13.00

The San Francisco Hilton has five dining facilities: The Gazebo Restaurant (a coffee shop), which is open from 6:30 a.m. to 9:30 p.m.; the California Wine Garden, which is open from 6:30 a.m. to 2:00 a.m.; the Chef's Table, which is open from 5:30 p.m. to 11:00 p.m.; Henri's Room at the Top, which is open from 11:00 a.m. to 2:00 a.m. and offers a luncheon and dinner buffet as well as cocktail service; and KiKu of Tokyo, which is open for lunch and dinner from 11:45 a.m. to 2:00 p.m. and from 5:30 p.m. to 10:30 p.m., respectively.

NATIONAL SCIENCE FOUNDATION INFORMATION CENTER

NSF staff members will be available to provide counsel and information on all NSF programs of interest to mathematicians from 9:00 a.m. to 5:00 p.m. on January 16, 17, and 18, in the Rosewood Suite, Section A. This room is located on the fourth floor of the main building at the Hilton Hotel.

ENTERTAINMENT

There will be a No-Host cocktail party from 4:30 to 6:00 p.m. on Thursday, January 17, in the California Room at the Hilton Hotel. This will be the only major social function of the meeting, and everyone is invited to attend.

There are many things to see and much to do in the San Francisco Bay area. Brochures describing various tours around the city will be available at the Local Information Desk. These will include walking and automobile trips, Gray Line bus rides, and Harbor Tours by boat. There will also be brochures in the registration area describing some of the major attractions of San Francisco, such as Chinatown, North Beach, Golden Gate Park, and Nob Hill.

San Francisco has numerous museums and art galleries. At night there is entertainment available to suit all tastes, from jazz and "highly original" nightclubs to legitimate theater and classical musical events.

Some of the finest restaurants in the nation are located in San Francisco. The Convention Bureau will provide a list of outstanding dining places in the city. There will also be a guide to dining near the Hilton available at the Local Information Desk. This will list the places to eat (both plain and fancy) which are located within a few blocks of the San Francisco Hilton

TRAVEL AND LOCAL INFORMATION

Airlines serving San Francisco include American, Delta, Hughes Airwest, National, Pan American, Trans World, United, Western, and various international carriers. There is bus transportation from the San Francisco International Airport to the downtown airport bus terminal which is next door to the Hilton. The fare is approximately $1.15. Taxi fare into the city is considerably higher—approximately $11.

Railroad service to San Francisco is offered by the Northern Pacific, Santa Fe, Southern Pacific, and Western Pacific Railroads. Taxi service is available from the various railroad depots to the Hilton Hotel.

The bus lines serving San Francisco include the Continental Trailways and the Greyhound Bus Lines. The bus terminals are located within a few blocks of the Hilton Hotel.

The San Francisco Hilton has a parking garage within the hotel. Free parking is available to registered guests of the hotel who are staying on floors 5–11 of the main building. Fees for registered guests on floors other than those mentioned above are $4.25 for 24 hours (with unlimited in-and-out privileges) or $0.50 for each 1/2 hour for less than 24-hour service.

WEATHER

During the month of January, San Francisco's average maximum temperature is 55° and the minimum is 45°. There is a likelihood of encountering some rain, so that rain coats, umbrellas, and rubbers or overshoes may prove useful. For clothing, medium weight wool suits or dresses are recommended.

MAIL AND MESSAGE CENTER

All mail and telegrams for persons attending the meetings should be addressed in care of Mathematics Meetings, San Francisco Hilton Hotel, Mason and O'Farrell Streets, San Francisco, California 94102. Mail and telegrams so addressed may be picked up at the Mail and Information Desk located at the registration area in the Tower Lobby located on the lobby level of the hotel.

A message center will be located in the same area to receive incoming calls for all members in attendance. Messages may be left for registrants during the hours the registration desk is open, cf. the section entitled MEETING PREREGISTRATION AND REGISTRATION, above. Messages will be recorded, and the name of any member for whom a message has been received will be posted until the message has been picked up at the Message Center. Members are advised to leave the following number with anyone who might want to reach them at the meeting (415) 771-1400, Extension 304.

LOCAL ARRANGEMENTS COMMITTEE

H. L. Alder (ex officio), William G. Badé, William G. Chinn, Newman H. Fisher (chairman), Daniel Gallin, Estelle M. Goldberg, R. Sherman Lehman, Kenneth A. Ross (ex officio), and Gordon L. Walker (ex officio).

SUMMARY OF ACTIVITIES

The AMS Committee to Monitor Problems in Communication has recommended that a Summary of Activities appear in the issue of the NOTICES which contains a reservation form for either an annual or a summer meeting. The purpose of this summary is to provide assistance to registrants in the selection of arrival and departure dates. The program, as outlined below, is based on the information available at press time.

	AMERICAN MATHEMATICAL SOCIETY
	PRECEPTORIAL INTRODUCTION TO COMPUTER SCIENCE FOR MATHEMATICIANS
SUNDAY, January 13	
8:30 a.m. - 2:00 p.m.	REGISTRATION
9:15 a.m. - 10:30 a.m.	Pragmatic and theoretical considerations concerning programming, I Jacob T. Schwartz
11:00 a.m. - 12:15 p.m.	Lower and upper bounds on the computational complexity of combinatorial problems, I Richard M. Karp
2:00 p.m. - 3:15 p.m.	Discrete computation: Theory and open problems, I Albert R. Meyer
MONDAY, January 14	
8:30 a.m. - 1:00 p.m.	REGISTRATION
9:15 a.m. - 10:30 a.m.	Pragmatic and theoretical considerations concerning programming, II Jacob T. Schwartz
11:00 a.m. - 12:15 p.m.	Lower and upper bounds on the computational complexity of combinatorial problems, II Richard M. Karp
2:00 p.m. - 3:15 p.m.	Discrete computation: Theory and open problems, II Albert R. Meyer

	AMS - MAA ANNUAL MEETINGS	
	American Mathematical Society	Other Organizations
MONDAY, January 14		
2:00 p.m.	COUNCIL MEETING	
2:00 p.m. - 8:00 p.m.	REGISTRATION	REGISTRATION
TUESDAY, January 15		
8:00 a.m. - 5:00 p.m.	REGISTRATION	REGISTRATION
8:00 a.m. - 10:45 a.m.	SESSIONS FOR CONTRIBUTED PAPERS	
8:00 a.m. - 10:45 a.m.	SPECIAL SESSIONS Functional analysis methods in nonlinear differential equations, ordinary and partial I Distribution modulo 1 and random number generation I Algebraic semigroups I Representations of finite groups I	
8:30 a.m. - 9:30 a.m.	INVITED ADDRESS: Intrinsic distances, measures and geometric function theory Shoshichi Kobayashi	
9:45 a.m. - 10:45 a.m.	INVITED ADDRESS: Lagrangian submanifolds Alan D. Weinstein	
11:00 a.m. - 12:00 noon	COLLOQUIUM LECTURES: Selected topics in partial differential equations Louis Nirenberg, Lecture I	
12:00 noon - 5:00 p.m.	EXHIBITS	EXHIBITS
1:30 p.m. - 2:30 p.m.	COLLOQUIUM LECTURES: Finite simple groups John G. Thompson, Lecture I	
2:45 p.m. - 6:00 p.m.	SESSIONS OF CONTRIBUTED PAPERS	
2:45 p.m. - 6:00 p.m.	SPECIAL SESSIONS Functional analysis methods in nonlinear differential equations, ordinary and partial II Distribution modulo 1 and random number generation II	

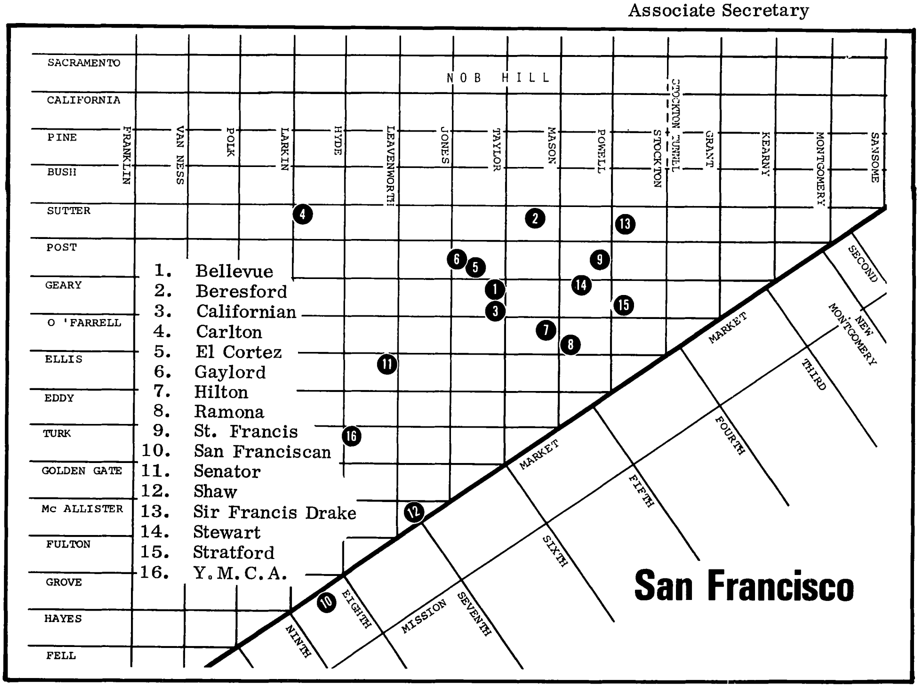
	American Mathematical Society	Other Organizations
2:45 p.m. - 6:00 p.m.	SPECIAL SESSIONS Representation of finite groups II Ring theory I Topological semigroups Set theory and the axiom of choice I Special operators	
2:45 p.m. - 3:45 p.m.	INVITED ADDRESS: Some recent developments in combinatorics Dijen K. Ray-Chaudhuri	
2:45 p.m.		Mathematicians Action Group BUSINESS MEETING
4:00 p.m. - 5:00 p.m.	INVITED ADDRESS: Combinatorial inequalities and smoothness of functions Adriano M. Garsia	
8:30 p.m.	GIBBS LECTURE Economics and mathematical analysis Paul A. Samuelson	
WEDNESDAY, January 16		
8:00 a.m. - 12:00 noon	SESSIONS FOR CONTRIBUTED PAPERS	
8:00 a.m. - 12:00 noon	SPECIAL SESSIONS Representations of finite groups III Ring theory II Algebraic semigroups II Crash course on Kleinian groups I Non-Abelian harmonic analysis I	
8:30 a.m. - 4:30 p.m.	REGISTRATION	
8:30 a.m. - 9:30 a.m.	INVITED ADDRESS: Some problems of mathematics and science Richard J. Duffin	
9:00 a.m. - 4:00 p.m.	EMPLOYMENT REGISTER	
9:00 a.m. - 5:00 p.m.	EXHIBITS	
9:00 a.m. - 4:00 p.m.		Mathematical Association of America BOARD OF GOVERNORS MEETING
9:45 a.m. - 10:45 a.m.	INVITED ADDRESS: Combinatorial game theory Elwyn R. Berlekamp	
11:00 a.m. - 12:00 noon	COLLOQUIUM LECTURES II Louis Nirenberg	
1:00 p.m. - 2:00 p.m.	COLLOQUIUM LECTURES II John G. Thompson	
2:15 p.m. - 2:45 p.m.	BÔCHER PRIZE SESSION	
2:45 p.m. - 3:45 p.m.	RETIRING PRESIDENTIAL ADDRESS Some groups and Lie algebras defined by Jordan algebras Nathan Jacobson	
4:00 p.m.	BUSINESS MEETING	
8:30 p.m.	PANEL DISCUSSION: Nonacademic employment of Ph.D.'s Edward E. David, Jr. John McQuown Carroll V. Newsom Henry O. Pollak (moderator)	
THURSDAY, January 17		
8:30 a.m. - 4:30 p.m.	REGISTRATION	
9:00 a.m. - 5:00 p.m.	EXHIBITS	
9:00 a.m. - 5:40 p.m.	EMPLOYMENT REGISTER	
9:00 a.m. - 10:20 a.m.	AMS-MAA PANEL DISCUSSION: The problem of learning to teach H. L. Alder (moderator) P. R. Halmos E. E. Moise George Piranian	

	American Mathematical Society	Other Organizations
10:30 a.m. - 11:50 a.m.		MAA PANEL DISCUSSION: On problem solving A. P. Hillman Murray S. Klamkin Lester H. Lange (moderator) George Polya John E. Wetzel
12:00 noon - 2:30 p.m.		Rocky Mountain Mathematics Consortium ANNUAL MEETING
1:00 p.m. - 6:00 p.m.	SESSIONS FOR CONTRIBUTED PAPERS	
1:00 p.m. - 6:00 p.m.	SPECIAL SESSIONS Set theory and the axiom of choice II Crash course on Kleinian groups II Non-Abelian harmonic analysis II Abelian groups I Geometrical topology I	
1:00 p.m. - 2:00 p.m.	COLLOQUIUM LECTURES III Louis Nirenberg	
2:15 p.m. - 3:15 p.m.	COLLOQUIUM LECTURES III John G. Thompson	
2:30 p.m. - 4:30 p.m.		Conference Board of the Mathematical Sciences PANEL DISCUSSION: Mathematics and society Donald L. Thomsen, Jr. (moderator) Biological, social, and cultural evolution Luigi L. Cavalli-Sforza Deployment of emergency vehicles Jan M. Chaiken Mathematical approach to ecosystem problems Simon A. Levin Discrete mathematics applied to environmental problems Fred S. Roberts
3:30 p.m. - 4:30 p.m.	INVITED ADDRESS: Representations of finite Chevalley groups Louis Solomon	
4:30 p.m. - 6:00 p.m.	NO-HOST COCKTAIL PARTY	
8:30 p.m.	PANEL DISCUSSION: The role of the dissertation in the Ph.D. program William Browder Karel deLeeuw Israel N. Herstein Saunders Mac Lane P. Emery Thomas (moderator)	
FRIDAY, January 18		
8:30 a.m. - 4:30 p.m.	REGISTRATION	
9:00 a.m. - 12:00 noon	EXHIBITS	
9:00 a.m. - 5:40 p.m.	EMPLOYMENT REGISTER	
9:00 a.m. - 9:50 a.m.		MAA - INVITED ADDRESS: The logic of equality Leon Henkin
10:00 a.m. - 10:50 a.m.		MAA - BUSINESS MEETING
11:00 a.m. - 11:50 a.m.		MAA - RETIRING PRESIDENTIAL ADDRESS Interval graphs and allied systems, with relations to molecular genetics, psychophysics, archaeology, ecology, and the inversion of sparse matrices Victor L. Klee, Jr.
1:00 p.m. - 6:00 p.m.	SESSIONS FOR CONTRIBUTED PAPERS	
1:00 p.m. - 6:00 p.m.	SPECIAL SESSIONS Non-Abelian harmonic analysis III Abelian groups II Geometric topology II	

	American Mathematical Society	Other Organizations
1:00 p.m. - 2:00 p.m.	COLLOQUIUM LECTURES IV Louis Nirenberg	
2:00 p.m.		CBMS - COUNCIL MEETING
2:15 p.m.	COLLOQUIUM LECTURES IV John G. Thompson	
3:30 p.m. - 4:30 p.m.	INVITED ADDRESS: Some applications of module theory to functor categories Barry M. Mitchell	
4:00 p.m.		Association for Women in Mathematics SESSION
7:30 p.m. - 10:29 p.m.		MAA - Films
8:00 p.m.		MAG - PANEL DISCUSSION
SATURDAY, January 19		
8:30 a.m. - 2:30 p.m.	REGISTRATION	
9:00 a.m. - 5:40 p.m.	EMPLOYMENT REGISTER	
		Sessions of the MAA
9:00 a.m.		APPLICATIONS OF MATHEMATICS TO THE BIOLOGICAL SCIENCES
9:00 a.m. - 9:50 a.m.		Mathematics in pulmonary physiology John W. Evans
10:00 a.m. - 10:50 a.m.		Graphs and molecules Joshua Lederberg
11:00 a.m. - 11:50 a.m.		Manifolds, machines, models and computability Hans Bremermann
1:30 p.m. - 2:20 p.m.		Some modern work on determinants Olga Taussky-Todd
2:30 p.m. - 3:20 p.m.		What is infinite dimensional topology? R. D. Anderson
3:30 p.m. - 4:20 p.m.		Computer science and its relations to mathematics Donald E. Knuth

Eugene, Oregon

Kenneth A. Ross
Associate Secretary



The Seven Hundred Twelfth Meeting
The University of Florida
Gainesville, Florida
March 7 – 8, 1974

The seven hundred twelfth meeting of the American Mathematical Society will be held at the University of Florida, in Gainesville, Florida, from noon Thursday, March 7, until noon Friday, March 8, 1974. The meeting of the American Mathematical Society will be followed by the regular annual meeting of the Florida section of the Mathematical Association of America.

By invitation of the Committee to Select Hour Speakers for Southeastern Sectional Meetings, two one-hour addresses will be presented. Professor A. T. Bharucha-Reid of Emory University will give an address entitled "Probabilistic operator theory," and an address entitled "The decision problem for recursively enumerable degrees" will be presented by Professor J. R. Shoenfield of Duke University.

There will be a special session on Dynamical Systems, Flows and One-parameter Semigroups of Transformations, to be organized by Professor John Neuberger of Emory University. Any member of the AMS who would like to have his or her paper considered for inclusion in the special session should have his or her abstract so marked and in Providence at least two weeks before the regular closing date for contributed papers (January 15, 1974).

There will also be sessions for contributed papers on Thursday and Friday. Abstracts for contributed papers should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the deadline of January 15, 1974.

O. G. Harrold

Tallahassee, Florida

CHAIRMEN AND TOPICS OF SPECIAL SESSIONS

Minneapolis, Minnesota, November 1973

Albert Marden and Edgar Reich, Complex analysis
Robert F. Craggs and Peter P. Orlick, Compact transformation
Robert W. Carroll and Walter Littman, Partial differential equations
Robert Ellis and Harvey B. Keynes, Topological dynamics

Atlanta, Georgia, November 1973

John W. Heidel, Applications of ordinary differential equations
M. D. Plummer, Graph theory

Tucson, Arizona, November 1973

Paul C. Fife and Wilfred M. Greenlee, Singular perturbations

San Francisco, California, January 1974

Frank W. Anderson, Ring theory
Lipman Bers, Crash course on Kleinian groups
Lamberto Cesari (assisted by Jack K. Hale), Functional analysis methods in nonlinear differential equations, ordinary and partial
Laszlo Fuchs, Abelian groups
Leslie C. Glaser, Geometrical topology
Ray A. Kunze, Non-Abelian harmonic analysis
William J. LeVeque, Distribution modulo 1 and random number generation
P. S. Mostert (assisted by Alfred H. Clifford), Topological and algebraic semigroups
Jean E. Rubin, Set theory and the axiom of choice
Gary M. Seitz, Representations of finite groups
Lawrence J. Wallen, Special operators

INTERNATIONAL CONGRESS OF MATHEMATICIANS
Vancouver, British Columbia, Canada
August 21 – 29, 1974

The Organizing Committee is pleased to announce that the next International Congress of Mathematicians will be held in Vancouver during August 21–29, 1974.

SCIENTIFIC PROGRAM

The work of the Congress will be divided into twenty sections. There will be approximately sixteen invited one-hour expository lectures and approximately one hundred fifty invited 45-minute specialist talks. Members of the Congress will be given an opportunity to present 15-minute oral communications of contributed papers and to organize small informal mathematical seminars on their own initiative either in advance or on the spot. See the following page for rules and format for submission of abstracts.

All formal lectures will be given at the University of British Columbia but many of the informal seminars are expected to take place at Simon Fraser University and the University of Victoria.

LANGUAGES

English, French, German and Russian are the designated languages of the Congress.

LOCAL ARRANGEMENTS

Accommodations for approximately 3,000 persons will be available in residences at the University of British Columbia. Based on single occupancy, the daily rate for these is expected to be approximately \$13 (Canadian) including meals. In addition, rooms in hotels primarily located in downtown Vancouver will be available. The rate for single rooms without meals in hotels is expected to be in the range of \$12–\$35 (Canadian).

A special bus service connecting the University of British Columbia with downtown and Simon Fraser University will be arranged for the members of the Congress. This is in addition to the regular public bus service.

Vancouver is considered one of the most scenic cities in the world with magnificent views of mountains and sea. A series of local tours, day excursions and more extended trips intended to show some of the spectacular scenery is being arranged.

Details and reservation forms will accompany the second announcement.

TRAVEL

Group charter flights will be available to the Congress from a number of centers. They are being coordinated by World Tours Ltd., 425 Howe St., Vancouver 1, Canada (affiliated with American Express), who may be contacted for further information. Details on travel and pre- and post-convention tours will also accompany the second announcement.

ADDRESS

International Congress of Mathematicians
The University of British Columbia
Vancouver 8, British Columbia, Canada

Cable address: Mathematix

TRAVEL GRANTS FOR THE INTERNATIONAL CONGRESS OF MATHEMATICIANS

The National Research Council has announced that travel grants will be made to about eighty U.S. mathematicians attending the International Congress of Mathematicians in Vancouver, British Columbia, August 21–29, 1974, if funding is received as expected. Because many senior mathematicians will have funds to attend the Congress, the NRC program is aimed primarily towards younger mathematicians. It has been announced that at least sixty percent of the awardees will be thirty-five or younger at the time of the Congress. Selection of the grantees will be made by a special committee. Applications for travel grants may be obtained from the Division of Mathematical Sciences, National Research Council, Washington, D.C. 20418. The Division's deadline for receipt of completed applications is December 31, 1973.

INTERNATIONAL CONGRESS OF MATHEMATICIANS
Submission of Abstracts

Members of the Congress will have the privilege of presenting fifteen-minute oral communications of their mathematical work. Those who intend to give such talks are requested to submit abstracts (in English, French, German, or Russian) to the Organizing Committee, International Congress of Mathematicians, The University of British Columbia, Vancouver 8, British Columbia, Canada; the deadline is April 15, 1974. Abstracts will be reproduced photographically from the copy submitted by the author, and will be available at the Congress. The abstract is to be typed within a rectangle 8" by 4" (20 cm by 10 cm). The abstract should be typed on good quality, heavy white paper using a black ribbon. If symbols are added by hand, black ink must be used. Name (in full upper case), university or institution, country, and title (in that order) and indented 1/2" (1.25 cm) should be typed (single-spaced) on the first two lines. A subject classification number should appear at the top of the page separate from the abstract, using the AMS (MOS) Subject Classification Scheme (1970) which appears in the Index of Volume 39 of Mathematical Reviews, June 1970. See format below.

Subject classification: 10G05

JOHN H. KRANZER North Dakota State University, Bottineau Branch, Bottineau, North Dakota, U. S. A.

Dirichlet characters and polynomials

The text of the abstract will appear here. It may be single-spaced, but double-spacing is preferable if the abstract contains formulas with subscripts and superscripts. The abstract will appear exactly as it is submitted by the author (no re-typing will be done), except that it will be reduced photographically to 67 percent (5 1/3" by 2 3/4" or 13.33 cm by 6.67 cm). The abstract may be as short as the author desires, but it may be no longer than will fit into a rectangle 8" by 4" (20 cm by 10 cm).

INVITED SPEAKERS AT AMS MEETINGS

This section of these *Notices* lists regularly the individuals who have agreed to address the Society at the times and places noted below. For some future meetings, the lists of speakers are incomplete.

Minneapolis, Minnesota, November 1973

Charles L. Feffermann

Frank A. Raymond

Atlanta, Georgia, November 1973

Ben Fitzpatrick

Paul Hill

M. Zuhair Nashed

Tucson, Arizona, November 1973

Andrew P. Ogg

Robert R. Phelps

San Francisco, California, January 1974

Elwyn R. Berlekamp
Richard J. Duffin
Adriano Garsia
Nathan Jacobson (Retiring Presidential Address)
Shochichi Kobayashi
Barry M. Mitchell
Louis Nirenberg (Colloquium Lectures)
D. K. Ray-Chaudhuri
Paul A. Samuelson (Gibbs Lecture)
Louis Solomon
John G. Thompson (Colloquium Lectures)
Alan D. Weinstein

Gainesville, Florida, March 1974

A. T. Bharucha-Reid

J. R. Shoenfield

Washington, D.C., January 1975

Linda Keen

Wilfried Schmid

REPRESENTATIVES OF THE AMS

John J. Sopka represented the Society at the inauguration of Donald Ezzell Walker as president of Southeastern Massachusetts University.

ERRATA

Item (46), page 243, volume 20, should read "Topological dynamics on C*-algebras. Professor GARY LAISON, Lehigh University, and Professor DIANE LAISON, Temple University (708-G13)."

SEVENTEENTH ANNUAL AMS SURVEY

Mathematical Science Faculty and Enrollments

by John Jewett

This article is to report on certain aspects of the 17th Annual AMS Survey. It supplements the salary data and other related information contained in the October issue of these *Notices*. This year's survey was more extensive than in previous years, largely in order to investigate various conjectures and anecdotal reports of increased class size, rising teaching loads, and various conflicting opinions concerning enrollment trends.

The results can be summarized succinctly as follows: There has been no significant evidence of increased teaching loads; average class size has gone down slightly; mathematical science classes have declined in enrollment by two percent at the undergraduate level and seven percent at the graduate level; faculty size is essentially unchanged; there has been a very slight increase in the number of graduate students in mathematics and a larger increase in the number of graduate students in other mathematical science departments.

Our results are based on data collected in the summer of 1973 from chairmen of mathematical science departments. The data were collected and tabulated under the direction of Dr. Lincoln K. Durst. The numbers appearing in the tables are estimates of national totals as derived from sample data. Any number representing some national total and appearing in one of the tables which follow was estimated separately for each of seven groups of U. S. departments* by multiplying the total reported by the actual respondents in that group by the reciprocal of the response ratio. If, as we believe, the larger institutions within a given group are more likely to respond than smaller institutions, the data reported would tend to be larger than actual values. The lower the response rate the more pronounced this effect would be. Where possible we have checked the data reported here with data from other sources. In particular, our data seemed to be consistent with the CBMS survey of 1970.

In analyzing data, U. S. doctorate-granting mathematics departments were divided into three groups:

Group 1. The 27 leading mathematics departments as ranked in the ACE Survey of 1969. **
Group 2. The 38 other mathematics departments which were rated in the ACE Survey.
Group 3. The 89 doctorate granting mathematics departments not ranked in the ACE Survey.

Included in our totals for the mathematical sciences are returns from statistics and computer science departments and from non-doctorate-granting mathematics departments. Canadian doctorate granting departments are treated separately in order that U. S. data can be compared with other data produced by N. S. F. or other government surveys. The response rate for U. S. doctorate granting departments was just over 50 percent, while for some other groups it was as low as 25 percent. Therefore, our estimates of, say, faculty size cannot be expected to agree precisely with estimates derived from other sources. However, if a figure is given for successive years, it is derived from data furnished by exactly the same set of respondents in both years, so that information about trends is probably more reliable than estimates of national totals.

ENROLLMENTS IN MATHEMATICAL SCIENCE COURSES

The general picture of enrollments and class size in all mathematical science courses is presented in Table 1. Since 93 percent of all mathematical science enrollments are in mathematics departments, a table for mathematics only would not differ significantly except for the entries for statistics and computer science. Even there, due to the infrequency of statistics and computer science departments outside doctorate granting universities, over half of the national enrollment both in statistics and in computer science is in mathematics departments.

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*The same seven groups of U. S. departments used in the report of salary data in the October issue of these *Notices*.

**The findings were published in "A Rating of Graduate Programs" by Kenneth D. Roose and Charles J. Anderson, American Council of Education, Washington, D. C., 1969, 115 pp. The information on mathematics was reprinted by the Society and can be found on pages 338–340 of the February 1971 issue of these *Notices*.

Table 1

Fall Enrollments in U.S. Mathematical Science Departments

	1971	1972	Percent Change	Avg. Class Size 1971	Avg. Class Size 1972
Courses Below Level of Calculus	652,000	651,000	—	32.4	31.5
Calculus Courses	409,000	387,000	Down 5%	26.2	25.8
Undergraduate Courses in Statistics	93,000	101,000	Up 9%	31.0	30.1
Undergraduate Courses in Computer Science	109,000	120,000	Up 10%	29.8	31.8
Undergraduate Math. Courses Above Calculus	252,000	229,000	Down 9%	20.9	19.6
Total Undergraduate	1,515,000	1,488,000	Down 2%	27.8	27.3
Graduate Courses					
Mathematics Departments	52,000	48,000	Down 8%	11.5	10.9
Statistics Departments	10,000	10,000	—	14.3	14.3
Other Math. Science Departments	9,000	8,000	Down 11%	13.6	11.9
Total Graduate Courses	71,000	66,000	Down 7%	12.0	11.4
All Courses	1,586,000	1,554,000	Down 2%	26.3	25.8

In 1972, a total of 44 percent of all undergraduate mathematical science courses were in precalculus courses and an additional 26 percent were in calculus. The total enrollment in statistics and computer science courses was only slightly less than the enrollment in upper division mathematics courses.

The decrease of four percent in enrollment in mathematics courses from 1971 to 1972 is significant and, to a certain extent, unexpected. The largest decrease is in undergraduate courses above calculus. This decrease of nine percent was most marked in the top ACE group of 27 leading departments where it amounted to a one year decline of 13 percent. In other categories of mathematics departments the decline, although present, was not so great, reaching a low of only 1.4 percent in bachelor's granting departments. It is interesting to note that the decrease of 23,000 in upper division mathematics enrollment was comparable in size to the combined increases of 19,000 in undergraduate statistics and computer science courses.

A decrease in calculus enrollments occurred in all groups of institutions except the two groups of ACE rated departments. An increase of four percent occurred in the highest group, while calculus enrollment in other ACE rated departments increased by less than one percent.

The last two columns of Table 1 give information on average class size. A modest decrease in class size was reported by every type of institution in courses of almost every level, the only exceptions being computer science courses generally and calculus courses in the highest ACE group.

MATHEMATICAL SCIENCE FACULTY

The data presented in Table 2 on number of mathematical science faculty shows a slight increase from the Fall of 1971 to the Fall of 1972 with a report of essentially no change from the Fall of 1972 to the expected numbers for the Fall of 1973. The chairmen reported that they expected a decrease of 11 percent in the number of part-time faculty for the Fall of 1973. Since plans for part-time staff may not yet have been too firm in the summer, when chairmen completed the questionnaires, it is not clear that this large a decline actually materialized.

The respondents reported not only on the number of part-time staff and graduate assistants but also on the full-time equivalents which these numbers represented. On the average each part-time staff member represented .43 full-time equivalents with the same ratio holding for graduate assistants. On this basis we have calculated the total full-time equivalent staff and have entered it in the last row of Table 2.

Table 2

Faculty and Staff in U.S. Mathematical Science Departments

	Number Fall 1971	Number Fall 1972	Estimated Fall 1973
Number of Full-Time Teaching Staff	17,906	18,177	18,169
Number of Part-Time Staff	3,220	3,430	3,054
Number of Graduate Teaching Assistants	9,384	9,582	9,545
Total Full-Time Equivalent Staff	23,438	23,800	23,577

Table 3

Full-Time Faculty in U.S. Mathematical Science Departments

Type of Department	Number Fall 1971	Number Fall 1972	Estimated Fall 1973
Doctorate Granting			
Mathematics			
Top 27 ACE Ranked	1,563	1,572	1,575
Other ACE Rated	1,464	1,471	1,441
Unrated	2,519	2,467	2,450
Statistics	573	580	577
Other Mathematical Sciences	868	987	1,031
Master Degree Granting	5,099	5,196	5,155
Bachelor Degree Granting	5,820	5,904	5,940
Total (U.S.)	17,906	18,177	18,169

Table 3 shows that the essential constancy of faculty size is reflected in the data for full-time faculty in specific types of mathematical science departments.

TEACHING LOADS AND STUDENT-FACULTY RATIOS

One major purpose of this year's survey was to test the hypothesis that there have been significant recent increases in teaching load. The respondents were asked the average teaching load of their faculty in hours per week for the academic year 1971–1972 and for 1973–1974, two years later. Despite small changes reported by individual schools, the median teaching loads were identical for the two years for each category of mathematical science departments. The median teaching loads were six hours for ACE ranked mathematics departments and also for statistics and for computer science departments. For unranked Ph.D. granting mathematics departments the median teaching load was seven hours in both years, while for both master's degree granting and bachelor's degree granting departments, the median was 12 hours.

More details are given in Table 4. Each entry in this table is the number of departments reporting a given average teaching load in a given year.

Table 4

Frequency of Given Teaching Loads in U. S. Mathematical Science Departments

Type of Department		Average Teaching Load in Hours per Week								
		<6	6	7	8	9	10	11	12	>12
ACE Rated Mathematics	1971–1972	4	15	14	1					
	1973–1974	3	15	13	3					
Unrated Ph. D. Granting Mathematics	1971–1972		18	10	8	5	2	1	2	
	1973–1974		13	11	9	8	3	0	2	
Other Ph. D. Granting Mathematical Sciences	1971–1972	8	20	3	3	3				
	1973–1974	9	18	3	3	4				
Master Degree Granting Mathematics	1971–1972		4	4	4	13	16	5	42	5
	1973–1974		3	3	4	15	13	4	46	5
Bachelor Degree Granting Mathematics	1971–1972		3	2	7	27	25	16	129	44
	1973–1974		5	3	5	26	26	18	121	49

Although the average number of hours spent in class is the most customary measure of teaching load, another and perhaps more significant measure is the ratio of the number of students to the number of teachers. The ratio of total mathematical science enrollments to full-time faculty members declined from 88.6 in the Fall of 1971 to 87.3 in the Fall of 1972 while the ratio of enrollments to full-time equivalent faculty declined from 67.7 to 65.3. These ratios both declined in almost every type of department considered except for ACE ranked mathematics departments, in which the increase was less than one percent, and for statistics departments, in which the first ratio increased by nine percent and the second by four percent.

The slightly different ratio of undergraduate enrollment to the number of full-time faculty was reported in the CBMS survey of 1970 to be 81.3. From the present data, this ratio can be calculated to be 81.8 for the fall of 1972.

GRADUATE ENROLLMENTS

It has been observed above that mathematics enrollments declined slightly from 1971 to 1972 and that faculty size has been essentially constant. In the light of this and other discouraging projections of job opportunities both for Ph. D.'s and non-Ph. D.'s, it is both interesting and sobering to observe that the total number of graduate students in mathematics was expected to increase by one percent from 1972 to 1973. This increase moreover comes about through a decrease in ACE rated departments combined with an increase in the number of graduate students in master's granting and unrated doctorate granting departments. The number of graduate students in statistics departments increased by six percent from 1,805 to 1,906. The low response rate to this question from other doctorate granting mathematical science departments precludes making any reliable estimates. The details for mathematics departments are given in Table 5.

Table 5

Number of Graduate Students in U. S. Mathematics Departments

Type of Mathematics Department	Number 1972–1973	Estimate 1973–1974	Percent Change
Top 27 ACE Ranked	3,119	2,943	Down 6%
Other ACE Rated	2,690	2,602	Down 3%
Unrated Ph. D. Granting	3,373	3,452	Up 2%
All Ph. D. Granting	9,182	8,997	Down 2%
Master Degree Granting	3,849	4,159	Up 8%
All Mathematics Departments	13,031	13,156	Up 1%

The questionnaire requested chairmen to estimate, during the summer of 1973, the number of first year graduate students expected in the fall. Although these estimates cannot be expected to be extremely accurate, indications are that the number of first year graduate students in mathematics was expected to be about the same for the fall of 1973 as the fall of 1972 both in doctorate granting and nondoctorate granting departments. Within this, the group of 27 highest ranked mathematics departments expected an increase in first year graduate students of almost ten percent. Statistics departments anticipated an increase of almost 20 percent and there were indications of a sizable increase for other mathematical science departments.

CANADIAN DEPARTMENTS

The same general trends were evident in Canadian doctorate granting mathematical science departments as in U.S. departments. A much smaller percent of their teaching is in precalculus courses; the typical class size is larger (but declining) and faculty size seems to now be fairly well stabilized. The enrollment decline in upper division mathematics courses matched that in the United States and there were also declines in enrollments in graduate courses. The details are summarized in Tables 6 and 7.

Table 6

Fall Enrollments in Canadian Doctorate Granting
Mathematical Science Departments

	1971	1972	Percent Change	Avg. Class Size 1971	Avg. Class Size 1972
Courses Below Level of Calculus	3,900	3,600	Down 8%	54.5	46.5
Calculus Courses	31,100	32,600	Up 5%	54.1	56.2
Undergraduate Courses in Statistics	10,000	10,100	Up 1%	44.6	46.1
Undergraduate Courses in Computer Science	22,000	19,400	Down 12%	67.7	59.4
Undergraduate Math. Courses Above Calculus	52,500	47,900	Down 9%	40.6	36.3
All Graduate Courses	4,900	4,500	Down 8%	9.6	8.7
Total	124,400	118,100	Down 5%	41.4	38.9

Table 7

Faculty and Staff in Canadian Doctorate Granting
Mathematical Science Departments

	Number Fall 1971	Number Fall 1972	Estimated Fall 1973
Full-Time Teaching Staff	1,208	1,234	1,245
Part-Time Staff	176	164	184
Graduate Teaching Assistants	1,112	988	994
Total Full-Time Equivalent Staff	1,649	1,640	1,695

Jobs and Ph.D.'s in the Mathematical Sciences, I.

By R. D. Anderson

This report consists of four sections:

1. The current job market situation,
2. Job prospects for September 1974,
3. Job prospects for the next 20 years,
4. Tenure, moral tenure, and job retention.

The first two sections comprise Part I. The third and fourth sections will be published as Part II in the December issue of these *Notices*.

The report has been prepared by the author on behalf of the AMS Committee on Employment and Educational Policy, but the responsibility is the author's. Other reports already prepared or planned on behalf of the Committee are on Mathematical Science Faculty and Enrollments (by John Jewett, this issue), Part-Time Jobs, The Economics of Early Retirement, and Ph.D.'s in Two Year Colleges. In addition, Committee plans call for the publication of reports on the panel discussions at San Francisco, as well as one or more articles on role models for graduate programs.

The members of the AMS Committee on Employment and Educational Policy are Richard D. Anderson (chairman), Michael Artin, John W. Jewett, Calvin C. Moore, Richard S. Palais, and Martha K. Smith.

1. THE CURRENT JOB MARKET SITUATION

Data collected by the AMS this summer from department chairmen, from new Ph.D.'s for the starting salary survey, and from a sample of nonretained Ph.D.'s in a special pilot mobility study, show a number of things about the current job market for Ph.D.'s in the mathematical sciences.

There were probably about 150 to 200 Ph.D.'s who graduated this past year or who graduated earlier and were employed in 1972–1973 who were still seeking professional employment on September 1. Data collected principally in June and July showed about 118 new Ph.D.'s not then employed and perhaps 210 nonretained Ph.D.'s who did not then have jobs for 1973–1974. But there were considerable employment opportunities in July and August and extrapolating from several recent reports, it seems likely that the not-yet-employed figure on September 1 was between 150 to 200. The comparable figure a year ago was probably about 200. Somewhat higher estimates from a year ago included continuing unemployed, i.e. Ph.D. mathematicians who had been professionally unemployed earlier and who were still seeking mathematical employment. This year we are not citing a figure for the continuing unemployed as it is very hard to give a reliable estimate of that number.

The accompanying table on 1973 doctorates with degrees granted from July 1, 1972, to June 1973 in the United States and Canada and their employment for 1973–1974 was prepared by Lincoln K. Durst of the AMS on the basis of advice from the AMS Committee on Employment and Educational Policy. It is similar to tables published in the *Notices* in November 1971 and October 1972 and in the Monthly in June 1970.

The data for the table were collected primarily from chairmen of doctorate producing departments in the mathematical sciences on forms listing new doctorates with their dissertation titles. The information was updated during the late summer and early fall of 1973 by data submitted by 593 of the new doctorates on starting salary survey forms. A summary of starting salaries was published in the October *Notices*.

The table includes information on all 1973 doctorates listed in the October and November issues of these *Notices*. It should be observed that this year 1,270 total Ph.D.'s are reported in the table, whereas, in 1971, 1,356 were reported and, in 1972, 1,375 were reported. For 1972, 83 additional Ph.D.'s were listed in the January and February 1973 *Notices* and, based on experience and on counts of degrees from departments which reported 1972 doctorates and have not yet reported 1973 doctorates, it is estimated that perhaps 50 to 100 additional doctorates will be reported later. The data are almost complete (> 95%) for departments of mathematics but are less complete for doctorates in statistics and computer related subjects. Indeed, due to lack of response to AMS requests and due to the absence of any clear basis on which to identify some departments as well as some dissertations as being properly in the mathematical sciences, there is no valid basis except perhaps the fact of listing in the *Notices* as a criterion for actually counting dissertations and thus total doctorates in the mathematical sciences. It should be noticed that only 20 degrees in mathematics education are listed, whereas Office of Education figures have listed over 100 such doctorates annually. Many of these not listed in AMS data are in elementary or secondary school education but some do take or retain positions in college mathematics departments.

1973–1974 EMPLOYMENT STATUS OF NEW DOCTORATES IN THE MATHEMATICAL SCIENCES

	PURE MATHEMATICS											
Type of Employer	Algebra and Number Theory	Analysis and Functional Analysis	Geometry and Topology	Logic	Probability	Statistics	Computer Science	Operations Research	Applied Mathematics	Mathematics Education	Other	Totals
University	40	53	48	5	12	44	46	7	17	2	7	281
College	71	81	44	10	8	40	39	5	17	11	7	333
Two-year colleges and high schools	7	4	3		1		1			2	1	19
Business and industry	15	15	3	2	7	18	46	7	10	1	6	130
Government	4	6	3	2	1	12	8	5	12		4	57
Research insts.	2	5	3			4	10	2	3		2	31
Canada	22	21	7		3	8	15	3	6		3	88
Foreign	17	32	11	4	4	20	18	6	14	3	4	133
Not seeking employment			1					2			1	4
Not yet employed	28	37	22	4	1	8	3		11	1	3	118
Unknown	5	15	8	1	2	4	26	1	14			76
Totals	211	269	153	28	39	158	212	38	104	20	38	1,270

Comparing the data in this table with the October 1972 table, there are several facts that stand out. The total number of Ph. D.'s listed is down by 105, the number of pure math. Ph. D.'s (the first five columns this year) being 35 below the pure math. totals of 1972. The number of those employed in universities (i. e. Ph. D. producing schools) is down by 85. The number of those employed in business, industry, government, and research institutions is up slightly from 206 to 218. The number listed as unemployed went from 103 to 118. In fact, because of reported hiring in late summer, it is believed that the number of 1973 Ph. D.'s still seeking employment on September 1 was probably 50 to 60.

Not revealed in the published tables is a rather sharp change in the distribution of the Ph. D.'s between the July–December and January–July halves of the year. In 1971–1972 sixty percent of the degrees were conferred in the latter half of the academic year, which was comparable to the 1971 figure of 62%, whereas in 1973 it was down to 53%. Thus, the reduction of doctorates this summer may actually be more pronounced than the fiscal year figures indicate. The figures for degrees from Canada were 99 in 1972 and 85 in 1973. This year, for the first time, data on citizenship of Ph. D. recipients was requested and revealed that about 21% of the U. S. degrees were earned by non U. S. citizens. Of the noncitizens receiving degrees in the U. S., about 100 got positions in the U. S. for 1973–1974.

According to data submitted to the AMS by department chairmen, the total numbers of beginning graduate students and of total graduate students in mathematics departments has been approximately stable over the past three years and the numbers of graduate students and of beginning graduate students has gone up at least 20% to 30% over the past three years in other mathematical sciences, such as statistics and computer science. Some NSF data show modest reductions in the figures between 1971 and 1972. It seems clear that the observed reduction in the number of Ph. D.'s this year over last year is a result either of a higher attrition rate or of a rather sudden stretching out of the time taken to get degrees. While available AMS data does not appear to give definitive evidence on which cause is more important, the author feels that it is very likely that the primary cause is a higher attrition rate. This is probably due in part to slightly higher standards but mainly to voluntary withdrawals from Ph. D. programs by students prior to getting their doctorates. Completing graduate training means long, hard and dedicated work for most students. If the prospects for continued (post-doctoral) research activity and economic well-being are dismal, a number of students can be expected to feel that the effort isn't worth it. Perhaps the earlier observed lower levels of attrition were artificially affected by apparent good job prospects and we are now seeing a return to a more "normal" phenomenon. However, the patterns reported above of the numbers of graduate students suggest that we cannot soon anticipate further sizeable reductions in Ph. D. production. In fact, we should experience an increase in the number of Ph. D.'s produced in mathematical science departments other than traditional mathematics departments. We now have about 150 departments of mathematics and another 150 departments in other mathematical sciences which produce doctorates. It seems highly unlikely that 150 departments of mathematics which in 1972 produced about 900 doctorates in pure and classical applied mathematics can early or easily cut the annual production to the 200, 300 or 400 who are likely to be able to get long term mathematical employment in our economy in a steady state situation five or ten years from now.

Another factor affecting employment prospects for individuals but hard to assess is the relatively large number of Ph. D. level mathematicians who have left or are leaving the profession. The author personally knows at least 10: one in law school, one in medical school, one working for a brokerage house, one (announced as) a truck driver, one operating a marina, two in communes (at an earlier time) and three in other jobs about which he does not have detailed information. In addition, he knows one who is employed as an engineer and he has been told about others in farming and such activities.

In counting the number of professionally unemployed, it is almost impossible to keep track of those who were not either in school or employed last year. Some of the earlier unemployed have left the profession (voluntarily, semivoluntarily or involuntarily), some look for positions only in the areas where their spouses have jobs, some have occasional part-time jobs in mathematics, and some are self-employed. It seems better and more accurate only to keep a running count on those who are newly professionally unemployed. However, the existing rather large reservoir (several hundred?) of Ph. D. mathematicians who are not employed professionally and those who are expected to join this reservoir over the next several years make it almost certain that even if production is curtailed drastically we shall have a supply of Ph. D.'s sufficient for anticipated national needs. Indeed, if 1,000 additional academic jobs in mathematics with decent pay and some prospects of permanency could suddenly materialize for next year, it is very probable that Ph. D.'s would be available for most or even almost all of them.

The accompanying flow diagram is based on data from the faculty mobility survey and the annual salary survey. This diagram refers to U. S. faculty only and represents extrapolations from all data available at AMS headquarters on September 13. The data involved responses from departments having about 50% of the total national faculty. The figures in the flow diagram that do not have question marks involve direct extrapolation from data collected. Those with question marks are deduced or estimated indirectly or from other sources. The net increase (300) in doctorates and net decrease (300) in nondoctorates on the total faculty was computed from the salary survey. There was essentially no change in the total national four-year college and university mathematics faculty (including members of statistics and computer science departments).

The percentage of the faculty having doctorates went up almost 2%, from about 70% to about 72%.

The rather sizeable number 200 of those who got doctorates and stayed in their positions presumably includes many (perhaps 100) who got degrees outside the mathematical sciences as tabulated by the AMS, e.g. in some aspects of mathematics education. The employment of new Ph. D.'s in the AMS lists shows only about 600 getting academic positions in the U. S. and not all of these were in departments in the mathematical sciences.

The number of positions available due to death and retirement is up to about 240 this summer from 200 last summer and, according to CBMS Survey figures, from about 200 in the summer of 1970. Figures on age distribution of the faculty suggest that under stable conditions this 200 figure should be expected for another 15 years. Perhaps we are experiencing some early retirement or perhaps there is a statistical abnormality.

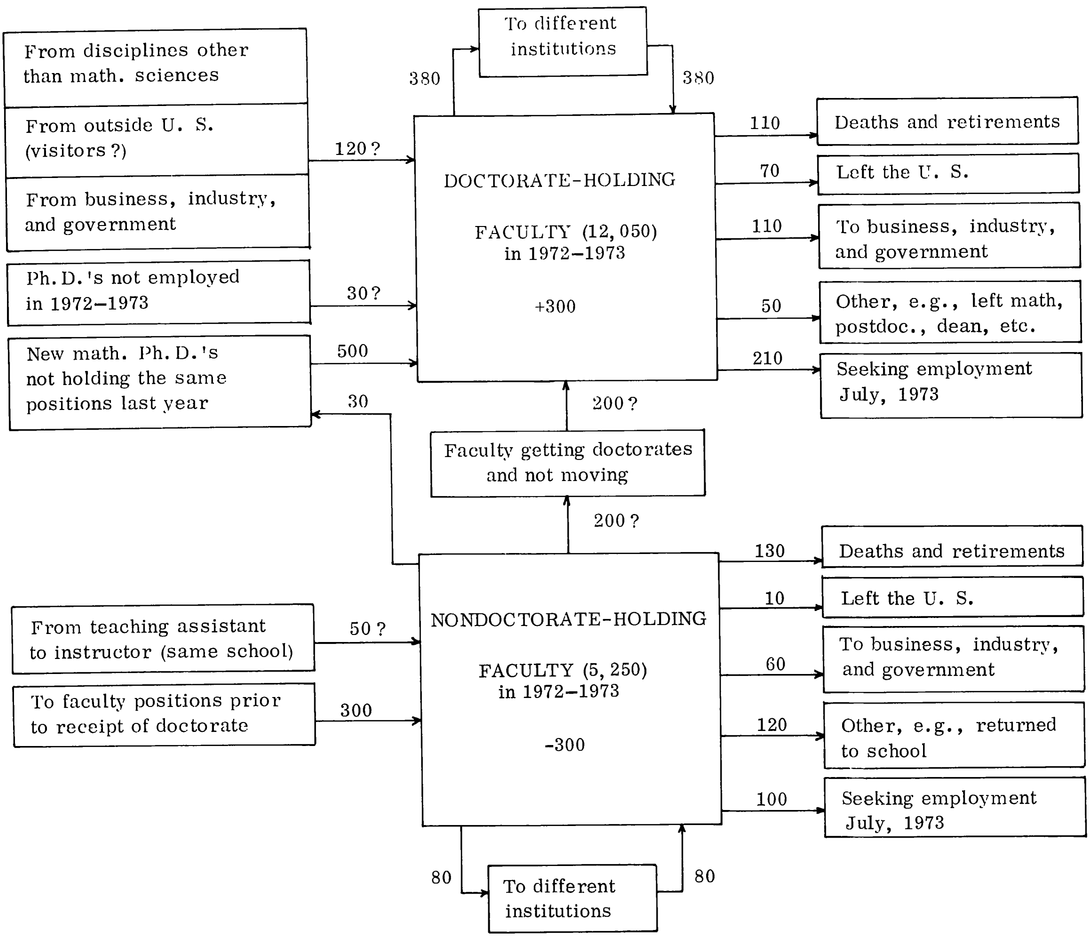
In comparing this flow diagram with that of last year (page 278, October *Notices* 1972), most figures are rather comparable. The replacement of master's level faculty by doctorates was down from 450 to 300. The doctorate level faculty leaving the U. S. was down from 120 to 70 but the number getting positions in business, industry and government was up from 80 to 110. Data of a type collected initially this year show a larger employment, 300, of new nondoctorates than had been supposed.

The number of nonretained faculty seeking employment is recorded as shown by the data which were collected chiefly in June and July. It seems probable that a third to a half of the 210 doctorates shown as seeking employment found positions by September.

The data published in the October *Notices* on faculty size, tenure and women in mathematics in connection with the annual salary survey are quite detailed and revealing for the interested reader. Returns were received from departments having 40%–50% of the total mathematics faculty (U. S. and Canada). As should be expected, a substantially greater percentage of returns were received from Ph. D. producing mathematics departments than from departments in the other mathematical sciences and master's and bachelor's level departments. In extrapolating to total figures, weights by known or estimated total faculty size in the various categories are probably better than weights by number of departments. For example, last year on another form returns from 50% of all departments showed about 62% of the total student population. Such weights by categories are used in computing figures for the flow diagram given above.

Some of the noteworthy figures concerning faculty size and distribution are the very large percentages of nondoctorates with tenure among all nondoctorates in group III (ACE unrated doctorate producing U. S. mathematics departments),

FACULTY FLOW DIAGRAM 1972–1973 to 1973–1974
Full-Time Mathematics Faculty in Four-Year Colleges and Universities in the U.S.



master's and bachelor's level departments. Omitting instructorships (where the pay is low and the positions are more frequently temporary) the percentages were 86%, 84% and 68% respectively. Thus we must anticipate very little further replacement of master's level faculty by doctorates except for death and retirement reasons or by nondoctorate faculty themselves receiving doctorates. The "moral tenure" study reported in Part II shows that the tenure figures are probably deceptively indicative of much more flexibility that exists in the system.

Whereas the total women reported on the faculty in the sample changed very little over the past year, the total women with doctorates on the faculty rose from 290 to 325, a rather surprising 12%, while the total men with doctorates rose only from 6, 000 to 6, 104 or 1. 7%. Whether such figures reflect reverse discrimination or an adjustment to a nondiscriminatory selection process is debatable and needs further study.

2. JOB PROSPECTS FOR SEPTEMBER 1974

At this time it is impossible to be certain about the employment status for next fall, but the author is pessimistic. There are at least two partially conflicting bases for estimates.

On the one hand, unemployment for September 1973 was probably lower than that for September 1972. A major factor was that production of Ph. D.'s in pure and classical applied mathematics was lower than that for the preceding year. Also, there were more jobs available in late July and August than experience had indicated should be expected. There is no known reason to believe that production will change very much—except that it probably will go up somewhat in statistics and computer related mathematics and jobs are more available in these areas. The economic prospects for the next year as they affect money both for higher education and for nonacademic employment appear generally comparable to this past year's phenomena except possibly for the effects of inflation. Thus, we might expect an employment situation like that for September 1973.

On the other hand, the tenure and long range retention phenomena are rapidly headed toward a state of inflexibility that will make it very difficult for nonretained Ph. D.'s to get academic positions, particularly those nonretained Ph. D.'s who have had their degrees for several years. Also, the replacement of faculty with master's degrees seems to be slowing down (as it must, soon). The number of graduate students is stable in traditional mathematics and is rising in newer applied areas. Employment outside academia is rising, but not rapidly. The numbers of students in higher education generally and in mathematics courses specifically have been almost stable over the past two or three years leading to a lack of pressure for more faculty positions (except possibly in statistics or computer related areas).

There are two numerical indicators, not yet known, which may foretell the employment situation for next fall. However, neither is likely to be very favorable and, indeed, both are expected to be slightly unfavorable.

(1) Freshmen or first time students in four-year colleges and universities this fall. Most indications are for the same or a slightly smaller number than that of last fall—thus, indicating almost no pressure for increased mathematics faculty for September 1974 (there being some time lag between enrollment changes and faculty size changes). National data come out about January from the Office of Education and independently from a survey organized by Dr. Garland Parker of the University of Cincinnati. Even a 2% increase in numbers of freshmen would have only limited positive effect on employment since there was a decrease of almost 3% a year ago.

(2) Applications for admissions for next fall. Such data are available in admissions offices during the spring. With the individual's apparent need for multiple applications still dropping, and the age group population increasing only slightly, there is little prospect for a national pattern of enough increased applications to force increased hiring next spring and summer. Indeed, again, the recent trends are in the opposite direction.

If either or both of these indicators is negative, then there will be very little, if any, increase in the national faculty next fall and therefore a potentially serious unemployment problem. A fuller discussion of the role of the first year student is included in section 3, to be published in Part II.

Another potentially negative factor affecting prospective employment next fall is the rather sharp recent increase in the inflation rate. The need for the higher educational establishment to use much of whatever extra money it gets to meet higher costs other than that for faculty expansion is obvious. Whether available funds will go up faster than the rate of inflation seems very doubtful.

All things considered, it seems probable that unemployment for Ph. D.'s will be somewhat worse next fall, particularly for nonretained Ph. D.'s who have had their degrees for several years. It is possible that it will be much worse. We can anticipate some yearly fluctuations in market conditions based on economic and other factors. But, the trend appears definitely unfavorable. In light of the prospective stability of the freshman-sophomore student population over the next several years and of the data on tenure and moral tenure discussed in section 4, the evidence points to a generally deteriorating academic employment situation, with the possibility of a rather sharp decline in employment opportunities at some time in the next few years.

NEWS ITEMS AND ANNOUNCEMENTS

HOLBROOK M. MACNEILLE

Professor Holbrook M. MacNeille of Case Western Reserve University died on September ber 30, 1973. Professor MacNeille served as the first Executive Director of the American Mathematical Society from 1949 through 1954. He was born in 1907 in New York City and received his B.A. and M.A. from Oxford University. In 1935, he was granted the Ph.D. from Harvard University. He served on the faculties of Harvard University, Kenyon College, Washington University, Case Institute of Technology, and Case Western Reserve University, as well as devoting many years to government service. Among the honors conferred upon him was the Certificate of Merit presented to him by President Truman in 1948.

Professor MacNeille devoted his life to mathematics, as a teacher, an administrator, and as an active member of numerous committees of mathematical organizations. The mathematical community has suffered a great loss and is deeply saddened by his death.

THE LEROY P. STEELE PRIZES

The LeRoy P. Steele Prizes, which were established in 1970, are awarded at summer meetings of the Society for expository papers which make accessible to other mathematicians an area of mathematics to which the author has made substantial contributions. The Committee on Steele Prizes will welcome nominations for these prizes. Please send suggestions to the chairman of the committee, Professor Hans F. Weinberger, School of Mathematics, University of Minnesota, Minneapolis, Minnesota 55455.

NATIONAL MEDAL OF SCIENCE

On October 10, 1973, eleven scientists and engineers were awarded the National Medal of Science. This is the Federal Government's highest award for distinguished achievement in science, mathematics, and engineering. Among the recipients of the National Medal was Professor John W. Tukey of Bell Laboratories and Princeton University.

NEW AMS PUBLICATIONS

REVIEW VOLUMES

Three sets of collected reviews are now being prepared for publication in the very near future: Reviews of Papers on Infinite Groups edited by Gilbert Baumslag, Reviews of Papers on Finite Groups edited by Daniel Gorenstein, and Reviews of Papers in Number Theory edited by William J. LeVeque. Professors Baumslag, Gorenstein, and LeVeque have compiled, edited, and classified the reviews on their particular topics which have been published in Mathematical Reviews since 1940.

The purpose of these collections is to assist those who search the literature to find what has been done on a certain topic. In addition to subject classifications, the volumes contain an author index, listing not only the author but the title, journal or publisher, and year of publication. Of special interest are the forward citations which are given at the end of many reviews. These citations take note of any references made to the article at a later date.

Reviews of Papers in Infinite Groups will appear in two volumes, 512*pages each, and the publication date is set for January 1974. The reviews are classified under twenty-four major headings: books and survey articles; the axioms for groups and some algebraic systems related to them; groups given by generators and defining relations; algorithmic and other problems relating to logic; decompositions of groups and group-theoretical constructions; varieties of groups; nilpotent groups and their generalizations; commutator calculus, commutators and related topics; locally finite and periodic groups; classes of groups; systems, series, and subgroup lattices; special subgroups and topics connected to subgroups; equations in groups and various embedding theorems; ordered, partially ordered, and lattice ordered groups; group rings and group algebras; morphisms and mappings; homology and cohomology of groups; abelian groups (further broken down under eight sub-headings); some unrelated topics; discrete linear groups; the classical groups; representation theory of discrete groups; groups of knots, braids, and links; groups connected to geometry, topology, and analysis. The index appears in the second volume.

Reviews of Papers in Finite Groups will appear in one volume of 736*pages, and will be published in February 1974. The twenty-one major headings are books and survey articles; ordinary representations and characters of groups; modular representations and characters; integral and p-adic representations; group rings and algebras; applications of character theory; symmetric groups; permutation groups; groups of Lie type and the sporadic groups; groups and geometries; classification of simple and nonsolvable groups; some unrelated structural problems; nilpotent groups; solvable groups; characterizations of solvable groups; π-structure of groups; some internal properties of groups; automorphisms of groups; arithmetic and combinatorial problems; cohomology of groups; miscellaneous problems related to rings.

Reviews of Papers in Number Theory will appear in six volumes, to be published during 1974. Following are the major headings, publication dates, and number of pages for each volume. Volume 1, 640*pages, February 1974: congruences, arithmetical functions, primes and factorization, continued fractions, and other expansions; sequences and sets; polynomials and matrices. Volume 2, 736*pages, April 1974: Diophantine equations; forms and linear algebraic groups; discontinuous groups and automorphic forms; Diophantine geometry. Volume 3, 424*pages, May 1974: geometry of numbers; Diophantine approximation; distribution modulo 1, metric theory of algorithms. Volume 4, 672*pages, June 1974: exponential and character sums; zeta functions and L-functions, analysis related to multiplicative and additive number theory; multiplicative number theory; additive number theory, lattice point problems; miscellaneous arithmetic-analytic questions. Volume 5, 480*pages, June 1974: algebraic number theory–global fields; algebraic number theory–local and p-adic fields; finite fields and finite commutative rings; connections with logic. Volume 6, 424*pages, February 1974: general; subject index; author index.

Orders are now being accepted for these three sets of review volumes. It should be noted that the two collections on group theory can be purchased as a set, and that the number theory volumes may be purchased individually, although it is suggested that those purchasing single volumes of the number theory set also purchase Volume 6 which contains the indexes. Prices are listed on the following page.

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* Approximate number of pages

	List	Institutional Member	Individual Member	Student
Reviews of Papers on Infinite Groups, two volumes	$ 70	$ 49	$ 28	$ 14
Reviews of Papers on Finite Groups, one volume	50	35	20	10
Combined set of above three volumes	100	70	40	20
Reviews of Papers in Number Theory, six volumes	190	133	76	38
Volume 1 (congruences, arithmetical functions, primes and factorization, continued fractions, and other expansions; sequences and sets; polynomials and matrices)	50	35	20	10
Volume 2 (Diophantine equations; forms and linear algebraic groups; discontinuous groups and automorphic forms; Diophantine geometry)	50	35	20	10
Volume 3 (geometry of numbers; Diophantine approximation; distribution modulo 1, metric theory of algorithms)	40	28	16	8
Volume 4 (exponential and character sums; zeta functions and L-functions, analysis related to multiplicative and additive number theory; multiplicative analytic questions)	50	35	20	10
Volume 5 (algebraic number theory–global fields; algebraic number theory–local and p-adic fields; finite fields and finite commutative rings; connections with logic)	40	28	16	8
Volume 6 (general; subject index; author index)	40	28	16	8

NEWS ITEMS AND ANNOUNCEMENTS

NSF GRADUATE FELLOWSHIPS AND NATO POSTDOCTORAL FELLOWSHIPS

Competition is now open for 500 National Science Foundation Graduate Fellowships and forty North Atlantic Treaty Organization Postdoctoral Fellowships. These fellowships are open to citizens or nations of the United States, and they are awarded for full-time study leading to the master's or doctor's degree in science. The deadline for the Graduate Fellowships applications is November 26; for the Postdoctoral Fellowships, the deadline is October 29.

The stipend for the NATO Postdoctoral Fellowships is $9,000 per year plus a limited travel and dependency allowance. The usual tenure is nine or twelve months. Graduate Fellows receive stipends of $3,600 for a twelve-month tenure, or $300 per month. No dependency allowances are paid. Graduate Fellowships are awarded for a three-year period, dependent on the student's satisfactory progress and availability of NSF funds. Awards will be made only to students who have completed not more than one year of graduate studies.

Announcement and application materials for NSF Graduate Fellowships may be obtained from the Fellowship Office, National Research Council, 2101 Constitution Avenue, N.W., Washington, D.C. 20418. Information on the NATO Postdoctoral Fellowships may be obtained from the Fellowship Section, Division of Higher Education in Science, National Science Foundation, Washington, D.C. 20550.

OVERSEAS OFFICE OF THE CALCUTTA MATHEMATICAL SOCIETY

The Calcutta Mathematical Society, which maintains a reciprocity agreement with the American Mathematical Society, has announced the establishment of an office in the U.S. to handle editorial work for the CMS Bulletin and membership in that society. The annual dues for AMS members is $2 (plus $1 admission fee at the time of joining), and the privilege of membership is a subscription to the CMS Bulletin, published four times a year. For submission of papers to the CMS Bulletin and information on membership, please write to Dr. Lokenath Debnath, Overseas CMS Office, Department of Mathematics, East Carolina University, Greenville, North Carolina 27834.

SPECIAL MEETINGS INFORMATION CENTER

The purpose of this center is to maintain a file on prospective symposia, colloquia, institutes, seminars, special years, meetings of other associations, and to notify the organizers if conflicts in subject matter, dates, or geographical area become apparent. An announcement will be published in these *Notices* if it contains a call for papers, place, date, subject (when applicable), and speakers; a second announcement will be published only if changes to the original announcement are necessary, or if it appears that additional information should be announced. In general, SMIC announcements of meetings held in the United States and Canada carry only date, title of meeting, place of meeting, speakers (or sometimes general statement on the program), deadline dates for abstracts or contributed papers, and name of person to write for further information. Meetings held outside the North American area may carry slightly more detailed information. Information on the pre-preliminary planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society. Deadlines for particular issues of the *Notices* are the same as the deadlines for abstracts which appear on the inside front cover of each issue.

January 1 - December 14, 1974
MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH (Mathematics Research Institute of Oberwolfach)
Information: Attendance at the sessions is by invitation only. Those wishing to attend should write directly to the chairmen of individual sessions requesting an invitation.

January 1–5
Arbeitstagung
Chairman: H. Salzmann, Tubingen

January 6–12
Mengenlehre und Modelltheorie
Chairmen: G. H. Müller, Heidelberg; A. Oberschelp, Kiel; K. Potthoff, Kiel

January 13–19
Zahlentheorie, insbesondere elementare und analytische Zahlentheorie
Chairmen: H.-E. Richert, Ulm; W. Schwarz, Frankfurt; E. Wirsing, Marburg

January 20–26
Arbeitsgemeinschaft uber C*-Algebren
Chairmen: H. Behncke, Bielefeld; A. Bergmann, Düsseldorf; G. Michler, Tübingen

January 27 - February 2
Intuitionistische Metamathematik
Chairmen: G. M. Muller, Heidelberg; A. S. Troelstra, Amsterdam

February 3–9
Spezielle Funktionen
Chairmen: C. Meyer, Köln; F. W. Schäfke, Konstanz

February 10–16
Wahrscheinlichkeitsmasse auf Gruppen
Chairmen: H. Heyer, Tübingen; L. Schmetterer, Wien

February 17–23
Funktionentheorie
Chairmen: Ch. Pommerenke, Berlin; K. Strebel, Zürich; H. Wittich, Karlsruhe

February 24 - March 2
Biomathematik und Medizinische Statistik
Chairmen: R. Repges, Aachen; E. Walter, Freiburg

March 3–9
Theoretische und experimentelle Behandlung instabiler Grenzschichten
Chairman: R. Eppler, Stuttgart

March 10–16
Mathematische Stochastik
Chairmen: V. Baumann, Hohenheim; W. Bühler, Mainz

March 17–23
Gewöhnliche Differentialgleichungen
Chairmen: H.-W. Knobloch, Würzburg; R. Reissig, Bochum

March 24–29
Regelungstheorie
Chairman: M. Thoma, Hannover

March 30 - April 6
Operatortheorie und Approximation
Chairmen: P. Butzer, Aachen; J. P. Kahane, Paris; B. Sz.-Nagy, Szeged

April 7–13
Seminar uber Darstellungstheorie Liescher Gruppen
Chairman: G. Schiffmann, Strassburg

April 14–20
Arbeitstagung
Chairmen: M. Kneser, Gottingen; P. Roquette, Heidelberg

April 21–27
Mathematische Logik
Chairmen: H. Hermes, Freiburg; K. Schütte, München

April 28 - May 4
Methoden und Verfahren der mathematischen Physik
Chairman: E. Meister, Tübingen

May 5–11
Allgemeine Gruppentheorie
Chairmen: W. Gaschütz, Kiel; K. W. Gruenberg, London

May 12–18
Ringe, Moduln und homologische Methoden
Chairmen: F. Kasch, München; A. Rosenberg, Ithaca

May 19–25
Konvexe Körper. Geometrische Ordnungen
Chairmen: O. Haupt, Erlangen; R. Schneider, Berlin

May 26 - June 1
Finite Geometries
Chairmen: D. R. Hughes, London; H. Lüneburg, Kaiserslautern

June 2–8
Differentialgeometrie im Grossen
Chairmen: M. Barner, Freiburg; S. S. Chern, Berkeley; W. Klingenberg, Bonn

June 9–15
Numerik der Differentialgleichungen
Chairmen: R. Ansorge, Hamburg; L. Collatz, Hamburg; G. Hämmerlin, München; W. Törnig, Darmstadt

June 16–22
Potentialtheorie
Chairman: H. Bauer, Erlangen

June 23–28
Algebraische K-Theorie
Chairman: A. Dress, Bielefeld
June 30 - July 6
Nonstandard Analysis
Chairmen: D. Laugwitz, Darmstadt; W. A. J. Luxemburg, Pasadena
July 7–13
Grundlagen der Geometrie
Chairmen: F. Bachmann, Kiel; A. Barlotti, Bologna; E. Sperner, Hamburg
July 14–20
Zahlentheorie (Diophantische Approximationen)
Chairman: Th. Schneider, Freiburg
July 21–27
Variationsrechnung
Chairmen: E. Heinz, Gottingen; S. Hildebrandt, Bonn; W. Jäger, Münster
July 28 - August 3
Spectral and Scattering Theory
Chairmen: K. Jörgens, München; P. Werner, Stuttgart; C. Wilcox, Salt Lake City
August 4–10
Kategorien
Chairmen: J. W. Gray, Urbana; H. Schubert, Düsseldorf
August 11–17
Unternehmensforschung
Chairmen: R. Henn, Karlsruhe; H. P. Künzi, Zürich; H. Schubert, Düsseldorf
August 18–24
Endliche Gruppen und Permutationsgruppen
Chairman: B. Huppert, Mainz
August 25–31
Fragen des Mathematikunterrichts an allgemeinbildenden Schulen
Chairman: (not named)
September 1–7
Komplexe Analysis
Chairmen: J. Grauert, Gottingen; R. Remmert, Münster; K. Stein, München
September 8–14
Topologie
Chairman: (not named)
September 15–21
Spezialtagung uber Topologie
Chairman: (not named)
September 22–25
Intervallrechnung
Chairman: K. Nickel, Karlsruhe
September 29 - October 5
Geometrie
Chairmen: P. Dombrowski, Köln; K. Leichtweiss, Stuttgart
October 6–12
Funktionalanalysis
Chairmen: H. König, Saarbrücken; G. Köthe, Frankfurt; H. H. Schaefer, Tübingen; H. G. Tillmann, Mainz
October 13–19
Arbeitstagung
Chairmen: M. Kneser, Göttingen; P. Roquette, Heidelberg
October 20–26
Problemgeschichte der Mathematik
Chairmen: H. Gericke, München; C. J. Scriba, Berlin
October 27 - November 2
Algorithmen und Komplexitatstheorie
Chairmen: C. P. Schnorr, Frankfurt; A. Schönhage, Tübingen; V. Strassen, Zürich
November 3–9
Fortbildungslehrgang für Studienräte
Chairman: (not named)
November 3–9
Logic Group
Chairman: G. H. Müller, Heidelberg
November 10–16
Spezialtagung uber Mathematische Stochastik
Chairman: (not named)
November 17–23
Optimierungstheorie und optimale Steuerungen
Chairmen: R. Bulirsch, München; W. Oettli, Mannheim; J. Stoer, Würzburg
November 24–30
Automatentheorie und formale Sprachen
Chairmen: G. Hotz, Saarbrücken; H. Langmaack, Saarbrücken; H. Walter, Darmstadt
December 1–7
Numerische Behandlung kombinatorischer und graphentheoretischer Probleme
Chairmen: L. Collatz, Hamburg; G. Meinardus, Erlangen; H. Werner, Münster
December 8–14
Fragen des Mathematikunterrichts an allgemeinbildenden Schulen
Chairman: (not named)

December 27–31, 1973
HOLIDAY SYMPOSIUM
New Mexico State University, Las Cruces, New Mexico
Program: Series of lectures on recent research on the application of Gel'fand-Fuks cohomology to foliations by Raoul A. Bott; contributed papers
Information: Professor Robert J. Wisner, Department of Mathematical Sciences, New Mexico State University, Las Cruces, New Mexico 88003

January 24–26, 1974
URBAN SERVICES SEMINAR
New York City Rand Institute, New York, New York
Sponsors: New York City Rand Institute and SIAM Institute for Mathematics and Society (SIMS)
Program: Review of the development and implementation in New York City of new approaches to fire protection and sanitation. Emphasis will be placed on mathematical aspects of the varied methodologies that were applied.
Information and applications: SIMS, 33 South 17th Street, Philadelphia, Pennsylvania 19103

April 1–3, 1974
SYMPOSIUM ON FINITE ELEMENTS AND PARTIAL DIFFERENTIAL EQUATIONS
Madison, Wisconsin
Program: Fourteen invited lectures dealing with the mathematical aspects of the use of finite elements in the numerical solution of partial differential equations
Information: Professor C. de Boor, Mathematics Research Center, University of Wisconsin–Madison, 610 Walnut Street, Madison, Wisconsin 53706

April 12–13, 1974
1974 SPRING MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC
(in conjunction with spring meeting of the American Mathematical Society)
Deadline for abstracts: January 25, 1974
Information: Dr. Paul C. Gilmore, IBM Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598

June 19–21, 1974
INTERNATIONAL SYMPOSIUM ON SATELLITE DYNAMICS: ORBIT AND ATTITUDE
São Paulo, Brazil
Program: Orbit dynamics (analytical theories, computer applications, numerical theories, surface forces); attitude dynamics (motion around center of mass, spin orbit coupling)
Contributed papers: Detailed summaries to COSPAR Secretariat, 55, Boulevard Malesherbes, Paris 8e, France, with copy to Professor G. E. O. Giacaglia, Instituto Astrônomico e Geofísico, Universidade de São Paulo, Av. Miguel Stefano, s/n, Parque do Estado, Caixa Postal nº 30.627, Sao Paulo, Brazil. Deadline March 1, 1974
Information: Professor G. E. O. Giacaglia at address above

July 17 - August 3, 1974
1974 EUROPEAN MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC AND INTERNATIONAL UNION OF HISTORY AND PHILOSOPHY OF SCIENCE
Kiel, Federal Republic of Germany
Organizer: Deutsche Vereinigung für Mathematische Logik und Grundlagenforschung
Program: Summer Institute, July 17–31, on mathematical logic, including model theory, set theory, recursion theory, computation theory, contributed papers; logic colloquium August 1–3, with invited lecturers and contributed papers.
Contributed papers to logic colloquium: Abstracts, 300 words or less, by May 15, 1974, to Professor Arnold Oberschelp, Philosophisches Seminar der Universitat, Olshausenstrasse, 23 Kiel, Federal Republic of Germany
Information: Professor Arnold Oberschelp at above address

1974–1976
MITTAG-LEFFLER INSTITUTE
Djursholm, Sweden
Program: General field of partial differential operators, including applications in physics. Organized by L. Carleson, L. Gårding, and L. Hörmander as an extended symposium on an invited basis with series of lectures and seminars.
Participation: Number of participants will be strictly limited but a few stipends for postgraduate work will be available, and some members with independent support can be accepted.
Information: Professor L. Carleson, Institut Mittag-Leffler, Auravägen 17, S-182 62 Djursholm, Sweden

NEWS ITEMS AND ANNOUNCEMENTS

PANEL OF VOLUNTEERS FOR CAREER INFORMATION

The American Mathematical Society and the many students who ask for career information from the Society are indebted to the volunteers listed below, who have, with impressive care and thoughtfulness, encouraged students in mathematics by answering their letters. An average of approximately seventy requests for career information are received every month. Most of these are routine in nature and are answered by sending the correspondent a brief brochure on careers in mathematics. Some of the letters, however, show a real interest in mathematics; it is these that are sent to the panel of volunteers to be answered. During the past year, many letters were turned over to these volunteers. The present roster includes Richard A. Alo (Carnegie-Mellon University), Richard V. Andree (University of Oklahoma), William F. Atchinson (University of Maryland), Gail H. Atneosen (Western Washington State College), Prem N. Bajaj (Wichita State University), Thomas L. Bartlow (Villanova University), Barnard H. Bissinger (Pennsylvania State University), Wray G. Brady (Slippery Rock State College), Robert C. Carson (University of Akron), Srisakdi Charmonman (University of Missouri at Columbia), Daniel Clock (Northern Michigan College), Romae J. Cormier (Northern Illinois University), Raymond F. Coughlin (Temple University), Charles H. Cunkle (Slippery Rock State College), John M. Danskin (University of California, Santa Cruz), Richard C. DiPrima (Rensselaer Polytechnic Institute), Underwood Dudley (DePauw University), Joseph H. Engel (The Franklin Institute), F. A. Ficken (New York University), Harry A. Gehman (State University of New York at Buffalo), Herbert A. Gindler (Tulane University), A. K. Gupta (University of Arizona), Deborah T. Haimo (Washington University), Franklin Haimo (Washington University), R. G. Helsel (Ohio University), Hurbert L. Hunzeker (Michigan Technological University), C. Ionescu Tulcea (University of Illinois), John Kenelly (Clemson University), Ladis D. Kovach (Naval Postgraduate School), David M. Krabill (Bowling Green State University), Herbert C. Kranzer (Adelphi University), George R. Kuhn (Northwestern Michigan College), John B. Lane (Edinboro State College), Kotik K. Lee (Syracuse University), Eugene H. Lehman (Université du Québec à Trois-Rivières), William J. LeVeque (Claremont Graduate School), William F. Lucas (Cornell University), Eugene Lukacs (Bowling Green State University), Kenneth O. May (University of Toronto), Bernard McGovern (RCA), Robert A. Melter (Southampton College), Sanford S. Miller (SUNY, College at Brockport), Paul D. Minton (Richmond, Virginia), Richard C. Morgan (St. John's University), Weston I. Nathanson (California State University at Northridge), Abraham Nemeth (University of Detroit), Sam Newman (Federal Aviation Agency), Michael Olinick (Middlebury College), Malcolm W. Oliphant (Hawaii Loa College), Otway Pardee (Syracuse University), George Piranian (University of Michigan), Lyle E. Pursell (University of Missouri at Rolla), Gordon Raisbeck (Arthur D. Little, Inc.), Stewart M. Robinson (Cleveland State University), Ervin Y. Rodin (Washington University), Alex Rosenberg (Cornell University), Paul Rotter (The Mutual Benefit Life Insurance Company), Jules P. Russell (Polytechnic Institute of Brooklyn), I. Richard Savage (Florida State University), Albert Soglin (Chicago City College), Thomas H. Southard (California State University at Hayward), Raymond A. Spong (General Dynamics), Nancy Tapper (Empire State College), Charles J. Thorne (U.S. Navy Missile Center), H. Wescott Vayo (University of Toledo), Daniel H. Wagner (Daniel H. Wagner Associates), Myron E. White (Stevens Institute of Technology), W. Thurmon Whitley (Marshall University), John W. Young, Jr. (Martin Marietta).

Special thanks are extended to Mr. J. A. Spencer, Magrath, Alberta, Canada, for his thoughtful answer to many communications concerning angle trisection.

Anyone who would be willing to be a part of this service is invited to send his name, address, and field of interest to Career Information, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940.

QUERIES

Edited by Wendell H. Fleming

The QUERIES column is published in each issue of these *Notices*. This column welcomes questions from AMS members regarding mathematical matters such as details of, or references to, vaguely remembered theorems, sources of exposition of folk theorems, or the state of current knowledge concerning various conjectures. When appropriate, replies from readers will be edited into a definitive composite answer and published in a subsequent column. All answers received to QUERIES will ultimately be forwarded to the questioner. Consequently, all items submitted for consideration for possible publication in this column should include the name and complete mailing address of the person who is to receive the replies. The queries themselves, and responses to such queries, should be typewritten if at all possible and sent to Professor Wendell H. Fleming, American Mathematical Society, Post Office Box 6248, Providence, Rhode Island 02940.

28. D. S. Mitrinović (Smiljanićeva 38, 11000 Beograd, Yugoslavia). The paper On inequalities of certain type in general linear integral equation theory by Mary Evelyn Wells was published in the American J. Math. 39(1917), 163–184. To the best of my knowledge these general inequalities in Hilbert space have not been considered between 1917 and 1973. Professor P. R. Beesack from Carleton University (Ottawa, Canada) confirmed my observation concerning the paper of M. E. Wells. I would like to know whether my knowledge is exact, and in the negative case which are the references where the paper in question is used. I need this information in connection with the preparation of a new edition of my book Analytic inequalities (Springer-Verlag, 1970).

29. J. J. Malone (Department of Mathematics, Worcester Polytechnic Institute, Worcester, Massachusetts 01609). In Gorenstein's book, Finite groups, the concept of a special group is treated. Where in the literature is there more information on (finite) special p-groups? (I am not interested in extra-special groups).

30. Hugh M. Edgar (Department of Mathematics, California State University, San Jose, California 95192). Let n be an odd perfect number so that we necessarily have $n = p^{\alpha} q_1^{2\beta_1} \ldots q_t^{2\beta_t}$ where p, $q_1, \ldots, q_t$ are distinct odd primes, $p \equiv \alpha \equiv 1 \pmod 4$, $t \geq 6$, etc., etc.

(a) Does it necessarily follow that there exist at least two pairs of subscripts $i < j < k < \ell$ for which $\sigma(q_i^{2\beta_i}) = q_j^{2\beta_j}$ and $\sigma(q_k^{2\beta_k}) = q_\ell^{2\beta_\ell}$?

(b) Does it necessarily follow that there exists exactly one pair of subscripts $i < j$ for which $\sigma(q_i^{2\beta_i}) = q_j^{2\beta_j}$?

RESPONSES TO QUERIES

Several responses have been received to queries published in recent issues of these Notices. The editor wishes to thank all those who have responded. The following summarizes information given therein, arranged according to query number and name of the questioner.

19. (Salzer, June 1973) a. S. A. Burr has communicated the following clarification of the point at issue in J. V. Uspensky and M. A. Heaslet's book:

"Note that $\omega_i(n)$ depends on i only through the function ψ_i. Note also that the function $\psi_0 = 2f$ can become any even function whatever. Therefore, for any n the statement that $\omega_0(n) = 0$ for every choice of even f implies that $\omega_i(n) = 0$ for that n and for every i and even f. Since it is easy to see that $\omega_0(1) = 0$ for every even f, the desired result now follows by induction on n." For another treatment see B. A. Venkov, Elementary number theory, Wolters-Noordhoff, Groningen, 1970, MR42 #178.

22. (Gilmer, June 1973) a. There is no good survey article on the question "what finite groups can be Galois groups of extensions of Q?". A well known guess is: every group can be. However, this is not supported by much evidence. The only general result is Safarevic's: every solvable group can be such a Galois group (Ref.: Izvestia Akad. Nauk SSSR, 1954). As for non-abelian simple groups, one knows that A_n can occur, $PSL_2(Z/pZ)$ when 2, 3 or 7 is a quadratic non-residue modulo p (Ph.D. thesis of K.-Y. Shih, Princeton University, 1972), also $PGL_2(Z/nZ)$ (see A. M. Macbeath, Bull. London Math. Soc. 1 (1969), 332–338, MR41 #3447). For a few other scattered results, see H. Zimmer, Computational Problems, Methods and Results in Algebraic Number Theory, Springer Lecture Notes in Math., No. 262, §3, New York, 1972.

PROBLEM LISTS

Problems in Commutative Harmonic Analysis

The following is an answer to problem 6:

The answer is negative. For example, E. Borel ("Les probabilites denumbrables et leurs applications arithmetiques." Palermo Rend. 27 (1909), 247–271) has shown that almost all idempotents are Cesaro summable to $\frac{1}{2}$ (i.e. $x_1 + \ldots + x_n \to \frac{1}{2}$ as $n \to \infty$).

Let
$$A = \left\{ x \in \ell^\infty : \frac{x_1 + \ldots + x_n}{n} \longrightarrow 1/2 \right\};$$

then it is easily seen that A is closed in ℓ^∞, not dense. The space of all idempotents has recently been studied in the papers of G. Bennett and N. J. Kalton: "FK-spaces containing c_0" (Duke Math. J. 39 (1972), 561–582) and "Inclusion theorems for K-spaces" (Canad. J. Math. 25 (1973), 511–524).

DOCTORATES CONFERRED IN 1972-1973

Supplementary List

The following are among those who received doctorates in the mathematical sciences and related subjects from universities in the United States and Canada during 1972–1973. This is a supplement to the list printed in the October 1973 issue of these *Notices*. The numbers appearing in parentheses after each university indicate the following: the first number is the total number of degrees listed for that institution; the next seven numbers are the number of degrees in the categories of 1. Pure Mathematics, 2. Statistics, 3. Operations Research, 4. Computer Science, 5. Applied Mathematics, 6. Mathematics Education, 7. Other. Sixteen universities are listed with a total of 77 individual names. This total, combined with the original list, includes doctorates from 147 universities and a total of 1,270 individual names. *

CALIFORNIA

CALIFORNIA INSTITUTE OF TECHNOLOGY (6;6, 0, 0, 0, 0, 0, 0)

Aliprantis, Charalambos Dionisios
On order and topological properties of Riesz spaces
Gray, Leonard Jeffrey
Essential central spectrum and range in a W^*-algebra
Landauer, Christopher Allen
Simple groups with 9, 10, and 11 conjugate classes
Lyford, William Carl
Scattering theory for the Laplacian in perturbed cylindrical domains
Nation, James Bryant
Varieties of algebras whose congruence lattices satisfy lattice identities
Stonesifer, John Randolph
Combinatorial inequalities for geometric lattices

UNIVERSITY OF SOUTHERN CALIFORNIA (2;2, 0, 0, 0, 0, 0, 0)

Nipp, Gordon Louis
The spinor genus of quaternion orders
Thurber, Edward Gerrish
The Scholz-Brauer problem on addition chains

GEORGIA

GEORGIA INSTITUTE OF TECHNOLOGY (5;0, 0, 0, 5, 0, 0, 0)

Department of Information and Computer Science

Baird, Michael L.
A paradigm for semantic picture recognition
DeMillo, Richard A.
Formal semantics and the logical structure of programming languages
Dexter, Margaret E.
A study of information control in computer-aided instruction
Horgan, Joseph R.
Abstract computers and degrees of unsolvability
Pass, Edgar M.
An adaptive microscheduler for a multi-programmed computer system

ILLINOIS

UNIVERSITY OF CHICAGO (12;12, 0, 0, 0, 0, 0, 0)

Alvarenga, Celio W. M.
Nonlinear mapping of orientation preserving type in Banach spaces
Flynn, Thomas E.
The complex bordism of cyclic groups
Ho Chat-Yin
On the classical case of the quadratic pairs for 3 whose root group has order 3
Isnard, Carlos A.
Degree theory in Banach manifolds
Kwong Man-Kam
Asymptotic partition of energy property of solutions of symmetric hyperbolic linear differential equations
Matijevic, Jacob R.
Some topics in graded rings
Powls, Keith W.
A boundary behavior problem in pseudo-differential operators
Schneider, Harold W.
Free involutions of homotopy $S^{[N/2]} \times S^{[(N+1)/2]}$'s
Specht, William H.
The quadratic pairs theorem in local analysis
Voreadou, Rodiani
A coherence theorem for closed categories
Wang Kai
Free and semi-free smooth actions of S^1 and S^3 on homotopy spheres
Ward, Coburn C.
Convergence of the empirical process for certain dependent sequences and resulting applications to Diophantine approximation

UNIVERSITY OF ILLINOIS (1;1, 0, 0, 0, 0, 0, 0)

Pelletier, Donald Hector
Boolean valued models of set theory when the Boolean algebra is a proper class in the ground model

KANSAS

KANSAS STATE UNIVERSITY (1;1, 0, 0, 0, 0, 0, 0)

John, Chester Charles, Jr.
The construction of finite commutative semigroups

MARYLAND

JOHNS HOPKINS UNIVERSITY (1;0, 1, 0, 0, 0, 0, 0)

Department of Biostatistics

Norris, Douglas Arnold
Two-sex population models: A cohort approach with emphasis on nuptiality

MASSACHUSETTS

CLARK UNIVERSITY (1;1, 0, 0, 0, 0, 0, 0)

Redding, Robert W.
A study of convex functions of order α, α-spiral-like functions of order β, and typically real functions of order α

MISSOURI

UNIVERSITY OF MISSOURI-COLUMBIA (5;5, 0, 0, 0, 0, 0, 0)

Ettling, William A.
Arc length in metric spaces

*The October 1973 list contained 1,193 names, not 1,183 as stated.

Kishta, Mahmoud A.
Generalized indefinite and Einstein Finsler spaces
Mosiman, Steven E.
Strict topologies and topological measure theory
Stephens, Charles R.
Wedderburn principal theorem
Stockton, Raymond E.
On simple totally antiflexible algebras

NEW YORK

POLYTECHNIC INSTITUTE OF BROOKLYN
(3;2, 0, 0, 0, 1, 0, 0)

Berenbom, Joshua
Topological algebras over valued fields
Flynn, Robert
The dynamic complexity of networks
Stoller, Gerald
A generalization of Ulm's theory

SYRACUSE UNIVERSITY (7;0, 0, 0, 7, 0, 0, 0)

Department of Systems and Information Science
Brown, Dale
Algebraic descriptions of general block designs
Folk, Michael
Influencies of development level on a child's ability to learn concepts of computer programming
McGill, Michael
Applications of networks of dynamically changing automata to the study of human communication behavior
Posdamer, Jeffrey L.
A computer organization for large graphics systems
Riesenfeld, Richard
Applications of B-spline approximation to geometric problems of computer-aided design
Shah, Dineshchandra
Models for the economic design of some Markovian control chart schemes for a finite jump process
Webb, Douglas
The development and application of an evaluation model for hash coding systems

PENNSYLVANIA

UNIVERSITY OF PITTSBURGH (2;0, 1, 0, 0, 0, 0, 1)

Department of Biostatistics
Lawson, Alton
Cytogenetic approach to the clonal nature of hematopoietic cells
Lee Tzuo-Yan
Some statistical models of survivorship and life-testing in biomedical experiments

TENNESSEE

UNIVERSITY OF TENNESSEE (7;7, 0, 0, 0, 0, 0, 0)

Amsbury, Wayne P.
The product of null-sequence arc-decompositions is euclidean
Dent, William H., Jr.
Extended decomposition of a 3-manifold M^3 so as to yield M^3
Lawson, Linda Marie
Computational methods for generalized inverse matrices
Lewis, Roger T.
Oscillation criteria for fourth order linear differential operators
Ridenhour, Jim R.
Tauberian theorems for Hankel transform in a class of convolution transforms
Simpson, Robert J.
The application of rectangular relations to the study of binary relations on a set
Webber, Robert P.
Semigroups determined by matrix norms

TEXAS

UNIVERSITY OF HOUSTON (7;6, 0, 0, 0, 1, 0, 0)

Bandy, Carroll L.
On M-spaces and Δ-spaces
Engvall, John L.
The analysis of a boundary value problem arising in laminar flow in the entrance region of ducts
Karvellas, Paul H.
Algebraic and topological semirings
Lea, Robert N.
Mappings of H-spaces
Read, David R.
Confluent, weakly confluent, and locally confluent maps
Shih Chao-Dung
Behavior of solutions of third order linear differential equations
Tefteller, Steve C.
Oscillation of second order nonhomogeneous differential equations

WISCONSIN

UNIVERSITY OF WISCONSIN-MADISON
(10;0, 10, 0, 0, 0, 0, 0)

Department of Statistics
Agresti, Alan
Bounds of extinction time distribution of branching processes
Chang Yu-Chi
Multivariate linear regression subject to zero constraints
Cleveland, William
Analysis and forecasting of seasonal time series
Hsu Der-Ann
Stochastic instability and the behaviour of stock prices
Hwang Tea-Yuan
Chernoff efficiency of linear rank statistics
Kim, Bock Ki
On the histogram type probability density estimates
Lee. Austin F. S.
Inference concerning means of two normal populations
MacGregor, John
Topics in the control of linear processes subject to stochastic disturbances
Philippou, Andreas
Asymptotic inferences in the independent not identically distributed case
Viort, Bernard
Design of experiments and dynamic models

CANADA

UNIVERSITY OF BRITISH COLUMBIA (7;6, 0, 0, 0, 1, 0, 0)

Body, Richard A.
H* the supernice spaces
Buckley, Albert Grant
Numerical simulation of a nonlinear wave equation and recurrence of initial states
Choo Koo-Guan
On the Whitehead groups of semi-direct product of free groups
Goodaire, Edgar
Irreducible representations of algebras
Lam Che-Bor
On non-linear time-lag evolution equations
Mosevich, Jack Walter
Differential geometry of tubular spaces
Snell, Roy C.
Invariant means on locally compact groups and transformation groups

VISITING MATHEMATICIANS

Supplementary List

The list of visiting mathematicians includes both foreign mathematicians visiting in the United States and Canada, and Americans visiting abroad. Note that there are two separate lists.

American and Canadian Mathematicians Visiting Abroad

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Bartle, Robert A. (U.S.A.)	Rumanian Academy of Sciences, Rumania	Functional Analysis and Operator Theory	1/74 - 6/74
Coburn, Lewis (U.S.A.)	Institut des Hautes Études Scientifique, France	Functional Analysis	10/73 - 12/73
Cornette, James Lawson (U.S.A.)	University Kebangsaan, Malaysia	Topology, Curriculum Development	5/73 - 1/74
Diamond, Harold G. (U.S.A.)	University of Nottingham, England	Analytic Number Theory	9/73 - 6/74
Ejike, Uwadiegwu (U.S.A.)	University of Glasgow, Scotland	Applied Mathematics	9/73 - 6/74
Hastings, Stuart (U.S.A.)	Oxford University, England	Ordinary Differential Equations	9/73 - 5/74
Hofmann, Karl H. (U.S.A.)	University of Paris, France	Topological Algebra	9/73 - 9/74
Isbell, John R. (U.S.A.)	Instituto di Matimatica, Universite di Perugia, Italy	Algebra, Category Theory	9/73 - 5/74
McFadden, Robert (U.S.A.)	Monash University, Australia	Algebraic Theory of Semigroups	9/73 - 5/74
Minty, George J. (U.S.A.)	Universität Hamburg, Fed. Rep. of Germany	Real Functions	9/73 - 9/74
Monk, J. Donald (U.S.A.)	Mathematics Research Institute, Switzerland	Algebraic Logic; General Algebra	8/73 - 5/74
Randolph, Paul (U.S.A.)	University of Oslo, Norway	Statistics, Applied Mathematics	8/73 - 8/74
Robinson, Derek J. S. (U.S.A.)	Swiss Federal Institute of Technology, Switzerland	Group Theory	9/73 - 6/74
Walter, John H. (U.S.A.)	Cambridge University, England	Group Theory	9/73 - 12/73
Watkins, Mark E. (U.S.A.)	Technische Hochschule, Austria	Combinatorics	9/73 - 5/74
Weinberg, Elliott (U.S.A.)	Hebrew University, Jerusalem	Ordered Algebraic Structures	9/73 - 12/73
Zarrow, Robert (U.S.A.)	Institut für Mathematik, Fed. Rep. of Germany	Complex Analysis	9/73 - 5/74

Foreign Mathematicians Visiting in the United States

Braess, Dietrich (Fed. Rep. of Germany)	University of Texas	Approximation Theory	9/73 - 12/73
de Figueiredo, Djairo G. (Brazil)	Tulane University	Functional Analysis and Differential Equations	1/74 - 2/74
Jakimovski, A. (Israel)	York University, Canada	Summability and Approximations	10/73 - 11/73
Knast, R. (Poland)	Waterloo University, York University, Canada	Automata	10/73 - 5/74
Leviatan, D. (Israel)	York University, Canada	Summability and Approximations	9/73 - 10/73
Leviatan, T. (Israel)	York University, Canada	Probability and Statistics	9/73 - 10/73
Malliavin, Paul (France)	Yeshiva University	Fourier Analysis	10/73 - 11/73
Medeiros, Luiz A. (Brazil)	Tulane University	Partial Differential Equations	1/74 - 3/74
Offord, A. C. (United Kingdom)	York University, Canada	Classical Analysis	10/73 - 5/74
Oo Kian Kee, William (Philippines)	Fayetteville State University	Differential Equations	9/73 - 6/74
Peterson, Hans (Fed. Rep. of Germany)	University of Wisconsin	Modular and Automorphic Functions	1/74 - 5/74
Thayer, F. Javier (Chile)	Tulane University	Functional Analysis	1/74 - 12/74
Zaremba, Stanislaw (United Kingdom)	Northern Illinois University	Probability, Analysis	8/73 - 5/74

LETTERS TO THE EDITOR

Editor, the *Notices*

I would like to bring to the attention of the mathematical community that December 7, 1973, is the 100th birthday of Set Theory.

"Nevertheless, Cantor did further busy himself with the mapping of sets, and by 7 December 1873 he was able to write Dedekind that he had succeeded in proving that the 'aggregate' of real numbers was uncountable. That date can probably be regarded as the day on which set theory was born."--H. Meschkowski, Dictionary of Scientific Biography, Vol. III, p. 54.

Although it is probably too late to organize any official observance of the centennial of Set Theory, we suggest that Set Theorists observe the anniversary by working on the continuum hypothesis on December 7, 1973.

Rudy Rucker

Editor, the *Notices*

In the August Notices, E. E. Moise wrote eloquently on "Jobs, Training, and Education for Mathematicians" (p. 217), making in particular some very thoughtful points about teaching, learning, and training Mathematicians to teach and also taking issue with some current proposals to weaken (or abolish) the thesis requirement for the Ph. D. He also proposes that possible past neglect of applications be rectified, by suggesting a requirement for the Ph. D. of a one-year course in applications of Mathematics. I agree, but my enthusiasm for all these good suggestions is lost when he tries to make room for this course. He proposes (p. 220), "The basic graduate courses should be no more learned than they need to be, to prepare the student for the qualifying examination; the qualifying examination should be such that the members of the graduate faculty can pass it easily."

Let me express emphatic dissent. The basic graduate courses are there for basic training; the examination is incidental. The qualifying examination is for the next generation, not this. For example, I like my students to know (and to be tested on) much more functional analysis and differential geometry than I might know myself.

Moise is dealing here with "leading departments (where) students are required to take highly sophisticated contemporary courses in many fields". He questions the necessity of this combination of breadth and depth. I believe the combination is essential: The leading departments are, or at least ought to be, in the business of catching the outstanding Mathematicians of the future. I believe that they must combine breadth and depth—and sophistication to boot. The sophistication is a necessary part of the enthusiasm (teaching and learning) for something new—and today's sophistication well understood is tomorrow's basics.

A problem remains: How to find room in the graduate program for a one-year course on applications. Let's not find the room by watering down present courses. A better move would be to have one less advanced course in the candidate's specialty—he will learn that later anyhow, and he's likely, at present, to be too specialized. Moise and I both admire those who "can make serious use of deep knowledge of many fields". I claim we should then expose students to deep knowledge in many fields.

Saunders Mac Lane

Professor Moise replies:

After re-examining my article, in the light of Professor Mac Lane's criticisms, I must confess that the criteria given at the bottom of column 1, p. 220, August 1973 *Notices*, were badly stated. On a straightforward reading, they may easily suggest that courses ought to be designed to fit examinations, rather than vice versa, and that introductory courses ought to be governed by a frozen tradition. Both of these suggestions are as objectionable to me as to anybody that I know of. For better explanations of what I had in mind, see my other remarks on the same page.

Apparently, Professor Mac Lane and I agree that a year-course in applied mathematics ought to be required, and that some other requirement ought accordingly to be reduced. In my article, I proposed a reduction of requirements in depth of study in fields outside the student's specialty. If introductory courses were taught "in a more classical spirit", then they might convey less information. But if time were taken to explain the connections with more elementary mathematics, this might give the student a better conception of the standards of motivation that ought to govern his future research.

Professor Mac Lane proposes, alternatively, that the student's advanced course work in his own specialty be reduced. This worries me. Plenty of Ph. D.'s turn out to be unable to do any research not directly connected, both in substance and in methodology, with their dissertations. In many cases, no doubt, this is due to a lack of creative talent. But in cases where it is due to dificient training, it is hard to believe that the sutdent's handicap is an insufficiently deep and contemporary knowledge of fields other than his own. More plausibly, the trouble is that the student knows so narrow a sector of the research frontier that he cannot find problems to work on without the help of a teacher. If his contact with the frontier is unduly brief, then even the dissertation may be written under conditions of tutelage. For this reason, it seems to me that requirements in depth are the second-to-last that we ought to consider reducing (the last being the requirement of a research dissertation.)

Editor, the *Notices*

In a letter published in the October *Notices*, Professor Daniel Pedoe of the University of Minnesota has raised serious questions about the operation of the merit system in American mathematics departments. He has beclouded the general issues by making some references to his own university. As one who served on a National Science Foundation team which evaluated the Minnesota mathematics department in connection with its successful application for a building grant and who has had many occasions to look into the professional standing of mathematicians at Minnesota in connection with his chairmanship of the AMS Committee to Select Hour Speakers for Western Sectional Meetings, I can say with certainty that Professor Pedoe's apparent low estimate of the mathematical accomplishments of his most eminent colleagues is not widely shared. Furthermore, I do not believe that the columns of the *Notices* should be used even implicitly for the purpose of discussing the standing of individual mathematicians and individual mathematics departments.

Paul T. Bateman

EDITORIAL COMMENT: Despite occasional lapses, the editors of the *Notices* intend to continue the existing policy that letters to the editor not contain material which could be interpreted as unfounded criticisms of particular individuals or institutions.

NEWS ITEMS AND ANNOUNCEMENTS

SIAM INSTITUTE FOR MATHEMATICS AND SOCIETY

Under the 1974 SIMS Transplant Program, mathematicians may move into centers where interdisciplinary projects, involving societal problems, are under study. In the first seven months of the program, one Transplant has been established at Harvard Center for Population Studies. In addition to the Transplant, SIMS is able to assist in arranging sabbaticals at participating centers. One has been arranged for the early part of 1974 at the Cornell Program on Science and Society. To date, twelve centers have agreed to participate in the program. They are Environmental Engineering Sciences and Environmental Quality Laboratory (Caltech); Program on Science, Technology, and Society Center for Environmental Quality Management (Cornell); Institute of Behavioral Science (Colorado); Center for Population Studies and School of Public Health (Harvard); New York City Rand Institute; Institute for Environmental Medicine (New York University); Center for Environmental Studies (Princeton); Food Research Institute and Human Genetics (Stanford). Additional centers in the fields of energy (two) and health care (three) are now in the process of being added. For further information, write to SIMS, 33 South 17th Street, Philadelphia, Pennsylvania 19103.

CORPORATE MEMBERS AND INSTITUTIONAL ASSOCIATES

The Society acknowledges with gratitude the support rendered by the following corporations who held either Corporate Memberships or Institutional Associateships in the Society during this calendar year.

Corporate Members

Bell Telephone Laboratories, Incorporated
Ford Motor Company
General Motors Corporation
International Business Machines Corporation
Rockwell International

Institutional Associates

Chelsea Publishing Company
Princeton University Press
Shell Development Company
Springer-Verlag

ERRATUM TO BIOGRAPHICAL INFORMATION FOR 1973 ELECTION

The biographical information about LEE LORCH, candidate for member-at-large of the Council in the 1973 election, contained the following paragraph that was inadvertently omitted when the camera copy was prepared: "(Research in) higher monotonicities of oscillatory Sturm-Liouville differential equations and special functions. Summability of various expansions, Lebesgue constants, Gibbs phenomena."

PERSONAL ITEMS

JOSEPH ARKIN of Spring Valley, New York, has been elected to the New York Academy of Sciences.

MARTIN W. BARTELT of Rensselaer Polytechnic Institute will be on a leave of absence at the University of Rhode Island for the academic year 1973–1974.

JOHN L. GAMLEN of the University of Alberta has been reappointed to a visiting assistant professorship at Yale University.

HOWARD GARLAND of SUNY at Stony Brook has been appointed to a professorship at Yale University.

JO ANN S. HOWELL has joined the staff of the Los Alamos Scientific Laboratory. She will work as a postdoctoral fellow with the Computing Science and Services Division.

HENRY S. LEONARD of Northern Illinois University has been appointed to a visiting fellowship at Yale University for 1973–1974.

ANGUS MACINTYRE of Aberdeen University, Scotland, has been appointed to an associate professorship at Yale University.

CLEMENT A. McCALLA of the Massachusetts Institute of Technology has been appointed to a visiting assistant professorship at Rensselaer Polytechnic Institute.

FRANK S. QUINN III of the Institute for Advanced Study has been appointed to an assistant professorship at Yale University.

JOACHIM REINEKE of Technische Universität Hannover has been appointed to a postdoctoral research fellowship at Yale University.

HASKELL ROSENTHAL of the University of California, Berkeley, has been appointed to a visiting professorship at Ohio State University for the academic year 1973–1974.

LEE A. SEGEL of Rensselaer Polytechnic Institute has been appointed to a professorship and to the chairmanship of the Department of Applied Mathematics at the Weizmann Institute, Rehovot, Israel.

JAU-SHYONG SHIUE of the National Chengchi University, Taipei, Republic of China, has been awarded an Alexander von-Humboldt Foundation fellowship, he will be visiting the University of Gottingen, Federal Republic of Germany, from May 1973 to April 1974.

MARSHALL SLEMROD of Brown University has been appointed to an assistant professorship at Rensselaer Polytechnic Institute.

PROMOTIONS

To Academic Dean. Regis College: SUSAN WILLIAMSON.

To Professor. Temple University: JANOS GALAMBOS.

To Associate Professor. Mississippi State University: JOHN R. GRAEF, PAUL W. SPIKES.

To Assistant Professor. Rensselaer Polytechnic Institute: KAY B. SOMERS.

INSTRUCTORSHIPS

Yale University: WILLIAM G. DWYER, ANDREW J. SOMMESE.

DEATHS

Dr. TRINIDAD J. JARAMILLO of San Diego, California, died on April 19, 1973, at the age of 70. He was a member of the Society for 41 years.

Mr. EUGENE McDERMOTT of Texas Instruments, Inc., died on August 24, 1973, at the age of 74. He was a member of the Society for 31 years.

ABSTRACTS PRESENTED TO THE SOCIETY

Preprints are available from the author in cases where the abstract number is starred.

The papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

An individual may present only one abstract by title in any one issue of the *Notices* but joint authors are treated as a separate category. Thus, in addition to abstracts from two individual authors, one joint abstract by them may also be accepted for an issue.

Algebra & Theory of Numbers

*73T-A263. ALBERT A. MULLIN, 9213 Kristin Lane, Fairfax, Virginia 22030. On the fundamental theorem of the geometry of numbers. Preliminary report.

This note provides three extensions of Minkowski's theorem together with alternative formulations of a result on lattice points found by G. D. Birkhoff circa 1913. Recall that a lattice point $(x_1, \ldots, x_n)$ is said to be visible provided GCD $(x_1, \ldots, x_n) = 1$ and highly visible provided the mosaics of $x_1, \ldots, x_n$ have no prime number in common. Lemma. If the n-dimensional Lebesgue measure $V(B)$ of a convex body B, symmetrical about the origin, satisfies the inequality $V(B) \geqq 2^n$, then B contains at least one visible lattice point. Main lemma. There exists a recursive function $f(\cdot)$ such that if $V(B) \geqq f(n)$ then B contains at least one highly visible lattice point. Problem (extending Birkhoff). If M is a Lebesgue measurable set in n-dimensional real Euclidean space of measure $V(M) > 1$, then there exist distinct $x \in M$ and $y \in M$ such that $x - y$ is highly visible, although not necessarily belonging to M. (Received June 28, 1973.)

*73T-A264. SHAFAAT AHMAD, Université de Sherbrooke, Sherbrooke, Québec, Canada. Algebraic domains and Fermat's last theorem. Preliminary report.

Let V be a variety of groupoids and let the free monogenic groupoid $(F,+)$ of V have a unique generator, say x. In terms of the endomorphisms φ_a, $a \in F$, $\varphi_a(x) = a$, define $a \circ b = \varphi_b(a)$. An algebra of the form $(F,+,\circ)$ will be called an algebraic domain. Taking V to be the variety of groupoids, commutative groupoids, and semigroups, one gets the algebraic domains that will be denoted by $(W,+,\circ)$, $(E,+,\circ)$, and $(N,+,\circ)$ respectively. The algebraic domain $(N,+,\circ)$ is the algebra of positive integers under ordinary addition and multiplication. For every algebraic domain $(F,+,\circ)$ the groupoid $(F,\circ)$ is a semigroup. $(F,+,\circ)$ is said to have Fermat property if $u^n + v^n = w^n$ has no solutions in $(F,+,\circ)$ for sufficiently large n. Theorem 1. $(W,+,\circ)$ and $(E,+,\circ)$ have Fermat property. Question 1. Are there uncountably many algebraic domains with Fermat property? Theorem 2. $(W,\circ)$, $(E,\circ)$ are free semigroups. Question 2. For what algebraic domains $(F,+,\circ)$ is the semigroup $(F,\circ)$ intrinsically free, have a unique set of generators, and a solvable word problem? (Received July 19, 1973.) (Author introduced by Professor K. Srinivasacharyulu.)

*73T-A265. PHILIP A. LEONARD, Arizona State University, Tempe, Arizona 85281 and KENNETH S. WILLIAMS, Carleton University, Ottawa, Ontario K1S 5B6, Canada. Character sums and cyclotomic fields with unique factorization.

For q a rational prime and $\zeta = \exp(2\pi i/q)$ the ring $Z[\zeta]$ is a unique factorization domain precisely when q is one of 3, 5, 7, 11, 13, 17, 19. For these primes q and a prime $p \equiv 1 \pmod q$ the Jacobsthal sum $\varphi_q(a) = \sum_{x=0}^{p-1} x(x^q + a)/p$ $(a \not\equiv 0 \pmod p))$ is evaluated in terms of suitably normalized prime factors of p in $Z[\zeta]$, extending the evaluation of $\varphi_q(a)$ by Rajwade when $q = 3$ and 5 ("On rational primes p congruent to 1 (mod 3 or 5)", Proc. Cambridge Philos. Soc. 66(1969), 61-70). When $q = 7$ an interesting application is made relating the sum $\varphi_7(a)$ to the solutions of a certain triple of diophantine equations. (Received August 31, 1973.)

*73T-A266. BJARNI JONSSON, Vanderbilt University, Nashville, Tennessee 37235. Finite bases for certain sums of lattice varieties.

Theorem. If $\mathcal{N}$ is the variety generated by the pentagon (the five-element nonmodular lattice) and $\mathcal{V}$ is any finitely based variety of modular lattices, then their lattice sum (join) $\mathcal{V} + \mathcal{N}$ is finitely based. Corollary. The unique lattice variety covering the variety of all modular lattices is finitely based. The proofs rely heavily on the techniques developed in Kirby Baker's forthcoming paper, "Primitive satisfaction and equational problems for lattices and other algebras". (Received August 31, 1973.)

*73T-A267. BRIAN A. DAVEY, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. Some annihilator conditions on distributive lattices.

Throughout L denotes a distributive lattice and n a positive integer. The annihilator $\langle a,b\rangle$ of a relative to b is the ideal $\{x \in L | x \wedge a \leq b\}$. If L has a zero, $\langle a,0\rangle$ is written $(a]^*$. All joins and meets range from 0 to n. Theorem 1. Assume L has a zero. T.f.a.e.: (i) Any $n+1$ minimal prime ideals are comaximal; (ii) If $a_0,\dots,a_n \in L$ with $a_i \wedge a_j = 0$ $(i \neq j)$, then $\bigvee_j (a_j]^* = L$; (iii) $(\bigwedge_j a_j]^* = \bigvee_j (\bigwedge_{i\neq j} a_i]^*$ holds identically in L. If L satisfies the conditions of Theorem 1 it is called a B_n-lattice. Theorem 2. T.f.a.e.: (i) L is a relatively B_n-lattice; (ii) Any $n+1$ mutually incomparable prime ideals are comaximal; (iii) If $b, a_0,\dots,a_n \in L$ with $a_i \wedge a_j \leq b$ $(i\neq j)$, then $\bigvee_j \langle a_j, b\rangle = L$; (iv) $\langle \bigwedge_j a_j, b\rangle = \bigvee_j \langle \bigwedge_{i\neq j} a_i, a_j\rangle$ holds identically in L; (v) $\bigvee_j \langle \bigwedge_{i\neq j} a_i, a_j\rangle = L$ holds identically in L; (vi) If $b, a_0,\dots,a_n \in L$ with $a_i \vee a_j \geq b$ $(i \neq j)$, then $\bigvee_j \langle b, a_j\rangle = L$; (vii) $\langle b, \bigvee_j a_j\rangle = \bigvee_j \langle b, \bigvee_{i\neq j} a_i\rangle$ holds identically in L; (viii) $\bigvee_j \langle a_j, \bigvee_{i\neq j} a_i\rangle = L$ holds identically in L. For L pseudocomplemented Theorem 1 reduces to a known result, but for L relatively pseudocomplemented Theorem 2 reduces to a known result only for the case $n = 1$. (Received September 10, 1973.)

*73T-A268. M. BHASKARAN, 91 Carr Street, Perth, Western Australia 6000. Some remarks on the decomposition of a rational prime in a Galois extension. II. Preliminary report.

The first paper on the above title is to appear. Let k be a finite Galois extension of the rationals of degree n. Let D denote the product of the distinct prime factors of the discriminant of k. Let e_{p_i} denote the ramification index of a k-prime (prime ideal of k) lying above p_i and h denote the class number of k. Theorem 1. If f is the smallest positive integer such that $a^f \equiv x^n \bmod D$, then there is an infinite number of rational primes $\equiv a \bmod D$, the residue class degrees of whose k-prime divisors divide a power of f. Theorem 2. If f in Theorem 1 is relatively prime to $2nh/f$ and if $f \mid$ l.c.m. of e_{p_i}, then the residue class degrees mentioned above are the same and equal to f. Theorem 3. If in Theorem 1, $n \mid$ l.c.m. of $(e_{p_i}, p_i - 1)$ and $(f, 2n/f) = 1$, then f is the residue class degree of any k-prime lying above a rational prime $\equiv a \bmod D$ provided the order of the k - prime in the ideal class group is relatively prime to f. Theorem 4. Let t be an integer such that $(t, 2n/t) = 1$ and $t \mid (n/\text{l.c.m. of } e_{p_i}) \times$ l.c.m. of $(e_{p_i}/((p_j - 1)/\mathrm{ord}_{p_j} p_i, e_{p_i}), p_j - 1)$ $(i \neq j)$. Then $t \mid h$. (Received September 24, 1973.)

73T-A269. DAVID J. RODABAUGH, University of Missouri, Columbia, Missouri 65201. On generalizing alternative rings.

Consider a ring R that satisfies the identity $(x,x,x) = 0$ and any two of the three identities: $(wx,y,z) + (w,x,[y,z]) - w(x,y,z) - (w,y,z)x = 0$; $([w,x],y,z) + (w,x,yz) - y(w,x,z) - (w,x,y)z = 0$; $(w, x \cdot y, z) - x \cdot (w,y,z) - y \cdot (w,x,z) = 0$. In this paper we prove that if R has characteristic prime to 6, then R semiprime with idempotent e implies R has a Pierce decomposition in which the modules multiply as they do in an alternative ring. If, in addition, R is prime with idempotent $e \neq 0, 1$ then R is alternative. (Received September 13, 1973.)

*73T-A270. GEORGE A. GRÄTZER and CRAIG R. PLATT, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada and HERBERT S. GASKILL, Memorial University, St. John's, Newfoundland, Canada. A characterization of transferable lattices.

Let K be a finite lattice. K is transferable (for the concept see G. Grätzer, "Universal algebra", Trends in Lattice Theory, Sympos., U. S. Naval Academy, 1966, pp. 173-210, van Nostrand-Reinhold, New York, 1970) iff for any lattice L and any embedding φ of K into I(L) (the lattice of all ideals of L), there exists an embedding ψ of K into L satisfying $a\psi \in a\varphi - \bigcup(b\varphi | b < a)$. If X, $Y \subseteq K$ and for all $y \in Y$ there is an $x \in X$ with $x \leqq y$, then X is said to dominate Y. For $a \in K$ and $J \subseteq K$, $\langle a, J\rangle$ is a minimal pair iff (i) $|J| > 1$; (ii) $a \leqq \bigvee J$; and (iii) if $X \subseteq K$, J dominates X, and $a \leqq \bigvee X$, then $X = J$. The condition $(T_\vee)$ requires that K have a linear ordering $\prec$ such that for any minimal pair $\langle a, J\rangle$ and for any $b \in J$ the relation $a \prec b$ holds. (See H. S. Gaskill, Algebra Universalis, 2(1972), 303-316.) Let $(T_\wedge)$ be the dual of $(T_\vee)$. Finally, let (W) be the condition that $x \wedge y \leqq u \vee v$ implies that $x \wedge y \leqq u$ or $x \wedge y \leqq v$ or $x \leqq u \vee v$ or $y \leqq u \vee v$. (B. Jonsson, Canad. J. Math. 13(1961), 256-264.)
Theorem. The conditions $(T_\wedge)$, $(T_\vee)$, and (W) are separately necessary and jointly sufficient for the transferability of a finite lattice. (The necessity of $(T_\wedge)$ and $(T_\vee)$, and the joint sufficiency of $(T_\wedge)$, $(T_\vee)$, and (W) were stated in H. S. Gaskill, Ph.D. Thesis, Simon Fraser University, 1972.) (Received September 14, 1973.)

*73T-A271. ANNE P. GRAMS, University of Tennessee, Nashville, Tennessee 37203. Atomic rings and the ascending chain condition for principal ideals. Preliminary report.

An element d of an integral domain D with identity is called an atom if d is a nonunit that cannot be written as a product of two nonunits; D is atomic if each nonzero, nonunit element of D can be written as a finite product of atoms. We say that D has the a.c.c.p. if D satisfies the ascending chain condition for principal ideals. If D has the a.c.c.p., then D is atomic. One of our major results is an example of an atomic domain D_1 that does not have the a.c.c.p. (An incorrect example of this is given in Amer. Math. Monthly 80(1973), 1-18.) D_1 has the interesting property of containing infinitely many atoms but no prime elements. In addition, the following examples are constructed: (1) a one-dimensional, non-Noetherian Prüfer domain of finite character that has the a.c.c.p. (This contrasts with the fact that an atomic Bezout domain is a principal ideal domain.); (2) an almost Dedekind domain with the a.c.c.p. that is not Noetherian; (3) all almost Dedekind domains D_2 with the a.c.c.p. but such that $D_2[X]_N$ does not have the a.c.c.p. for some multiplicative system N of $D_2[X]$. (Received September 17, 1973.)

*73T-A272. A. HOLLEMAN, Free University, Amsterdam, The Netherlands. Modular algebras. Preliminary report.

Between 1965 and 1972, Weisfeld, Sweedler, Rasala, van Oystaeyen, Kime, and Bégueri studied definitions of a concept "modular field extension". To what degree do their concepts extend to arbitrary commutative algebras? These definitions can all be extended to algebras, but the new definitions become, to a large degree, independent. A general definition (which is rather much in line with the Sweedler-Rasala approach) is being given, such that (a) all former definitions (for field extensions) are included, (b) the concept "modularity" becomes a local property (i.e. A/K is modular iff A_x/K_x is modular for all $x \in \mathrm{Spec}(K)$). Thus, if $\mu: K \to A$ is a K-algebra (K a ring of prime characteristic p) then A/K is modular if $\forall i \in N$, the rings A^{p^i} and K are linearly disjoint over $K \cap A^{p^i}$. It is proved that there exists a "modular closure" for A/K (K a field and $A = A_{red}$), i.e. there exists a unique minimal K-algebra M(A/K) which contains A and is modular over K. This generalizes a recent result of L. A. Kime. In addition, a reduced, algebraic algebra A over a field K is the tensor product of the maximal separate K-subalgebra of A and the "maximal inseparable" K-subalgebra. (Received September 17, 1973.) (Author introduced by Professor Willem Kuyk.)

*73T-A273. J. BOGO and WILLEM KUYK, Rijksuniversitair Centrum, 2020 Antwerpen, Belgium. Hecke operators for $G(\sqrt{2})$ and $G(\sqrt{3})$. Preliminary report.

Let $G(\sqrt{\ell})$, $\ell = 2, 3$, be the Hecke groups generated by the matrices (linear notation) $(1, \sqrt{\ell}; 0, 1)$ and $(0, -1; 1, 0)$. Then, departing from the commensurability of these groups with $SL_2(\mathbb{Z})$, for $n \in \mathbb{N}$, the (commuting) Hecke operators $T(\ell; n)$ for $G(\sqrt{\ell})$ are being constructed. If $\ell \nmid n$, these operators satisfy precisely the same identities as the classical ones, $T(1; n)$, for the group $SL_2(\mathbb{Z})$. For $n = \ell^r$ they satisfy the relations $T(\ell; \ell^{r+1}) = T(\ell; \ell)T(\ell; \ell^r) - \ell \cdot (\lambda, 0; 0, \lambda)\ T(\ell; \ell^r) - \ell \cdot (\ell; 0; 0, \ell)\ T(\ell; \ell^{r-1})$. Owing to the fact that, with the Weil-definition of modular forms, and the Petersson inner product, the $T(\ell; n)$ are Hermitian on the spaces of forms of even weight, the analogue of the theory for $SL_2(\mathbb{Z})$ can be written for $G(\sqrt{\ell})$ (relation between coefficients, existence of a basis with integral Fourier coefficients, functional equation, the Petersson conjecture, etc....). (Received September 17, 1973.)

*73T-A274. ABRAHAM BERMAN, Technion, Israel Institute of Technology, Haifa, Israel and ROBERT J. PLEMMONS, University of Tennessee, Knoxville, Tennessee 37916. Matrix group monotonicity.

Matrices for which the group inverse exists and is nonnegative are studied. In general, these matrices are characterized using a generalization of the concept of matrix monotonicity. In particular, nonnegative matrices having this property are characterized in terms of nonnegative rank factorizations. (Received September 18, 1973.)

73T-A275. LAWRENCE L. WANG, University of Pittsburgh, Pittsburgh, Pennsylvania 15260. A test for the sequencing of a class of finite groups with two generators. Preliminary report.

E. G. Strauss posed the question as to whether the complete directed n-graph can be decomposed into n directed Hamiltonian paths. It is known that [N. S. Mendelsohn, "Hamiltonian decomposition of the complete directed n-graph", Theory of Graphs (Proc. Colloq., Tihany, 1966), pp. 237-241. Academic Press, New York, 1968] if a finite group of order n is sequenceable, then a complete latin square can be constructed and the Hamiltonian decomposition of the complete directed n-graph is possible. Gordon [Pacific J. Math. 11(1961), 1309-1313] showed that an abelian group is sequenceable if and only if it contains exactly one element of order 2. Also, if the abelian group is sequenceable, a sequencing can be constructed. Hence, if n is even, the cyclic group of order n is sequenceable, and the completed n-graph can be decomposed into n Hamiltonian paths. According to Mendelsohn, for $n = 3, 5, 7$, the dissection is impossible. However, he "heuristically" obtained five sequencings of a group of order 21. We wrote a computer program to test whether the following type of groups are sequenceable: Let G be a finite group generated by x and y with the defining relations, $x^p = y^q = e$, $x^{-1}yx = y^r$, where e is the identity of G, r is a positive integer, p, q are primes, $q \equiv 1 \bmod p$ and $r^p \equiv 1 \bmod q$. The program was tested for $n = 6$, 10, and 21, for which the answers were known. For $n = 39$ ($p=3$, $q=13$, $r=3$), 55 ($p=5$, $q=11$, $r=3$), and 57 ($p=3$, $q=19$, $r=7$), the groups are also sequenceable; these results seem to be new. (Received September 24, 1973.) (Author introduced by Professor C. Y. Chao.)

73T-A276. ALEXANDER ABIAN, Iowa State University, Ames, Iowa 50010. Direct product decomposition of alternative rings.

Let R be an alternative ring without nonzero nilpotent elements. Theorem. $(R, \leqq)$ is a partially ordered set where $\leqq$ is defined by $x \leqq y$ iff $xy = x^2$. Moreover, $(R, \leqq)$ is infinitely distributive, i.e., $b \sup r_i = \sup br_i$. Definition. A nonzero element a of R is called an atom iff a dominates (w.r.t. $\leqq$) only 0 and itself. The notion of an atomic ring is introduced and it is shown that the set of the atoms which do not annihilate a given atom, together with 0, form an alternative integral domain. A subset S of R is called orthogonal iff the product of every two distinct elements of S is zero. Theorem. R is isomorphic to a direct product of alternative integral domains iff R is atomic and orthogonally complete (w.r.t. $\leqq$). By the introduction of the notion of a hyperatom [cf. Abian, Proc. Amer. Math. Soc. 24(1970), 502-507. MR 41 #3461] the results are extended to the direct product decomposition of R into alternative division rings. The results are also generalized to the rings (not necessarily associative or commutative) in which for every product $r_1 r_2 \ldots r_m$ of the elements r_i of the ring, $r_1 r_2 \ldots r_m = 0$ iff the subproduct (in any association and permutation whatsoever) of all the distinct factors appearing in $r_1 r_2 \ldots r_m$ is equal to zero. (Received September 24, 1973.)

73T-A277. HYO CHUL MYUNG and LUIS R. JIMENEZ, University of Northern Iowa, Cedar Falls, Iowa 50613. Direct product decomposition of alternative rings without nilpotent elements.

Let A be an alternative ring without nonzero nilpotent elements and let $\leqq$ be a relation on A defined by $x \leqq y$ if and only if $xy = x^2$. It is shown that A is isomorphic to a direct product of alternative division rings if and only if $\leqq$ is a partial order on A such that A is hyperatomic and orthogonally complete. This result is proved without using the axiom of choice, and generalizes the result shown by Chacron for the associative case (Proc. Amer. Math. Soc. 29(1971), 259-262) to a nonassociative ring. (Received September 24, 1973.)

*73T-A278. DAVID ZEITLIN, 1650 Vincent Avenue North, Minneapolis, Minnesota 55411. An explicit formula for the coefficients of the chromatic polynomial of a complete bipartite graph.

Let S_n^k and $\mathfrak{S}_n^k$ denote Stirling numbers of the first and second kind, respectively. J. R. Swenson (Amer. Math. Monthly 80(1973), 797-798) has shown that the chromatic polynomial in t of a complete bipartite graph $K_{p,q}$ is monic, has coefficients symmetric in p and q, and is given by $F(p,q,t) \equiv \sum_{j=1}^{p+q} A_j t^j \equiv \sum_{r=1}^{p} \binom{t}{r} r! \,\mathfrak{S}_p^r (t-r)^q$, where $A_0 = 0$ and $A_{p+q} = 1$. One explicit formula for A_j is now given by Theorem 1. (*) $A_j = \sum_{m=j}^{p+q} \mathfrak{S}_p^{m-q} \cdot \sum_{k=0}^{j} \binom{q}{k} (q-m)^{q-k} \cdot S_{m-q}^{j-k}$ for $j = 0, 1, \ldots, p+q$. Theorem 2. From (*) we obtain $A_{p+q-1} = -pq$ and $A_{p+q-2} = \binom{pq}{2}$. Theorem 3. $-4A_{p+q-3} = 4\binom{pq}{3} - 2\binom{pq+1}{2} + pq(p+q)$. Theorem 4. $12A_{p+q-4} = 12\binom{pq}{4} - 18\binom{pq}{3} + (8p+8q-22)\binom{pq}{2} - pq(p-q)^2$. A second explicit formula for A_j is now given by (**) $A_j = \sum_{i=j}^{p+q} \mathfrak{S}_p^{i-q} \sum_{k=j}^{i} \mathfrak{S}_q^{k+q-i} \cdot S_k^j$, $(j = 0, 1, \ldots, p+q)$. Remarks. In the evaluation of A_j, (*) is preferred for general p and q; but for specific numerical values of p and q, (**) is preferred. I suspect that for general p and q, one has for $j = 0, 1, \ldots, p+q$ that $A_{p+q-j} = (-1)^j \binom{pq}{j} + \ldots$. For general values of p and q, additional values of A_j can be obtained but with increasing algebraic effort. (Received September 24, 1973.)

Analysis

*73T-B300. H. M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada and ROBERT G. BUSCHMAN, University of Victoria, Victoria, British Columbia, Canada and University of Wyoming, Laramie, Wyoming 82070. Some polynomials defined by generating relations. Preliminary report.

In an attempt to present a unified treatment of the various polynomial systems introduced from time to time, new generating functions are given for the sets of polynomials $\{S_{n,q}^{(\alpha,\beta)}(\lambda; x)\}$ and $\{T_{n,q}^{(\alpha,\beta)}(\lambda; x)\}$, defined in this paper, and for their natural generalizations in several complex variables. The present paper also indicates relevant connections of the results derived here with different classes of generating relations which have appeared recently in the literature [cf., e.g., Trans. Amer. Math. Soc. 168(1972), 73-84]. (Received May 7, 1973.)

*73T-B301. CHANDRA MOHAN JOSHI, University of Jodhpur, Jodhpur, Rajasthan, India 342001 and M. L. PRAJAPAT, Defence Laboratory, P. B. No. 136, Jodhpur, Rajasthan, India 342001. On some properties of the generalized Hermite polynomials. Preliminary report.

In a recent communication, Maya Lahiri (Proc. Amer. Math. Soc. 27(1971), 117-121) has introduced the generalized Hermite polynomials by means of the generating relation $\sum_{n=0}^{\infty} H_{n,m,\nu}(x)/n! = e^{\nu xt - t^m}$ where n is

a nonnegative integer and m is a positive integer. She has confined herself only to the derivation of its hypergeometric representations and the differential formulas. The object of this paper is to study a number of properties involving these polynomials and to examine if most of the well-known properties are carried over to the generalized case.

(Received August 29, 1973.)

*73T-B302. FRANCIS SULLIVAN, Catholic University of America, Washington, D.C. 20017. A property of best approximation in UR spaces.

Let X be a uniformly rotund (UR) Banach space. If M is a closed subspace of X then P(M) denotes the best approximation operator on X. Theorem. Let $\{x_i\}$ be a sequence in X with $\{\|x_i\|\}$ convergent, and let $\{M_i\}$ be a sequence of closed subspaces such that for some $K > 0$ and all i, $\|x_i - P(M_i)x_i\| > K$. The following are equivalent: (i) $\lim\|x_i\| = \lim\|x_i - P(M_i)x_i\|$, (ii) $\lim P(M_i)x_i = 0$. If X^* is also UR (i.e. X is uniformly smooth) and, for $y \in X$, $n(y)$ denotes the unique norm-1 linear functional such that $n(y)(y) = \|y\|$, then the following are also equivalent to (i) and (ii): (iii) $\lim\|n(x_i) - n(x_i - P(M_i)(x_i))\| = 0$, (iv) $\lim\|n(x_i) + M_i\| = 0$, (v) $\lim\|n(x_i)| M_i\| = 0$. Several consequences and applications are given including a simple proof of the theorem of R. B. Holmes [Nieuw Arch. Wisk. 14(1966), 106-113] on a uniform equicontinuity of best approximations on bounded subsets of UR spaces.

(Received August 1, 1973.) (Author introduced by Professor Charles L. Byrne.)

*73T-B303. JOAN WICK PELLETIER, York University, Downsview, Ontario, Canada. A categorical approach to the closed graph theorem.

Let $\mathcal{L}$ be the category of locally convex (Hausdorff) topological vector spaces and continuous linear maps, and $\mathfrak{B}$ the full subcategory of barrelled spaces. Two functors $S, T: \mathcal{L} \to \mathfrak{B}$ are considered, where S, T assign to X the space having X as its underlying set and endowed respectively with the strongest locally convex topology and the weakest barrelled topology stronger than X. Proposition. T is the right adjoint of the forgetful functor $\mathfrak{B} \to \mathcal{L}$. Given a linear map $f: B \to X$ with closed graph, $B \in \mathfrak{B}$, the pushout $P_{(B,f)}$ of $\{1: SB \to B,\ Sf: SB \to SX\}$ is formed in $\mathcal{L}$ (the latter map can be justified). The generalized pushout of $\{B \to P_{(B,f)}\}$ taken over all (B, f) has X as its underlying set and is denoted by X_α. Theorem. The following statements are equivalent: (i) X is a target of the closed graph theorem with respect to $\mathfrak{B}$; (2) $1: X_\alpha \to X$ is continuous; (3) $X_\alpha \cong TX$. Propositions relating the above theorem to known results on the closed graph theorem by Pták, Komura, and Adasch are given. (Received September 4, 1973.)

*73T-B304. EVELYN MARIE SILVIA, University of California, Davis, California 95616. A variational method on certain classes of functions of bounded boundary rotation. Preliminary report.

For $k \geqq 2$, let M_k denote the class of real-valued functions $m(t)$ of bounded variation on $[-\pi, \pi]$ which satisfy $\int_{-\pi}^{\pi} dm(t) = 2$, $\int_{-\pi}^{\pi} |dm(t)| \leqq k$. Let α be real and such that $|\alpha| \leqq \pi/2$. We say that $f(z)$ is α-spiral-like of bounded boundary rotation $k\pi$, denoted $f(z) \in S_p(\alpha, k)$, if for some $m(t) \in M_k$, $f(z) = z \exp \int_{-\pi}^{\pi} -\log(1 - ze^{-it})^{e^{i\alpha}\cos\alpha}\, dm(t)$. Since $S_p(\alpha, 2)$ is the class of α-spiral-like functions that was introduced by Špaček [Časopis Pěst. Mat. Fys. 62(1933), 12-19], this definition of $S_p(\alpha, k)$ clearly generalizes that for the class of α-spiral-like functions. Similarly, we say $f(z) \in K_p(\alpha, k)$ if $f'(z) = \exp \int_{-\pi}^{\pi} -\log(1 - ze^{-it})^{e^{i\alpha}\cos\alpha}\, dm(t)$ for some $m(t) \in M_k$. Note that for $k = 2$, $K_p(\alpha, k)$ consists of those functions $f(z)$ for which zf' is α-spiral-like which were defined by Robertson [Michigan Math. J. 12(1965), 385-387]. Using the variational method of G. M. Goluzin ["On a variational method in the theory of analytic functions", Leningrad. Gos. Univ. Učen. Zap. 144 Ser. Mat. Nauk 23(1952), 85-101; English transl., Amer. Math. Soc. Transl. (2) 18(1961), 1-14], we solve certain extremal problems for these classes of functions. (Received September 4, 1973.)

73T-B305. KIRTI K. OBERAI, Queen's University, Kingston, Ontario, Canada. On the Weyl spectrum.

Let T be a continuous linear operator on a Banach space X. Let $\sigma(T)$ and $\pi_{00}(T)$ be respectively the spectrum of T and the isolated points of $\sigma(T)$ which are eigenvalues of finite multiplicity. The Weyl spectrum $\omega(T)$ of T is defined by $\omega(T) = \{\lambda \in C : \lambda I - T$ is not a Fredholm operator of index $0\}$. We show that the mapping $T \to \omega(T)$ is upper semicontinuous while in general it fails to be lower semicontinuous, and we give some sufficient conditions for this mapping to be lower semicontinuous. Again, we say that Weyl's theorem holds for T if $\sigma(T) \sim \pi_{00}(T) = \omega(T)$. We show that Weyl's theorem holds for a spectral operator of finite type (in the sense of Dunford) (for definitions see e.g., N. Dunford and J. Schwartz, "Linear operators", Vol. III, John Wiley, New York, 1971) while it may not hold for a spectral operator in general. (Received September 4, 1973.)

*73T-B306. LEE A. RUBEL and SUBRAMANIAN VENKATESWARAN, University of Illinois, Urbana, Illinois 61801. Uniform approximation by splines of polynomials and of entire functions.

Definition. Let $\{E_i\}$, $i = 1, 2, \ldots$, be a countable class of compact nowhere dense sets such that $\bigcup_1^\infty E_i = E$ is compact. A function g is called an mth order spline of polynomials on $\{E_i\}$ if $g|_{E_i} = g_i|_{E_i}$ is the restriction of a polynomial g_i for each i, and if for all $z \in E_i \cap E_j$, $i \neq j$, we have $g_i^{(k)}(z) = g_j^{(k)}(z)$ for $k = 0, 1, 2, \ldots, m$. If we replace compact sets by closed sets and polynomials by entire functions, we arrive at the definition of a spline of entire functions. Necessary and sufficient conditions on the configuration of the $\{E_i\}$ are obtained, in order that (i) for every $\epsilon > 0$, $f \in C(E)$, and integer $m \geqq 0$, there exists an mth order spline of polynomials g on $\{E_i\}$ such that $\|f - g\|_E < \epsilon$ (the case when E is compact); (ii) for every $\epsilon > 0$, $f \in C(E)$, and integer $m \geqq 0$, there exists an mth order spline of entire functions g on $\{E_i\}$ such that $\|f-g\|_E < \epsilon$ (the case when E is closed). Here, $\|f\|_E = \sup\{|f(z)| : z \in E\}$. Along the way we prove a "Walsh lemma" in which we simultaneously approximate by entire functions and interpolate at an infinite set of points. (Received September 4, 1973.)

*73T-B307. PAUL T. SCHAEFER, State University College, Geneseo, New York 14454. Mappings of positive integers and subspaces of m.

Let N be the set of positive integers and let $\sigma : N \to N$ be a mapping. For each $x \in m$, the Banach space of bounded real sequences, set $Tx = \{x_{\sigma(n)}\}$, where $x = \{x_n\}$. Properties of σ with respect to the closed subspaces c, c_σ, and V_σ of m are investigated, where c denotes the set of convergent sequences, c_σ denotes the set of bounded sequences x such that $T^k x \to \{L, L, L, \ldots\}$ for some real number L, and V_σ is the set of $x \in m$ for which $(x+Tx+\ldots+T^k x)/(k+1) \to \{L, L, L, \ldots\}$ for some real L. When $\sigma(n) = n + 1$, it is well known that $c_\sigma = c$ and V_σ is the set of all almost convergent sequences. Condition A. $\sigma(n) \to +\infty$ as $n \to +\infty$. Condition B. $\sigma^k(n) \to +\infty$ uniformly in n as $k \to +\infty$. Condition C. For each nonempty $S \subset N$, $S \subset \sigma(S)$ is false. Condition D. $\sigma^k(n) \neq n$ for all n and k. Representative results. (1) σ satisfies A iff $\liminf x \leqq \liminf Tx \leqq \limsup Tx \leqq \limsup x$ for all $x \in m$ iff $Tc \subset c$ and $\lim Tx = \lim x$ for all $x \in c$. (2) σ satisfies B iff $c \subset c_\sigma$ and $\lim x = L$ for all $x \in c$. (3) B implies C. (4) C implies D. (5) σ satisfies D iff $c \subset V_\sigma$ and $\lim x = L$. (Received September 10, 1973.)

*73T-B308. JOHN C. MORGAN II, Syracuse University, Syracuse, New York 13210. On translation invariant families of sets.

In an earlier abstract (Abstract 72T-B95, these *Notices* 19(1972), A-436) it has been shown how several analogies between Baire category and Lebesgue measure can be unified under an abstract theory of Baire category. Within this framework the author, in the present note, unifies additional analogies which involve translation invariance of measure and category. (Received September 10, 1973.) (Author introduced by Professor Daniel Waterman.)

73T-B309. WILLIAM R. ZAME, State University of New York at Buffalo, Amherst, New York 14226. Holomorphic convexity of compact sets and cohomology vanishing theorems. Preliminary report.

Let X be a closed subvariety of the open set $\Omega \subset C^n$ and let K be a compact holomorphically convex subset of X; i.e., every nonzero homomorphism $\Gamma(K, \mathcal{O}_X) \to \mathbb{C}$ arises from evaluation at a point of K. Theorem 1. If $\mathfrak{F}$ is a coherent sheaf of $\mathcal{O}_X$-modules on K then: (A) ${}_x\mathfrak{F}$ is generated by $\Gamma(K, \mathfrak{F})$ for each $x \in K$; (B) $H^q(K, \mathfrak{F}) = 0$ for $q \geqq 1$. Property (B) characterizes holomorphically convex subsets of a variety in the following sense. Theorem 2. Let Y be an analytic space and L a compact subset of Y such that $H^1(L, \mathcal{A}) = 0$ for each coherent analytic subsheaf $\mathcal{A}$ of $\mathcal{O}_Y$. Then there are an open neighborhood Y' of L, an open set $U \subset C^k$, a closed subvariety V of U and a biholomorphism $\varphi : Y' \to V$ such that $\varphi(L)$ is holomorphically convex in V. (Received September 10, 1973.)

73T-B310. DANIEL MALCOLM OBERLIN, University of Maryland, College Park, Maryland 20742. Restriction to the diagonal in $H^1(U^n)$. Preliminary report.

Let U denote the open unit disc and let B^q $(0 < q < 1)$ denote the space of functions g analytic in U for which $\int_0^1 (1-r)^{(1/q)-2} M_1(r,g)\,dr$ is finite. For $f \in H^1(U^n)$, define $f_D(\lambda)$ to be $f(\lambda, \lambda, \ldots)$ for $\lambda \in U$. Then Rudin ["Function theory in polydiscs", Benjamin, New York, 1969, p. 69] has shown that $f \to f_D$ maps $H^1(U^2)$ into (a space equivalent to) $B^{1/2}$ and has remarked that it is not known whether this map is actually onto $B^{1/2}$. We prove the following Theorem. The map $f \to f_D$ takes $H^1(U^n)$ onto $B^{1/n}$. Related partial results are also obtained for other spaces $H^p(U^n)$. (Received September 10, 1973.)

*73T-B311. IH-CHING HSU, Northern Illinois University, DeKalb, Illinois 60115. On some functional inequalities.

The author investigates the following functional inequalities which correspond to some well-known functional equations: (1) $f(x+y) \geqq f(x)g(y) + f(y)g(x)$ (corresponding to the functional equation arising from the addition formula for the sine function); (2) $f(x+y) + f(x-y) \leqq 2f(x)f(y)$ (corresponding to D'Alembert's equation); (3) $f(x+y) \geqq f(x)g(y)$ (corresponding to the generalized Cauchy's equation). In these functional inequalities g occurs as a given function and f occurs as an unknown function. In solving for f, the basic technique used in this note is to derive differential equations or differential inequalities from functional inequalities. This derivation becomes possible after some local properties around 0 have been imposed on g or f. The Banach fixed-point theorem is used in proving the following Theorem. Suppose that f is a solution to the functional inequality $f(x+y) + f(x-y) \leqq 2f(x)f(y)$. Suppose further that $f(0) = 1$, $f'(0) = 0$ and, on a neighborhood of 0, $f''(x)$ exists continuously. Then f is even, and $f(x) \leqq \cosh\sqrt{K}x$ for all real numbers x, where $K = |f''(0)|$. Some of the other results in this paper generalize an earlier work of J. E. Wetzel ("On the functional inequality $f(x+y) \geqq f(x)f(y)$", Amer. Math. Monthly 74(1967), 1065-1068). (Received September 10, 1973.)

73T-B312. T. K. MUKHERJEE, University of Arkansas, Fayetteville, Arkansas 72701. Hereditary completeness and quasi-reflexivity.

In this note we show that the countable direct sum of quasi-reflexive Banach spaces is hereditarily complete, and hence the countable inductive hull of quasi-reflexive Banach spaces is always complete. This partially answers a question of Van Dulst in "A note on B- and B_r-completeness", Math. Ann. 197, 197-202. (Received September 4, 1973.)

73T-B313. ROLANDO B. CHUAQUI, Universidad Catȯlica de Chile, Santiago, Chile and University of California, Berkeley, California 94720. The existence of an invariant countably additive measure and paradoxical decompositions.

In "Cardinal algebras" (Oxford Univ. Press, New York, 1949), Tarski proved that a necessary and sufficient condition for the existence of a finitely additive measure defined on the power set of A, normed by a set

$U \subseteq A$ and invariant under a group G of permutations of A is: There are no disjoint sets U' and U'' such that $U = U' \cup U''$ and $U \simeq_G U' \simeq_G U''$, where $\simeq_G$ stands for equivalence under finite decomposition under G (see 16.12 in Tarski). In the author's paper "Cardinal algebras and measures" (Trans. Amer. Math. Soc. 142(1969), 61-79) it was conjectured that the same condition was necessary and sufficient for the existence of a countably additive measure when $\simeq_G$ is taken to be equivalence under countable decompositions. Lang proved in his Diplomarbeit "Kardinalalgebren in der Masstheorie" (Universität Heidelberg, 1970) that a similar conjecture was false assuming GCH. A counterexample for the original conjecture can be obtained without GCH, taking for A any set of power $\aleph_1$ and G the group of permutations of A that are different from the identity in at most a countable number of points. If the conjecture were true, $\aleph_1$ would be a measurable cardinal. (Received September 13, 1973.)

73T-B314. EDWARD M. LANDESMAN, University of California, Santa Cruz, California 95060 and ALAN C. LAZER, University of Cincinnati, Cincinnati, Ohio 45221. A Rayleigh-Ritz type algorithm for a class of "Max.-Min." problems. Preliminary report.

Let D be a bounded domain in R^n. Denote the boundary of D by ∂D. Consider the boundary-value problem $*$ $\Delta u + g(x,u(x)) = 0$ in D, $u = 0$ on ∂D. Suppose that there exist constants $\alpha, \beta > 0$ such that $\lambda_N < \alpha \leqq g_u(x,u(x)) \leqq \beta < \lambda_{N+1}$, where λ_N and λ_{N+1} are the Nth and (N+1)st eigenvalues of the problem $\Delta\varphi_n + \lambda_n\varphi_n = 0$ in D, $\varphi_n = 0$ on ∂D. There exists a weak solution of $*$ [see E. M. Landesman and A. C. Lazer, Pacific J. Math. 33(1970), 311-328]. This solution can be characterized as a saddle point of a "natural functional" corresponding to the problem. An algorithm similar to the well known Rayleigh-Ritz method for determining solutions to minimum problems is used to obtain a sequence which converges to the solution of $*$. In the linear case when $g(x,u(x)) = p(x)u - h(x)$, the procedure is entirely constructive, while for more general functions g, a solvability result for a finite system of nonlinear equations in finitely many variables is used. (Received September 20, 1973.)

*73T-B315. PRATIBHA GHATAGE, University of Toronto, Toronto, Ontario, Canada. Generalized algebraic operators. Preliminary report.

Let T be a polynomially bounded operator on a Hilbert space $\mathscr{H}$ and let A be the algebra of complex-valued functions which are continuous on the closed disk Δ and analytic in int Δ. Let φ denote the norm-continuous homomorphism from A to $B(\mathscr{H})$ which extends $p \to p(T)$, where p denotes a polynomial. Definition 1. T is said to be in class D_0 if $\ker\varphi \neq (0)$. In this case let F denote the g.c.d. of inner parts of nonzero elements of $\ker\varphi$. F is called the minimum function of T. Theorem 1. If $F = 1$ then T is similar to a unitary. Let f be the outer function in A such that fF generates $\ker\varphi$. Let $\ker f(t) = \mathscr{H}_0$ and closure $R(f(T)) = \mathscr{H}_1$. Theorem 2. If $x \in \mathscr{H}_0$ and $y \in \mathscr{H}$ then there exists a unique singular measure $\nu[x,y]$ on $\partial\Delta$ such that $(p(T)x,y) = \int p(Z)\, d\nu[x,y]$ for all polynomials p. If $x \in \mathscr{H}_1$ and $y \in \mathscr{H}$ then there exists an absolutely continuous measure $\mu[x,y]$ on $\partial\Delta$ such that $(p(T)x,y) = \int p(Z)\, d\mu[x,y]$. Moreover $\mathscr{H}_0 \dotplus \mathscr{H}_1 = \mathscr{H}$. Definition 2. If $\mathscr{H} = \mathscr{H}_0$ $(\mathscr{H}_1)$ then T is said to be singular (absolutely continuous). Theorem 3. Suppose T is absolutely continuous and satisfies an $\mathscr{H}^\infty$ function. Then $T^n \xrightarrow{st} 0$ and $T^{*n} \xrightarrow{st} 0$. Corollary (Nagy-Foiaş). $c_0 \subseteq c_{00}$. (Received September 10, 1973.)

*73T-B316. RAYMOND JOHNSON, University of Maryland, College Park, Maryland 20742. Convoluteurs of H^p spaces. Preliminary report.

Nonperiodic analogues and generalizations of some results of Duren and Shields (Pacific J. Math. 32 (1970), 69-78) are given, highlighting the key role played by the homogeneous Besov spaces and their images under the Fourier transform. In particular, the convoluteurs of H^p into H^q are characterized for $0 < p < 1 \leqq q < \infty$. (Received September 21, 1973.)

*73T-B317. HERBERT HALPERN, University of Cincinnati, Cincinnati, Ohio 45221. Quasi-equivalence classes of normal representations for a separable C^*-algebra.

A representation λ of a separable C^*-algebra A is said to be normal if the von Neumann algebra $\lambda(A)''$ generated by $\lambda(A)$ is a semifinite factor and the intersection of $\lambda(A)$ with the ideal of elements of $\lambda(A)''$ of finite trace generates $\lambda(A)''$. It is shown that the quasi-equivalence classes of normal representations of A form a standard Borel space in the quasi-dual taken with the Mackey-Borel structure, and that there is a Borel subset S of the space of factor states taken with the w*-topology such that the set of canonical representations of elements in S meets each quasi-equivalence class of normal representations in exactly one point. (Received September 24, 1973.) (Author introduced by Professor Charles W. Groetsch.)

*73T-B318. ATHANASSIOS G. KARTSATOS, University of South Florida, Tampa, Florida 33620. A stability problem of the solution of a boundary value problem on an infinite interval. Preliminary report.

Consider the boundary value problem: (I) $x' + A(t)x = f(t)$, (II) $Tx = r$, where $A(t)$ is a real $n \times n$ matrix defined and continuous on $R_+ = [0,\infty)$, $f \in C[R_+, R^n]$, $R = (-\infty, \infty)$, r is a fixed vector in R^n, and T is a bounded linear operator on the space C_ℓ of all functions $f \in C[R_+, R^n]$ such that $\lim_{t\to\infty} f(t)$ exists and is finite. Our purpose here is to establish conditions, under which the existence of a unique solution to the problem ((I), (II)) implies the same fact for the problem ((III), (II)), where (III) $x' + B(t)x = f(t)$, provided that the matrix $B(t)$ is "sufficiently close" to the matrix $A(t)$. (Received September 24, 1973.)

73T-B319. VASANT A. UBHAYA, Department of Applied Mathematics and Computer Science, Washington University, St. Louis, Missouri 63130. Infima of integrals involving mollifier functions.

A real valued, nonnegative, infinitely differentiable function φ defined on the real line is called a Friedrichs mollifier function if it has support in $(0,1)$ and $\int_0^1 \varphi(x)\,dx = 1$. Let Φ be the class of all the mollifier functions. We determine the values of $\inf_{\varphi\in\Phi} \int_0^1 |\varphi^{(k)}(x)|\,dx$, where $\varphi^{(k)}$ denotes the kth derivative of φ. This has applications to the results concerning the problem of monotone polynomial approximation announced by this author in Abstract 706-41-2, these *Notices* 20(1973), A-525. Theorem. $2k \inf_{\varphi\in\Phi} \|\varphi^{(k-1)}\| = \inf_{\varphi\in\Phi} \int_0^1 |\varphi^{(k)}(x)|\,dx = k!2^{(2k-1)}$, for $k = 1, 2, 3$ where $\|\varphi^{(k-1)}\| = \max_{x\in[0,1]} |\varphi^{(k-1)}(x)|$, $\varphi^{(0)} = \varphi$. The first equality of the theorem is not true for all $k \geqq 4$. We state the result for the case $k = 4$ only. Theorem. $\inf_{\varphi\in\Phi} \int_0^1 |\varphi^{(4)}(x)|\,dx = 96/p(y_1^*, y_2^*, y_3^*)$ and $\|\varphi^{(3)}\| = 24 \max(y_1^*, y_2^*) ((y_1^* + y_2^*) p(y_1^*, y_2^*, y_3^*))^{-1}$ where $p(y_1, y_2, y_3) = y_1^4 + 4y_1^3y_2 + 6y_1^2y_2^2 + 4y_1^3y_3 + 12y_1^2y_2y_3$ and (y_1^*, y_2^*, y_3^*) is the unique solution of $p(y_1, y_2, y_3) = 3y_2^4 + 12y_2^3y_3 + 5y_3^4$, $y_1y_2 + y_2^2 = y_3^2$, $y_1 + y_2 + y_3 = \frac{1}{2}$ with $y_i > 0$, $i = 1, 2, 3$. (Received September 24, 1973.)

Applied Mathematics

*73T-C61. OTOMAR HAJEK, Case Western Reserve University, Cleveland, Ohio 44106. Nondifferentiability of the minimal time function.

Consider any control system $\dot{x} = Ax - u$, $u(t) \in U$, in n-space; A is a real n-square matrix, and U is compact and contains 0. For points x which can be steered to 0 in finite time, let $T(x)$ be the least time. Theorem 1. T is not differentiable at 0. Theorem 2. If U has an n-dimensional corner (e.g. dim $U < n$), then T is not differentiable at any point of an analytic curve issuing from 0. Theorem 3. If U is not a neighborhood of 0, then the same conclusion as in Theorem 2; furthermore, the exceptional curve intersects every isochrone $\{x : T(x) = t\}$, $t \geqq 0$. (Received August 29, 1973.)

73T-C62. S. P. BHATTACHARYYA and S. K. JAIN, Indian Institute of Technology, Powai, Bombay 400076, India. Magnetohydrodynamic convection in an infinite channel.

The stability of an electrically conducting hot fluid heated from below in an infinite channel in the presence of a uniform magnetic field across the channel is discussed. It is found that the stabilizing influence of the magnetic field is more prominent for lower values of the aspect ratio a (the ratio of the width to the height) of the channel, and, as the ratio tends to infinity, the critical value of the Rayleigh number R tends to a limit which is the same for all the field strengths. By considering the disturbance with the lowest mode, it is observed that finite rolls aligned perpendicular to the side walls appear only if the magnetic field strength is greater than a certain definite value and the width of the rolls decreases with increasing field strength. (Received September 4, 1973.) (Authors introduced by Professor Padam C. Jain.)

73T-C63. MARVIN G. MOORE, Bradley University, Peoria, Illinois 61606. General relativity without tensors in a central field.

In the general theory of relativity, it is found possible completely to avoid tensor analysis by considering only those gravitational fields which have spherical symmetry. The principle of equivalence, illustrated by Einstein's elevator, is used to obtain Schwarzschild's equation, on which the three well-known tests of the general theory are usually based. The derivation is guided, as with Einstein, by Poisson's (Laplace's, in empty space) equation, which here can be solved by simple calculus. (Received September 17, 1973.)

Geometry

73T-D24. ROY OLSON, University of Hawaii, Honolulu, Hawaii 96822. On the six conics problem. Preliminary report.

Theorem. There do not exist six points in the plane and six distinct congruent hyperbolas such that each hyperbola contains exactly five of the points. J. Seidel and J. van Wollenhoven (Elem. Math. 17(1962), 85) have obtained a similar result for ellipses. After disposing of the other possibilities, the two above results yield the following: There do not exist six points in the plane and six distinct congruent conic sections such that each conic contains exactly five of the points. That solves a problem of L. M. Kelly (Math. Mag. 19(1944), 123-130) and J. J. Schäffer (Elem. Math. 17(1962), 65), which was presented by V. L. Klee in his film, "Shapes of the future—some unvolved problems in geometry" (1971). (Received September 17, 1973.)

Logic and Foundations

73T-E112. CARL G. JOCKUSCH, JR., LEE A. RUBEL and GAISI TAKEUTI, University of Illinois, Urbana, Illinois 61801. First order topology. II. Preliminary report.

Let $\mathcal{T}$ be the class of all lattices isomorphic to L(X) for some T_1 space X. Then the class $\mathcal{T}$ is not closed under elementary equivalence (e.e.) of lattices, and various Skolem-Löwenheim theorems fail for $\mathcal{T}$; i.e., it is not the case that every topological space is e.e. to a countable topological space or to a space with a countable basis. Furthermore $\mathcal{T}$ is not compact, i.e., there is a set Σ of sentences such that Σ has no model in $\mathcal{T}$ but every finite subset of Σ does. (The analogue of this result for closure algebras was obtained by Andrew Adler, and our proof uses his ideas and the observation that every space e.e. to the unit interval is compact.) Grzegorczyk has shown that the theory of the lattice of closed subsets of the Euclidean plane is undecidable. We sharpen this result to show that this theory is in fact recursively isomorphic to second order arithmetic. Finally we show that if X is a space having a countable basis of clopen sets, then L(X) shares some properties of the lattice of r.e. sets; e.g., every open but not closed set is the disjoint union of two such. These results fail if the assumption of a countable basis is removed. (Received August 24, 1973.)

*73T-E113. STEVEN K. THOMASON, Simon Fraser University, Burnaby 2, British Columbia, Canada. Reduction of second-order logic to modal logic.

A semantic interpretation is constructed of the classical monadic second-order logic S of a binary relation in propositional modal logic M (with the Kripke relational semantics). More precisely, there is a formula δ of M and an effective map ψ of formulas of S to formulas of M, such that $\Gamma \models \alpha$ in S iff $\{\delta\} \cup \{\psi(\gamma) | \gamma \in \Gamma\} \models \psi(\alpha)$ in M. ($\Delta \models \beta$ means β is valid in every structure (W, R) in which every $\delta \in \Delta$ is valid.) Roughly speaking, δ says of a structure (W, R) that an arbitrary (W', R') and the notion of validity of formulas of S in (W', R') are definable in (W, R) by certain formulas of M, and $\psi(\alpha)$ is the translation of "α is valid in (W', R')". The conclusion is that propositional modal logic is as powerful and comprehensive as it could possibly be, subject to the obvious condition that M is a fragment of S. From this viewpoint, the many completeness theorems for particular systems of modal logic appear to be not special cases of some unknown general principle but remarkable exceptions to the general principle. (Received September 6, 1973.)

*73T-E114. MIRIAM A. LIPSCHUTZ-YEVICK, Rutgers University, New Brunswick, New Jersey 08093 and Princeton University, Princeton, New Jersey 08540. Holographic deconvolution and decrosscorrelation. Preliminary report.

With the notation of previous abstracts (June 1973) we now define a multiplicative inverse $I_{A,R}$ of the object A to be an object such that one of the terms of $\mathfrak{F}\{A, I_{A,R}\}$ yields R_{ab}. This latter is the object with $R_{ab}(x,y) = R(x-a, y-b)$. We have $I_{A,R} = H_{A,R} \cdot |H_A|^{-2}$. The object $|H_A|^{-2}$ can be realized with coherent optical processes. Two objects which differ only by a linear translation have the same multiplicative inverse for every R. The operations $\mathfrak{C}(A,B)^{-A} = \mathfrak{F}\{C(A,B), I_{A,R}\}$ and $\mathfrak{T}(A,B)^{-A} = \mathfrak{F}\{T(A,B), I_{A,R}\}$, which yield C(R, B) and T(R, B), respectively, are the inverse operations of filtering $\mathfrak{F}\{A, H(B,R)\}$. When R is a "true reference" (i.e. a point source or a plane wave) these operations yield RB and R*B respectively, i.e. given an object which is a crosscorrelation of a known object A with an unknown B, we retrieve B. (Received August 29, 1973.)

*73T-E115. V. FREDERICK RICKEY, Bowling Green State University, Bowling Green, Ohio 43403. The one variable implicational calculus.

The axiomatization given by H. W. Johnson and R. Price in "Axioms for the implicational calculus with one variable" (Theoria 30(1964), 1-4), is shown to be inadequate. A correct presentation of the one variable implicational calculus based on a single (rejected) axiom and four rules of inference is given. This is akin to the computable protothetics of Lesniewski. This presentation cannot be much improved on, for we show that the one variable implicational calculus cannot be finitely axiomatized using the rules of substitution and detachment. This provides an elementary example of a nonaxiomatizable propositional calculus. (Received September 10, 1973.)

73T-E116. C. WARD HENSON III, Duke University, Durham, North Carolina 27706. The isomorphism property in nonstandard analysis. Preliminary report.

Definition. An enlargement *M of M has the κ-isomorphism property if for each first order language L with $< \kappa$ symbols, if $\mathcal{A}$ and $\mathcal{B}$ are elementarily equivalent structures for L whose domains, relations and functions are all internal (relative to *M), then $\mathcal{A}$ and $\mathcal{B}$ are isomorphic. Theorem 1. For each κ and M, there is an enlargement *M of M which has the κ-isomorphism property. Theorem 2. If *M has the κ-isomorphism property, then *M is κ-saturated. Theorem 3. Suppose *M has the $\aleph_0$-isomorphism property. If A is any internal set (which is not actually finite) and ω is an infinite integer, then (a) there is a bijection f of A onto $\{1,2,\dots,\omega\}$ such that for any $C \subset A$, C is internal iff $\{f(x) : x \in C\}$ is internal; (b) there is a bijection g of A onto $\{1,2,\dots,\omega\}$ such that for any $C \subset A$, one of C or $A \sim C$ is *-finite iff $\{g(x) : x \in C\}$ is internal. Theorem 4. Suppose *M has the $\aleph_0$-isomorphism property. For any normed space X in M there is a separable closed subspace Y of X such that the nonstandard hulls of X and Y are isometrically isomorphic. Example. Let m(c) be the space of bounded (convergent) scalar-valued sequences with the supremum norm. If *M has the $\aleph_0$-isomorphism property, then the nonstandard hulls of m and c are isometrically isomorphic. Moreover, they are also isometrically isomorphic to the subspace of $\hat{m}$ defined by elements of *m whose support is contained in $\{1,2,\dots,\omega\}$ (ω an infinite integer). (Received September 10, 1973.)

73T-E117. LEO A. HARRINGTON and ALEXANDER S. KECHRIS, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Spector-Gandy theorems for classes of inductively defined sets. Preliminary report.

Let ω be the set of natural numbers and $\mathcal{P}(\omega) = \{s : s \subseteq \omega\}$ its power set. Given an operator $\Theta : \mathcal{P}(\omega) \to \mathcal{P}(\omega)$ define Θ^ξ for ξ an ordinal by $\Theta^0 = \emptyset$, $\Theta^\xi = \bigcup_{\eta<\xi} \Theta(\Theta^\eta)$ and put $\Theta^\infty = \bigcup_\xi \Theta^\xi$. If Γ is a collection of relations on ω and $\mathcal{P}(\omega)$, Θ is called a Γ-operator if the relation $H(n,s) \Leftrightarrow n \in \Theta(s)$ belongs to Γ. Given Γ, let $\Gamma^\infty = \{s \subseteq \omega : s$ is 1-1 reducible to some Θ^∞, where Θ is a Γ-operator$\}$. Let also $\Delta^\infty = \{s \subseteq \omega : s \in \Gamma^\infty \ \&\ \omega - s \in \Gamma^\infty\}$. Γ has the Spector-Gandy property if for every $A \in \Gamma^\infty$ there is a $B \in \Gamma$ so that (*) $\forall n(n \in A \Leftrightarrow \exists s \in \Delta^\infty B(n,s))$. Γ has the dual Spector-Gandy property if for all $A \in \Gamma^\infty$ there is a $B \in \check{\Gamma}$ (= collection of the negations of the relations of Γ) satisfying (*). Theorem. Let $n \geqq 1$ below and assume projective determinacy if $n \geqq 2$. All Σ^1_n with n odd $\geqq 1$, all Π^1_n with n even and also Π^1_1 have the Spector-Gandy property. All Π^1_n with n odd > 1, all Σ^1_n with n even and also Σ^1_1 have the dual Spector-Gandy property. (Received September 10, 1973.)

*73T-E118. D. SARACINO and VOLKER B. WEISPFENNING, Yale University, New Haven, Connecticut 06520. Commutative regular rings without prime model extensions. Preliminary report.

It is known that the theory K of commutative regular rings with identity has a model completion K' (See L. Lipshitz and D. Saracino, "The model companion of the theory of commutative rings without nilpotent elements", Proc. Amer. Math. Soc. 38(1973), 381-388). Theorem. There exists a countable commutative regular ring R such that the isolated points are not dense in the Morley space S(R) of R. Corollary 1. R has no prime extension to a model of K'. Corollary 2. K' is not quasi-totally transcendental. Remarks. R can be chosen to have a minimal extension to a model of K'. Also it is easy to see that K' is $\varkappa$-unstable for all infinite cardinals $\varkappa$. (Received September 12, 1973.)

*73T-E119. GEORGE E. WEAVER, Department of Philosophy, Bryn Mawr College, Bryn Mawr, Pennsylvania 19010. The completeness of some systems of sentential logic.

For each $k \geqq 2$, the logic $\mathcal{L}_k$ is described. For $k = 2$, $\mathcal{L}_k$ is classical two-valued logic; and for $k = 3$, $\mathcal{L}_k$ is the many-valued logic studied by Łukasiewicz but unpublished. Let $\mathbb{L}$ be a sentential language over the set D of sentence letters and connectives C and N. For A in $\mathbb{L}$, let $N^nA = N\dots_n\dots N$. For p, q, r in D, let [2k] be $CCCpqrC_1Np\dots C_{k-1}N^{k-1}pr$; [3k] be $CC_1Np\dots C_{k-1}N^{k-1}prCCpqr$ and $S_k = \{CN^iCpqCN^ipN^iq,\ CCN^ipN^iqN^iCpq : 2 \leqq i \leqq k-1\}$. Let $\mathcal{L}_k$ be the logic with axioms $\{CCCpqrCpr, [2_k], [3_k]\} \cup S_k$, and substitution and detachment as rules of inference. T_k denotes the theorems of $\mathcal{L}_k$. M_k is the matrix $\langle A_k, B_k, C_k, N_k\rangle$ where (1) $A_k = \{1,\dots,k\}$; (2) $B_k = \{1,\dots,k-1\}$ (the designated values); (3) for all x, y in A_k, $C_{k(x,x)} = 1$ and $C_k(x,y) = y$ if $x \neq y$; (4) for all x in A_k, $N_k(x) = x - 1$ for $x \neq 1$ and $N_k(1) = k$. Theorem 1. For all k: (1) $\mathcal{L}_k$ is sound and complete relative to M_k;

(2) $\mathcal{L}_k$ is Post-complete; (3) classical C-logic is a subset of T_k; (4) for all $i > k$, $T_i \subsetneq T_k$. Theorem 2. For all k, M any matrix, if $\mathcal{L}_k$ is sound relative to M, then $\mathcal{L}_k$ is complete relative to M. (Received September 17, 1973.)

*73T-E120. ROHIT J. PARIKH, Boston University, Boston, Massachusetts 02215. $\aleph_0$-categorical theories.

Theorem. There exists a decidable, complete, $\aleph_0$-categorical, first order theory T such that the language of T is essentially infinite. More specifically, the language L(T) consists of infinitely many predicate symbols P_i (P_n is n-ary) such that P_i is definable from $P_{j_1}, \ldots, P_{j_k}$ iff $i \leqq \max(j_1, \ldots, j_k)$. (Received September 24, 1973.)

*73T-E121. HELMUT SCHWICHTENBERG, Westfälische Wilhelms-Universität, 44 Münster, Federal Republic of Germany. Functions definable by typed λ-terms.

In the typed λ-calculus one can represent natural numbers by terms $\lambda\alpha\alpha^n$ (α variable of type 00, $\alpha^n := \lambda x \cdot \alpha(\alpha(\ldots(\alpha x)..))$). Any closed term t of type $(\nu\ldots\nu)\nu$ with $\nu := (00)00$ then defines a total recursive function. Hindley, Lercher, and Seldin ["Introduction to combinatory logic", 1972, p. 72] ask which functions can be defined this way. Schütte (unpublished) gave the following examples: $n + m$ is defined by $\lambda FG\alpha \cdot (F\alpha) \circ (G\alpha)$, $n \cdot m$ by $\lambda FG \cdot F \circ G$, k (constant function) by $\lambda F\alpha \cdot \alpha^k$, and $d(n, m, i) := n$ if $i = 0$, and $= m$ if $i \neq 0$, by $\lambda FGH\alpha x \cdot H(\lambda y \cdot G\alpha x)(F\alpha x)$. The set of definable functions is clearly closed against substitution. Hence all polynomials are definable, and more generally all functions which can be defined from polynomials by cases, depending on some arguments being 0 or $\neq 0$. In the 2-place case these are the functions $f(n,m) = P_1(n,m)$ if $n \neq 0$ and $m \neq 0$, $= P_2(n)$ if $n \neq 0$ and $m = 0$, $= P_3(m)$ if $n = 0$ and $m \neq 0$, and $= k$ if $n = m = 0$, with polynomials P_i. It is shown that these are all the definable functions. (Received September 24, 1973.)

Statistics and Probability

73T-F21. THOMAS H. SAVITS, University of Pittsburgh, Pittsburgh, Pennsylvania 15260. Space-time harmonic functions applied to age-dependent processes. II. Preliminary report.

We consider an age-dependent model X which allows the parent to continue living with probability $\alpha(x)$ after it gives birth at age x. Let G be its lifetime distribution and $\pi(x,s) = \sum_{k=0}^{\infty} p_k(x)s^k$ its age-dependent generating function. We assume that G is continuous and set $T = \inf\{t \geqq 0 : G(t) = 1\}$. Define $\gamma(x) = \int_{(0,x]} \alpha(y)[1 - G(y)]^{-1} dG(y)$ and $m = \int_{(0,T]} m(y)e^{\gamma(y)}\, dG(y)$ where $m(y) = \sum_{k=0}^{\infty} kp_k(y)$. Then modulo some technical assumptions we prove the following. Theorem 1. Extinction occurs with probability one iff $m \leqq 1$. In the case where $\infty > m > 1$, define λ to be the unique (positive) root of $\int_{(0,T]} m(y)e^{-\lambda y}e^{\gamma(y)}dG(y) = 1$, and set $\varphi(x) = e^{\lambda x}e^{-\gamma(x)}[1 - G(x)]^{-1} \cdot \int_{(x,T]} m(y)e^{-\lambda y}e^{\gamma(y)}\, dG(y)$ on $S = [0,T)$. Theorem 2. $h(x,t) = e^{-\lambda t}\varphi(x)$ is a $\check{}$-space-time harmonic function of the process X. Theorem 3. The nonnegative martingale $W_t = e^{-\lambda t}\check{\varphi}(X_t)$ converges w.p.1. to a random variable W; moreover, W is nontrivial if $\int_{(0,T]} \pi''(y,1)\, dG(y) < \infty$. Let Z_t denote the number of particles at time t. Theorem 4. If $0 < \alpha \leqq \varphi \leqq \beta < \infty$ for some constants α, β, then $\beta^{-1}W \leqq \underline{\lim}_{t\to\infty} e^{-\lambda t}Z_t \leqq \overline{\lim}_{t\to\infty} e^{-\lambda t}Z_t \leqq \alpha^{-1}W$ w.p.1. (Received September 10, 1973.)

Topology

73T-G145. ROBERT R. KALLMAN, University of Florida, Gainesville, Florida 32601. Two theorems on CCR groups.

In what follows, all groups are locally compact and have a countable basis for their topologies. Theorem 1. Let G be a CCR group. Then any open subgroup of G is also a CCR group. Theorem 2. Let G be a group and H a closed subgroup such that G/H is finite. Then G is a CCR group if and only if H is a CCR group. (Received July 26, 1973.) (Author introduced by Professor Gian-Carlo Rota.)

73T-G146. LUDVIK JANOS, University of Newcastle, Newcastle, New South Wales, Australia 2308. Topologization of sets which are mapped into themselves.

Let X be an abstract set and $f: X \to X$ a mapping of X into itself. It is sometimes of interest to ask whether there exists a topology on X having certain prescribed properties and rendering f continuous. Theorem 1. If the cardinality $|X|$ of X does not exceed continuum there is always a separable metric topology on X with respect to which f is continuous. Theorem 2. If, in addition, f is such that all iterations f^n have a unique fixed point, then for every $c \in (0,1)$ there is a metric ρ on X such that (X,ρ) is separable and f is a contraction with Lipschitz constant c. Remark. The construction of ρ is a modification of that given by C. Bessaga and depends on the weak (countable) axiom of choice. It is an open question whether ρ can be claimed complete as it is by the Bessaga construction. (Received June 8, 1973.)

73T-G147. H. E. WHITE, JR., 251 North Blackburn Road, Route #5, Athens, Ohio 45701. An example involving Baire spaces.

Suppose $2^{\aleph_0} = \aleph_1$. Using a modification of an argument of J. C. Oxtoby ("Cartesian products of Baire spaces", Fund. Math. 49(1961), 157-166), it can be shown that there is a dense subspace Y of $(R, \mathcal{T})$, where $\mathcal{T}$ is the density topology on the real line R, such that (1) every subspace of Y is a Lindelöf Baire space, (2) Y is a homogeneous space, (3) $Y \times Y$ is of the first category. (Received July 13, 1973.)

*73T-G148. JAMES M. McPHERSON, Australian National University, P. O. Box 4, Canberra, A.C.T. 2600, Australia. The nullity of a tame knot in a compact 3-manifold.

Let G be a group; use free differential calculus to associate with G a matrix with entries in $Z(B_G)$, where B_G is the (Betti) nullity $\nu(G)$ of G. If X is a topological space use $\nu(X)$ to denote the nullity of $\pi_1(X)$. Let s(X) denote the minimum number of generators of $\pi_1(X)$; in the situation of van Kamphis' theorem, we have $\nu(X \cup Y) \geqq \nu(X) + \nu(Y) - s(X \cap Y)$. Let M be a compact 3-manifold and n(M) the maximum value that $\nu(M-k)$ can take if k is a tame knot in M. Theorem 1 below follows directly from free differential calculus, and this together with Theorem 2 (whose proof is a simple application of the inequality above, with $X = \{M - \text{a regular neighbourhood of } k\}$) answers some of the questions raised by me in "The nullity of a wild knot in a compact 3-manifold" (J. Austral. Math. Soc. (to appear)). Theorem 1. $\nu(G) \leqq \text{rank } B_G = p_1(G)$, hence $p_1(G) = 0 \Rightarrow \nu(G) = 0$, and $p_1(G) > 0 \Rightarrow 1 \leqq \nu(G) \leqq p_1(G)$. Theorem 2. $n(M) = 1 + \nu(M)$; $v(M-k) = n(M)$ if k lies in an open 3-cell in M. (Received August 31, 1973.)

73T-G149. JACQUELINE DEWAR, University of Southern California, Los Angeles, California 90007. A characterization theorem for set valued coincidences. Preliminary report.

Let (X,d) be a metric space, let $A \subset X$, and R_0 = the set of nonnegative real numbers. A function $f: X \to R_0$ is said to be positive definite mod A iff for every $\epsilon > 0$ there is a $u > 0$ so that $f(x) < u$ implies $d(x,A) < \epsilon$. Let Y be any topological space. Two set valued mappings S and $T: X \to$ (the set of subsets of Y) are said to have a coincidence x if $Sx \cap Tx \neq \emptyset$. Theorem 1. For S and T to have a coincidence it is sufficient that there exist a l.s.c. function $f: X \to R_0$ satisfying (1) $f(x) = 0$ implies $Sx \cap Tx \neq \emptyset$; (2) $\inf f(x) = 0$; (3) f is positive definite mod a compact set $A \subset X$. Moreover if Y is metrizable and S (or T) is u.s.c. the condition is necessary. Coincidence analogs of well-known fixed point theorems for distance diminishing mappings are proven via this result, and they reduce to the classical results in the point valued case. (Received September 4, 1973.)

*73T-G150. ADIL G. NAOUM, College of Science, University of Baghdad, Baghdad, Iraq. A note of free actions of S^1 on homotopy spheres.

Theorem. For each $k \geqq 1$, there exists a homotopy sphere Σ of dimension $8k+3$ such that Σ^{8k+3} bounds a π-manifold (in fact, it is an element of order 2 or 1 in $\theta^{8k+3}(\partial\pi)$) and there exists a free smooth action φ on Σ such that Σ/φ is almost diffeomorphic but not diffeomorphic to $CP(4k+1)$. Conjecture. Σ^{8k+3} is diffeomorphic to S^{8k+3}. (Received September 7, 1973.)

73T-G151. W. WISTAR COMFORT, Wesleyan University, Middletown, Connecticut 06457 and NEIL B. HINDMAN, California State University, Los Angeles, California 90032. Almost disjoint refining families for uniform ultrafilters. Preliminary report.

Throughout, $U(\alpha)$ is all uniform ultrafilters on α and $p = \{A_\xi : \xi < 2^\alpha\} \in U(\alpha)$. Definitions 1. p is a $\varkappa$-point if $\exists \varkappa$ disjoint open subsets of $U(\alpha)$ with p in the boundary of each. 2. $\mathscr{A}$ ($\subset \mathscr{P}(\alpha)$) is almost disjoint (a.d.) if $S, T \in \mathscr{A}$ and $S \neq T \Rightarrow |S \cap T| < \alpha$. Theorem 1. If $\alpha^+ = 2^\alpha$ and p is α-complete then p is a 2^α-point. Theorem 2. Assume α regular and $\alpha^+ = 2^\alpha$. These are equivalent: (a) $\exists$ a.d. $\mathscr{A}$ such that $\xi < 2^\alpha \Rightarrow \exists S \in \mathscr{A}$ with $|S \backslash A_\xi| < \alpha$; (b) $\exists$ a.d. $\mathscr{A} = \{S_\xi : \xi < 2^\alpha\}$ with $S_\xi \subset A_\xi$; (c) p is a 2^α-point; (d) p is a 2-point; (e) $\exists \mathscr{A}, \mathscr{T} \subset [\alpha]^\alpha$ such that $|S \cap T| < \alpha$ for $S \in \mathscr{A}$, $T \in \mathscr{T}$ and $\forall \xi < 2^\alpha \exists S, T$ such that $|S \cap A_\xi| = |T \cap A_\xi| = \alpha$; (f) $\exists$ a.d. $\mathscr{A} \subset [\alpha]^\alpha$ such that $\forall \xi < 2^\alpha \exists S \in \mathscr{A}$ with $|S \cap A_\xi| = \alpha$, and $\mathscr{U} \subset \mathscr{A}$, $|\mathscr{U}| < \alpha \Rightarrow \bigcup \mathscr{U} \notin p$; (g) $\exists$ maximal a.d. $\mathscr{A} \subset [\alpha]^\alpha$ such that $\mathscr{U} \subset \mathscr{A}$, $|\mathscr{U}| < \alpha \Rightarrow \bigcup \mathscr{U} \notin p$. Theorem 3. If $\alpha^+ = 2^\alpha$ or $\alpha = 2^{\underline{\alpha}}$, some $q \in U(\alpha)$ is a 2^α-point. Remarks. 3 extends and partially duplicates N. Hindman, Proc. Amer. Math. Soc. 21(1969), 277-280. Much of 2, including (b) $\Leftrightarrow$ (c) $\Leftrightarrow$ (d), and other equivalences, have been obtained independently by K. Prikry (to appear). (Received September 6, 1973.)

*73T-G152. JACK M. SHAPIRO, Washington University, St. Louis, Missouri 63130 and Israel Institute of Technology, Haifa, Israel. On the cohomology of the classical linear groups. Preliminary report.

Let G be one of the classical linear groups over the finite field with q elements, F_q, described in Chapter 1 of [R. Carter, "Simple groups of Lie type", Wiley, New York, 1972]. Suppose ℓ is an odd prime which divides $q-1$ if $G \neq U_n$ or $q+1$ if $G = U_n$, then $H^*(G)$ can be embedded in $H^*(T)$, where T is the "diagonal" subgroup of G (see Carter), and coefficients for cohomology are taken in $Z/_{\ell Z}$. Furthermore if we take entries for G in a subfield of the algebraic closure of F_q which contains all the ℓ^{rth} roots of unity, and if W is the "Weyl group" of G, then $H^*(G) \cong H^*(T)^W$. $H^*(G)$ is generated by Chern classes and in fact is a polynomial algebra with coefficients in $Z/_{\ell Z}$ over these Chern classes. These results are an extension of [D. Quillen, "The Adams conjecture", Topology 10(1971), 67-80] where the cases $G = GL_n$ and $G = O_n$ are discussed and in fact the methods used in this paper are those of Quillen's. (Received July 16, 1973.) (Author introduced by Professor Jack Sonn.)

73T-G153. ROBERT W. BUTTON, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. Monads for regular and normal spaces.

We shall assume that every enlargement is based on, among other things, $\mathbb{N}$. Theorem 1. For any enlargement, X is regular iff $\mu(x)$ is Q-closed for all x in X. Theorem 2. For any enlargement, X is normal iff $\mu(A)$ is Q-closed for all closed subsets A of X. Theorem 3. For any enlargement of any space X and subset B of

X, $\mu(B)$ is Q-open. The above results have led to nonstandard proofs for a number of standard theorems. (Received September 13, 1973.)

73T-G154. HSU-TUNG KU and MEI-CHIN HSIAO KU, University of Massachusetts, Amherst, Massachusetts 01002. Montgomery-Yang invariants and free smooth actions. Preliminary report.

Let (S^i, Σ^n) denote a free smooth action of S^i on the homotopy n-sphere Σ^n, $i = 1, 3$, and $(S^1, \Sigma^n) \not\subset (S^3, \Sigma^n)$ the action (S^1, Σ^n) unable to extend to (S^3, Σ^n). Let $D^2 \to W \to \Sigma^{4n-1}/S^1$ be the associated disk bundle corresponding to (S^1, Σ^{4n-1}). Let $\beta \in H^2(W)$ be a generator. One may use $(2m+1)\beta$ to define $\nu_{2m+1}(\Sigma^{4n-1})$ as the definition of $\nu(\Sigma^{4n-1})$ of Montgomery-Yang without taking modulo 1. Similarly one defines the Eells-Kuiper invariant $\mu(S^3, \Sigma^{4n-1})$ of (S^3, Σ^{4n-1}). Theorem. Given (S^3, Σ^{4n-1}) and $S^1 \subset S^3$, then 1. $\nu_1(\Sigma^{4n-1}) = a_n \mu(S^3, \Sigma^{4n-1})$ as rational numbers, where $a_n = 4/(3+(-1)^n)$. 2. $E(E^{1/2} - E^{-1/2})^{2(k+1)} = \langle \alpha^k \hat{\mathfrak{A}}(\Sigma^{4n-1}/S^3)\hat{\mathfrak{A}}(\rho^{-1})^k, [\Sigma^{4n-1}/S^3]\rangle$, where ρ is the canonical line bundle over Σ^{4n-1}/S^3, $\alpha \in H^4(\Sigma^{4n-1}/S^3)$ a generator, $E^t = -E^{-t} = \sum_{i=1}^t \nu_{2i-1}(\Sigma^{4n-1})$ for $t > 0$, $E^0 = 0$ and $n > k+1$ $(k \geqq 0)$ (express the left-hand side as a polynomial in E before applying the definition of E^t). Let $A = \nu_3(\Sigma^{11}) - \nu_1(\Sigma^{11})$ and $B = \nu_5(\Sigma^{11}) - 3\nu_3(\Sigma^{11}) + 2\nu_1(\Sigma^{11})$. Corollary 1. If n is odd, and $\nu_1(S^{4n-1}) \notin 2Z$, then $(S^1, S^{4n-1}) \not\subset (S^3, S^{4n-1})$. 2. $(S^1, \Sigma^{11}) \not\subset (S^3, \Sigma^{11})$ if one of the following conditions is satisfied: a. $31\nu_1(\Sigma^{11}) \notin 24Z$, b. $A \notin 3Z$, c. $B \notin 4Z$. Examples 1. For each $k \in Z$, there exist actions with $\nu_1(S^{11}) = 37k$, $A = k$ and $B = 2k$ respectively. 2. There are infinitely many (S^1, S^{4n-1}) with distinct $\nu_1(S^{4n-1})$ such that $(S^1, S^{4n-1}) \not\subset (S^3, S^{4n-1})$ $(n \geqq 2)$. (Received September 14, 1973.)

*73T-G155. DAVID A. EDWARDS, State University of New York, Binghamton, New York 13901. The structure of superspace. Preliminary report.

Let S be the set of isometry classes of compact metric spaces. A set theoretic map $f: X \to Y$ is an ϵ-isometry if $|d_X(a,b) - d_Y(f(a), f(b))| \leqq \epsilon$. Let $I_\epsilon(X, Y)$ be the set of ϵ-isometries from X to Y. Definition. $d(X, Y) = \inf\{\epsilon \mid I_\epsilon(X, Y) \neq \emptyset \neq I_\epsilon(Y, X)\}$. Structure theorem. (S, d) is a contractible, separable metric space. Density theorem. Finite metric spaces are dense in S. Compact, connected, metric polyhedra are dense in the subspace of S consisting of connected spaces. Existential theorem. There exists a computable metric d' on the natural numbers N such that the completion of (N, d') is isometric to the completion of (S, d). (Received September 14, 1973.)

*73T-G156. CHAMOND LIU, Fordham University, Bronx, New York 10458. Cohomology of locally compact groups. Preliminary report.

The purpose of this paper is to construct and study, in the context of a second countable locally compact Hausdorff group G acting continuously on a real Banach space B, a group cohomology theory in which, analogously to Eilenberg-Mac Lane cohomology for abstract groups, the defining cochains arise from the existence of (suitably defined) projective modules and projective resolutions. In this theory the principal example of a projective is $L^1(G)$, the space of real functions on G integrable with respect to Haar measure. Properties include: G is compact if and only if G has cohomology dimension 0. (The cohomology dimension of G is the least integer n for which $H^{n+k}(G, B) = 0$ for all coefficients B and all integers $k > 0$.) If H is a closed subgroup of G, the cohomology dimension of H is $\leqq$ the cohomology dimension of G. If G is discrete abelian and B is separable reflexive, $H^1(G, B) = 0$. If G is infinite discrete and acts by translation on K, the kernel of the integration map $L^1(G) \to R$, $H^1(G, K) \neq 0$. (Received September 19, 1973.) (Author introduced by Professor Robert R. Dobbins.)

*73T-G157. IVAN L. REILLY, University of Auckland, Auckland, New Zealand and IAN E. COOKE, Queen Elizabeth College, University of London, London, England. On bitopological compactness.

This paper discusses the idea of compactness for bitopological spaces. Six apparently different definitions of bitopological compactness have appeared in the literature, and this paper investigates the relationships between these definitions. In particular, it is shown that three of these definitions are in fact equivalent. (Received September 18, 1973.)

*73T-G158. RICHARD B. SHER, University of Georgia, Athens, Georgia 30602. Proper shape theory and neighborhoods of sets in Q-manifolds.

Theorem 1. Suppose M and N are Q-manifolds, X and Y are Z-sets in M and N, respectively, U is a neighborhood of X in M and V is a neighborhood of Y in N, and $Sh_p X \leq Sh_p Y$. Then there exist closed Q-manifold neighborhoods R and S of, respectively, X and Y in M and N, such that $R \subset U$, $S \subset V$, and $R \cong S$. Theorem 2. Suppose M is a Q-manifold, X is a Z-set in M, and P is a locally compact polyhedron such that $Sh_p X \leq Sh_p P$. Then there exists a cofinal system $\{U_\alpha | \alpha \in A\}$ of closed neighborhoods of X in M such that (1) if $\alpha \in A$, $U_\alpha \cong P \times Q$, (2) if $\alpha \in A$, $FrU_\alpha \cong P \times Q$, and (3) if $\alpha, \beta \in A$, there exists a homeomorphism from U_α onto U_β fixing X. Theorem 2 generalizes Chapman's result that if X is a compact Z-set of trivial shape in Q, then X has arbitrarily small closed neighborhoods in Q, each homeomorphic to Q, and yields the following analogous result for X an SUV^∞-space. Corollary. Suppose X is an SUV^∞-space embedded as a Z-set in the Q-manifold M. Then there exists a tree T such that X has arbitrarily close closed neighborhoods in M homeomorphic to $T \times Q$. (Received September 21, 1973.)

73T-G159. PETER J. KAHN, Cornell University, Ithaca, New York 14850. Mixing homotopy types of manifolds. Preliminary report.

We consider only based maps of connected CW complexes inducing π_1 isomorphisms and having torsion homotopy groups. Let P_1 and P_2 be complementary sets of primes. A (P_1, P_2) square for a map f consists of maps f_i, g_i, $i = 1,2$, such that the homotopy groups of f_i and g_i are P_i-torsion, and $f_2 f_1 = f = g_1 g_2$. We show that, given any f, there is a (P_1, P_2) square S for f, unique up to homotopy, thus extending a construction of Zabrodsky to the nonsimple case, and we study to what extent "geometric" conditions on any three of the four vertices of S impose similar conditions on the fourth. For example, we can show that if three of the vertices of S satisfy one of the following conditions, then the fourth satisfies that condition: (a) finite domination; (b) finite type; (c) Poincaré duality; (d) structure of a closed (topological, PL, or almost smooth) manifold of dimension ≥ 5. The results can be extended to maps of pairs, as well as to some maps not necessarily inducing π_1 isomorphisms. Surgery obstructions are encountered for the case of closed, smooth manifolds. (Received September 21, 1973.)

*73T-G160. ULRICH KOSCHORKE, Rutgers University, New Brunswick, New Jersey 08903. Concordance classes of line fields.

Let M be a closed, connected, smooth n-manifold with vanishing Euler number, and let b denote the $\mathbb{Z}_2$-dimension of $H^1(M;\mathbb{Z}_2)$. Theorem. There is only a finite number, $a(M)$, of concordance classes of line fields on M. If n is even, then $a(M)$ equals $2^b - 1$ plus the number of those $x \in H^1(M;\mathbb{Z}_2)$ such that $(1+x)^{-2}w(M)[M] = 0$. If n is odd, then there is a canonical one-to-one correspondence between concordance classes of line fields on M, and those $x \in H^1(M;\mathbb{Z}_2)$ for which $(1+x)^{-1}w(M)[M] = 0$. Corollary 1. If $n \equiv 0$ (4) and $w_1(M)^3 = 0$, or if $n \equiv 2$ (4) and $w_1(M) = 0$, then $a(M) = 2^{b+1} - 1$. Corollary 2. If $n = 4$ and $w_1(M)^3 \neq 0$, then $a(M) = 3 \cdot 2^{b-1} - 1$. Corollary 3. If n is even, $n \leq 20$, and if $w_1(M)^2 w_{n-14}(M) = 0$ (satisfied e.g. if $n < 14$), then $a(M)$ is odd or equal to 6, or, if $n \equiv 0$ (4), possibly equal to 2 or 4. Corollary 4. If $n \equiv 1$ (4) and $w_1(M)^2 = 0$, then $a(M) = 2^b$ if $w_{n-1}(M) = 0$, and $a(M) = 2^{b-1}$ if $w_{n-1}(M) \neq 0$. Corollary 5. If n is odd, $n < 10$, and if $w_1(M)w_{n-7}(M) = 0$ and $w_1(M) \neq 0$, then $a(M)$ is even or equal to 3. In the case of (even or odd) low n, further upper and lower bounds for $a(M)$ are obtained which depend on the (non-)vanishing behavior of certain polynomials in the Stiefel-Whitney classes $w_i(M)$. (Received September 24, 1973.)

73T-G161. TOM KNOBLAUCH, 8022 West Dixon Street, Milwaukee, Wisconsin 53214. Imbedding compact 3-manifolds in E^3.

A finiteness theorem of W. Haken ["Some results on surfaces in 3-manifolds", Studies in Modern Topology, P. J. Hilton, editor, Mathematical Association of America, 1968, p. 39] is used to prove that a large finite disjoint collection of compacta in a compact orientable 3-manifold M^3 contains a compactum with a neighborhood in M^3 that imbeds in E^3. Moreover, no compact 3-manifold serves to replace E^3 in the nonorientable case. (Received September 24, 1973.) (Author introduced by Professor Daniel R. McMillan, Jr.)

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Algebra & Theory of Numbers

*709-A1. ANDY N. C. KANG, Virginia Commonwealth University, Richmond, Virginia 23220 and C. K. KANG, Princeton University, Princeton, New Jersey 08540. A formula for the partition function. Preliminary report.

A partition of a positive integer n is a representation of n as the sum of positive integers. The number of partitions of n is denoted by $p(n)$, which is called a partition function. Let $Q(x)$ be a predicate, then $\mu\ell[Q(\ell)]$ is the least integer such that $Q(\ell)$ holds. $[x]$ is the largest integer less than or equal to x and ${}_nC_k$ is the binomial coefficient. Define $S_m(n) = n(n+1)\ldots(n+m)/(m+1)!$. The value of $p(n)$ can be calculated by the following formula: $p(n) = \sum_{k=1}^{n}(f_k(n))$, where $f_k(n)$ is the number of partitions of n into k parts. We have $f_1(n) = 1$, $f_2(n) = [n/2]$ and for $n \geqq k \geqq 3$, $f_k(n) = \sum_{\ell=1}^{[n/k]}(\sum_{i=1}^{\ell}((-1)^{i+1}(\sum_{j=1}^{\ell-1C_{i-1}}(f_{k-i}(n-\ell-r_{ij})))))$. In the formula we choose $r_{i1} = i(i-1)/2$, $r_{1j} = 0$, and for $i, j \geqq 2$, $r_{ij} = (i-1)(i-2)/2 + \sum_{k=0}^{i-2}(\ell_k)$, where $\ell_k = \mu\ell[j^{(k)} \leqq S_{i-k-2}(\ell)]$, $j^{(0)} = j$ and $j^{(k+1)} = j^{(k)} - S_{i-k-2}(\ell_k - 1)$. (Received April 23, 1973.) (Author introduced by Dr. Robert J. Schwabauer.)

*709-A2. ROBERT B. GARDNER, University of North Carolina, Chapel Hill, North Carolina 27514. A characterization of finitely generated modules whose exterior rank equals the minimal number of generators. Preliminary report.

Let R be a commutative ring and M a finitely generated R-module. Let $\{x^1, \ldots, x^n\}$ be a minimal generating set and let J be the ideal generated by $\{a_{i_j} | \sum a_{i_j} x_j = 0\}$. Theorem. $\wedge^n M \neq 0$ if and only if $1 \notin J$. Thus the exterior rank equals the minimal number of generators. Applications include classical results over local rings and principal ideal domains. (Received June 25, 1973.)

709-A3. PETER V. O'NEIL, College of William & Mary, Williamsburg, Virginia 23185. Nearly planar graphs and the reconstruction problem. Preliminary report.

The reconstruction problem for graphs (reconstructing a graph G from the subgraphs G - v obtained by removing vertices of G) leads in a natural way to the problem of classifying the nearly planar graphs (nonplanar graphs with each G - v planar). Results of Klaus Wagner (J. Combinatorial Theory 3(1967), 326-365) are discussed, and constructions are derived which lead to an explicit list of all nearly planar graphs. (Received August 3, 1973.)

*709-A4. JIMMY T. ARNOLD, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061 and ROBERT GILMER, Florida State University, Tallahassee, Florida 32306. Dimension theory of commutative rings without identity. Preliminary report.

A commutative ring S with identity e is said to be a unital extension of its subring R if $S = \{r + ne | r \in R, n \in Z\}$. This paper considers the relationship between the (Krull) dimensions of R and S if S is a unital extension of R, as well as the problem of determining the set of dimension sequences for commutative rings without identity (see Abstract 73T-A23, these *Notices* 20(1973), A-7, for the definition of the dimension sequence of a ring). Theorem 1. Let R^* be the ring obtained by the canonical adjunction of an identity of characteristic 0 to R. (a) If $\dim R = -1$, then R^* is the unique unital extension of R and $\dim R^* = 1$. (b) If $\dim R = 0$ and if $\dim R^* = 1$, then each unital extension of R distinct from R^* has dimension 1, or each unital extension of R distinct from R^* has dimension 0. (c) If $\dim R = n \geqq 1$, then all unital extensions of R have the same dimension and $\dim R^*$ is either n, n + 1, or n + 2. Theorem 2. Assume that the ring R has positive dimension and that $\{M_\lambda\}$ is the set of maximal prime ideals of R. If S is the weak direct sum of the family $\{R_{M_\lambda}\}$ of rings, then R and S^* have the same dimension sequence. Corollary. Except for the sequence $\{-1, -1, -1, \ldots\}$, the set of dimension sequences for rings without identity is the same as the set of dimension sequences for rings with identity. (Received August 16, 1973.)

*709-A5. CARL POMERANCE, University of Georgia, Athens, Georgia 30602. On a problem of Ore: Harmonic numbers.

A natural number n is said to be harmonic if $H(n) = nd(n)/\sigma(n)$ is an integer. Here d(n) is the number of divisors of n and $\sigma(n)$ is the sum of the divisors of n. Ore proved that H(n) is the harmonic mean of the divisors of n. Ore also showed that every perfect number is harmonic and that no prime power is harmonic. He raised the following question: Is there an odd harmonic number > 1? Garcia proved that there are no odd harmonic numbers in the range $1 < n < 10^7$. He also proved that a harmonic number of the form $p^a q$, where p and q are primes, must be an even perfect number. In the present paper we prove Theorem 1. A harmonic number of the form $p^a q^b$, where p and q are primes, must be an even perfect number. Theorem 2. A harmonic number n is divisible by the largest prime dividing $\sigma(n)$. (Received August 31, 1973.)

709-A6. RICHARD A. DUKE, Georgia Institute of Technology, Atlanta, Georgia 30332. Some Ramsey-type theorems for two-complexes. Preliminary report.

There has been considerable interest on the part of several authors recently in the Ramsey numbers for classes of graphs other than the classical case of complete graphs. Several papers by Chvátal and Harary dealing with such extensions are summarized in their note, "Generalized Ramsey theory for graphs" (Bull. Amer. Math. Soc. 78(1972), 423-426). A number of analogous results are derived for simplicial two-complexes. Use is made of the Ramsey numbers determined in works such as those mentioned above as well as of results on Steiner triple systems and other designs and of the incidence patterns for two-complexes. (Received September 10, 1973.)

*709-A7. CHARLES C. LINDNER, Auburn University, Auburn, Alabama 36830. Some remarks on the Steiner triple systems associated with Steiner quadruple systems.

Let (Q, q) be a Steiner quadruple system and x any element in Q. Set $Q_x = Q\backslash\{x\}$ and denote by q(x) the set of all triples $\{a,b,c\}$ such that $\{x,a,b,c\} \in q$. It is well known that $(Q_x, q(x))$ is a Steiner triple system. Two very interesting problems concerning Steiner quadruple systems are the following: (1) The construction of quadruple systems (Q, q) such that for some subset X of Q containing at least two elements, the Steiner triple systems $(Q_x, q(x))$ and $(Q_y, q(y))$ are nonisomorphic whenever $x \neq y \in X$. (2) The construction of a pair of nonisomorphic Steiner quadruple systems with the property that their associated Steiner triple systems can be isomorphically paired. N. S. Mendelsohn and H. S. Y. Hung ("On the Steiner systems S(3,4,14) and S(4,5,15)", Utilitas Mathematica 1(1972), 5-95) have constructed a quadruple system of order 14 having property (1) and a pair of nonisomorphic quadruple systems of order 14 having property (2). As far as the author can tell, these quadruple systems of order 14 are the only known systems having property (1) or (2). In this paper an infinite class of quadruple systems having property

(1) and an infinite class of pairs of nonisomorphic quadruple systems having property (2) are constructed. Several open problems are stated. (Received September 7, 1973.)

*709-A8. DONALD L. GREENWELL, Emory University, Atlanta, Georgia 30322. Odd cycles and perfect graphs. Preliminary report.

A proof of Toft's conjecture (G is an odd cycle if and only if $|G| = 2n+1$, $\alpha(G) = n$ and $\alpha(G-v-w) = n$ for all $v, w \in V(G)$) is given. This then is used to show that if G is an irreducible nonperfect graph and $\omega(G) = 2$ ($\alpha(G) = 2$), then G is an odd cycle (complement of an odd cycle). (Received September 5, 1973.)

*709-A9. ROBERT L. HAMMINGER, Vanderbilt University, Nashville, Tennessee 37235. Periodic line digraphs.

A digraph D is called periodic if two of its line digraph iterates are isomorphic, i.e. if $L^m(D) \simeq L^{m+k}(D)$ for some integers m and k, $m \geqq 0$, $k > 0$. We will discuss the characterization of such digraphs. The results are complete in the finite case and are nearly complete in the infinite case. (Received September 10, 1973.)

*709-A10. DAVID P. SUMNER, University of South Carolina, Columbia, South Carolina 29208. Graphs with 1-factors. Preliminary report.

Suppose that G is a connected graph of even order that does not contain a 1-factor. Then by Tutte's theorem on 1-factors, there exists a set $S \subseteq V(G)$ such that $G - S$ has more than $|S|$ odd components. Such a set S is called an antifactor set and if no proper subset of S is also an antifactor set, then S is termed a minimal antifactor set. Theorem 1. Every vertex in a minimal antifactor set S of a graph G is the center of an induced $K_{1,3}$ and if G is cubic, then S is an independent set. Corollary 1. A connected graph with no induced $K_{1,3}$ (in particular, a line graph) has a 1-factor iff it has even order. Theorem 2. If S is a minimal antifactor set for the graph G, then S is contained in a single block of G. Theorem 3. If S is a minimal antifactor set for the graph G, then $|S| \leqq \max_B\{q(B) - p(B) + 2\}$ where the maximum is taken over all the blocks of G and q(B) and p(B), respectively, denote the number of edges and vertices in B. Theorem 4. If S is a minimal antifactor set for the graph G, then $|S| \leqq \beta_0(G) - 2$ and $|S| \leqq \beta_1(G)$. Theorem 5. If G is a connected graph of order p that does not contain a 1-factor and k is an integer $4 \leqq k \leqq p$, then G contains an induced, connected, subgraph of order k that also fails to contain a 1-factor. (Received September 10, 1973.)

709-A11. RENU LASKAR and BRUCE AUERBACH, Clemson University, Clemson, South Carolina 29631. On r-partite graphs. Preliminary report.

Let $K_n r$ denote the complete r-partite graph, each vertex set containing n vertices. Hamilton circuits and paths of $K_n r$ are studied for different values of n and r. A 1-factorization $\mathfrak{F}$ of a graph G is called perfect iff $i \neq j$ and $F_i, F_j \in \mathfrak{F}$ implies $F_i \cup F_j$ is a Hamilton circuit. Perfect 1-factorizations of $K_n r$ for some values of n and r have been investigated. (Received September 10, 1973.)

*709-A12. ROBERT GOLD, Ohio State University, Columbus, Ohio 43210. Genera in abelian extensions.

Let K/F be an abelian extension of algebraic number fields; C, the ideal class group of K; and $\overline{K}$, the Hilbert class field of K. The subgroups of C correspond 1-1 with the fields between K and $\overline{K}$, and the principal genus H, of C, is that subgroup which corresponds to the maximal extension unramified over K and abelian over F. It has long been known that this definition extends Gauss' definition of H in terms of certain arithmetic characters on C in the case F = Q and K/Q quadratic. H. Hasse (J. Number Theory 1(1969), 4-7) has shown that for F = Q and K/Q abelian, H could be described by characters. More recently, F. Halter-Koch (J. Number Theory 4(1972), 144-156) achieved this result for arbitrary F and K/F cyclic. The result of this paper is that H can be described in terms of characters for arbitrary F and K/F abelian. The method of proof involves only the basic results of class field theory and, in particular, the norm limitation theorem. (Received September 14, 1973.)

*709-A13. RALPH J. FAUDREE and RICHARD H. SCHELP, Memphis State University, Memphis, Tennessee 38152. Path connected graphs.

Oystein Ore showed if $d(u) + d(v) \geqq |G| + 1$ for u and v not adjacent in a graph G (d denotes the degree), then G is Hamiltonian connected. For graphs satisfying these conditions precisely those path lengths which must exist between each pair of points in the graph are determined. For $u \neq v$ in the graph G, property $P_i(u,v)$ is said to hold if u and v are connected by a path of length i. If $P_i(u,v)$ holds for every $u, v \in G$ and all $2 \leqq i \leqq |G| - 1$, then G is called PLD-maximal. Theorem. If $d(u) + d(v) \geqq |G| + 1$ for u and v not adjacent, then $P_i(x,y)$ holds for all $x \neq y$ in G, $4 \leqq i \leqq |G| - 1$. Corollary. Let $|G| \geqq 4$ and $d(u) + d(v) \geqq [3|G|/2 - 1]$ for u and v not adjacent. Then G is PLD-maximal. Corollary. A $[|G|/2 + 1]$ connected graph is PLD-maximal. Each of these results is the best possible. (Received August 31, 1973.)

709-A14. KENNETH B. REID, Louisiana State University, Baton Rouge, Louisiana 70803. Some recent results on paths in tournaments.

A well-known result in graph theory is that every finite tournament contains a hamiltonian (complete) path. Using this theorem as our point of departure, we discuss several recent results on paths in tournaments. We mention Grünbaum's result on antidirected hamiltonian paths (B. Grünbaum, "Antidirected Hamiltonian paths in tournaments", J. Combinatorial Theory 11(1971), 249-257; also, M. Rosenfeld, "Antidirected Hamiltonian paths in tournaments", J. Combinatorial Theory 12(1972), 93-99), Forcade's results on generalized paths in tournaments (R. W. Forcade, "Hamiltonian paths in tournaments", Ph.D. Thesis, Univ. of Washington, 1971), the author's result on equivalence of n-tournaments via hamiltonian path reversals (K. B. Reid, "Equivalence of n-tournaments via k-path reversals", Discrete Math., to appear), and current work by Alspach, Reid, and Roselle on bypasses in tournaments (a k-bypass of an arc xy in a tournament is a k-path from x to y). (Received September 4, 1973.)

*709-A15. HAROLD S. FINKELSTEIN, Emory University, Atlanta, Georgia 30322. An elemental divisibility property of groups.

Let G be a finite group. Let $h[s,k](G)$ denote the number of elements in G whose order is a multiple of k and a divisor of sk, where sk divides $|G|$, the order of G. Let C denote a cyclic group of order $|G|$. Suppose p^r is the highest power of p dividing $|G|$. Theorem 1. If p is the smallest prime dividing $|G|$, then the only nonzero value of $h[|G|p^{-r}, p^r](G)$ is $h[|G|p^{-r}, p^r](C)$. Theorem 2. If p is the smallest prime dividing $|G|$ and the Sylow p-subgroups of G are normal in G, then $h[t, p^r](C)$ divides $h[t, p^r](G)$ for any divisor t of $|G|p^{-r}$. It is conjectured that the normality condition in Theorem 2 is not necessary. (Received September 17, 1973.)

709-A16. RONALD C. LINTON, University of South Alabama, Mobile, Alabama 36688. Fully invariant subgroups of totally projective abelian groups. Preliminary report.

Definition. F is a fully invariant subgroup of the group G if $\varphi(F) \subseteq F$ for all $\varphi \in \mathrm{End}(G)$. It is known that totally projective groups are fully transitive [P. Hill, "On transitive and fully transitive primary groups", Proc. Amer. Math. Soc. 22(1969), 414-417]. Thus, if F is fully invariant in the totally projective group G, there is a U-sequence for G, say $\underline{\alpha}$, such that $F = G(\underline{\alpha})$ [Kaplansky, "Infinite Abelian groups", Univ. of Michigan Press, Ann Arbor, Mich., 1954]. This characterization and the relation between Griffith's generalized Prüfer groups and the totally projective groups [Corollary 71, "Infinite Abelian group theory", Univ. of Chicago Press, Chicago, Ill., 1970] yield the following result. Theorem. If F is fully invariant in the totally projective group G, then both F and G/F are totally projective. (Received September 18, 1973.) (Author introduced by Professor Richard Vinson.)

*709-A17. JIMMY T. ARNOLD, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061, ROBERT GILMER, Florida State University, Tallahassee, Florida 32306 and WILLIAM J. HEINZER, Purdue University, West Lafayette, Indiana 47907. Some countability conditions in a commutative ring. Preliminary report.

Let R be a commutative ring with identity and let R[[X]] denote the formal power series ring in an indeterminate X. Theorem. If PR[[X]] = P[[X]] for each prime ideal P of R, then R is Noetherian. Here PR[[X]] denotes the ideal of R[[X]] generated by P, and $P[[X]] = \{\alpha = \sum a_i X^i \in R[[X]] \mid a_i \in P$ for each $i\}$. An ideal I of R is such that IR[[X]] = I[[X]] precisely if I satisfies (*): each countably generated ideal contained in I is contained in a finitely generated ideal contained in I. (*)-ideals need not be finitely generated, but if I is a (*)-ideal, then I satisfies (**): if $I_1 \subseteq I_2 \subseteq \ldots$ is an ascending chain of ideals of R such that $\bigcup_{n=1}^{\infty} I_n = I$, then $I = I_n$ for some n. It is shown that (**)-ideals need not be (*)-ideals, and that a ring in which each prime ideal is a (**)-ideal need not be Noetherian. Also, an example is given to show that a prime (*)-ideal P need not be finitely generated, even if P is the radical of a finitely generated ideal. (Received September 21, 1973.)

*709-A18. WILLIAM T. TROTTER, JR., Department of Mathematics and Computer Science, University of South Carolina, Columbia, South Carolina 29208. Graph coloring and dimension theory. Preliminary report.

In this paper we discuss the connections between chromatic numbers for graphs and dimension theory for posets. We describe several methods for associating with a poset X a graph G(X), so that the determination of dim X is related to the determination of $\chi(G(X))$. We also discuss the dual problem of associating posets with graphs. As a consequence of these associations, a family of regular graphs which includes for each $k \geqq 4$ infinitely many k-critical graphs with $q > cp^2$ can then be used to determine the dimension of posets called crowns. The crowns are used to show that for each $t \geqq 3$ there are infinitely many nonisomorphic, t-irreducible posets. We also give restrictions on the parameters of these structures so that a critical graph is associated with an irreducible poset. A construction for triangle free graphs with large chromatic number is then used to prove that the height of irreducible posets is not bounded even though no irreducible poset with height larger than 4 is known. (Received August 14, 1973.)

*709-A19. JAMES K. DEVENEY, Florida State University, Tallahassee, Florida 32306. An intermediate theory for a purely inseparable Galois theory.

Let K be a finite dimensional purely inseparable modular extension of F, and let L be an intermediate field. This paper is concerned with an intermediate theory for the Galois theory of purely inseparable extensions using higher derivations. If L is a Galois intermediate field and M is the field of constants of all higher derivations of L over F, we prove that every higher derivation on L over F extends to K if and only if $K = L \otimes_M J$ for some field J. Similar to classical Galois theory the distinguished intermediate fields are those which are left invariant under a standard generating set for the group of all rank t higher derivations on K over F. We prove: L is distinguished if and only if L is M-homogeneous (4.9). (Received September 24, 1973.)

709-A20. JACOB T. B. BEARD, Jr. and KAREN I. WEST, University of Texas, Arlington, Texas 76019. A software package for factoring in GF[q,x]. Preliminary report.

The capability of a computer program package for factoring polynomials over GF(q), $q = p^d$, is discussed and the extent of present output is indicated. The latter includes tables of prime polynomials, complete factorization tables, and factorization tables for $x^n - 1$, arbitrary binomials, and trinomials. The package is written in Zerox Extended FORTRAN IV and has minimal core storage requirements (12K). (Received September 24, 1973.)

*709-A21. ROY B. LEVOW, Florida Atlantic University, Boca Raton, Florida 33432. Realizable sets of boundary colorations. Preliminary report.

A set, C, of 4-colorations of a k-cycle is realizable as a set of boundary colorations if there is a planar graph G which can be drawn in the plane so that some face is bounded by a k-cycle and the set of colorations obtained by restricting the 4-colorations of G to that face boundary is precisely C. Two colorations of a k-cycle are said to be equivalent if one can be obtained from the other by a permutation of the colors. Theorem. Let c be a coloration of a k-cycle. If C is the set of colorations of a k-cycle containing all colorations not equivalent to c, then C is realizable. If it is assumed that the four-color conjecture is true, the realizable sets of colorations for $k = 5$ are determined with two possible exceptions. (Received September 26, 1973.)

*709-A22. JULIANA DOWELL, East Carolina University, Greenville, North Carolina 27834. Some analogs of arithmetic functions.

Let k be a nonnegative integer, and let $\sum_{i=1}^{m} a_i p^i$ $(0 \leq a_i < p)$ be the p-adic expansion of k for some prime p. A nonnegative integer d is a p-factor of k $(d|_p k)$ if $d = \sum_{i=1}^{n} b_i p^i$ with $n \leq m$ and $0 \leq b_i \leq a_i$ for $i = 1, \ldots, n$. If f,g are arithmetic functions defined on the nonnegative integers, whose ranges are contained in some field F, then the Lucas product of f and g can be expressed as $f * g(n) = \sum_{d|_p n} f(d)\, g(n-d)$. Several years ago L. Carlitz developed analogs of the τ-function (number of divisors), the μ-function, and the Möbius inversion formula using the Lucas product. Now analogs of the σ-function (sum of divisors), the Euler φ-function, the Ramanujan sum, and others are given. (Received September 26, 1973.)

*709-A23. EZRA BROWN, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Class numbers of real quadratic number fields.

This article is a study of congruence conditions, modulo powers of two, on class numbers of real quadratic number fields $Q(\sqrt{d})$, for which d has at most three distinct prime divisors. Techniques used are those associated with Gaussian composition of binary quadratic forms. (Received September 26, 1973.)

*709-A24. CHANG MO BANG and KENNETH I. MANDELBERG, Emory University, Atlanta, Georgia 30322. Finite basis theorem for rings and algebras satisfying a central condition.

Let R be a commutative Noetherian ring with 1, and let T be an R-algebra, not necessarily associative. It is shown that if, for some positive integer n, $T^n \subseteq Z(T)$, the center of T, then all the polynomial identities satisfied by T are finitely based, that is, logical consequences of a fixed finite subset of them. Immediate corollaries are: the varieties of rings generated by a nilpotent ring, an α_n-ring [for the definition of α_n-ring, see M. Putcha, and A. Yaqub, "Rings satisfying polynomial constraints", J. Math. Soc. Japan 25(1973), 115-124], or commutative rings are finitely based. (Received September 27, 1973.)

*709-A25. KENNETH I. MANDELBERG, Emory University, Atlanta, Georgia 30322. On the classification of quadratic forms over semilocal rings.

Let R be a commutative semilocal ring in which 2 is a unit. It is further assumed that either R has no residue fields with 5 or fewer elements, or squares of units may be lifted modulo the Jacobson radical of R. Let I(R) denote the ideal generated by the free quadratic forms of even rank in the Witt ring W(R). Theorem. If d is a unit of R which is not a square, then $I^3(R) = 0$ implies $I^3(R\sqrt{d}) = 0$. Corollary. Quadratic forms over R are characterized by their Hasse invariant, determinant and rank iff $I^3(R) = 0$, and the latter condition depends only on the ring structure of W(R). These results were originally obtained when R was a field by Elman and Lam. The present proof does not require several nonelementary techniques used in the original proof. (Received September 27, 1973.)

709-A26. DORIAN P. YEAGER, University of Tennessee, Knoxville, Tennessee 37914. Universal compact inverse semigroups.

There is a functor $\mathcal{J}$ from the category CS of compact semigroups into its subcategory CIS of compact inverse semigroups, and a natural transformation i from the identity functor $\mathcal{E}$ on CS to $\mathcal{J}$ such that the following holds. For any compact semigroup S, compact inverse semigroup I, and continuous homomorphism $f: S \to I$ there exists a continuous homomorphism $g: \mathcal{J}(S) \to I$ such that $gi_S = f$. $\mathcal{J}(S)$ is called the universal compact inverse semigroup for S. For any object S of CS, i_S is an imbedding if and only if S is imbeddable in a compact inverse semigroup. Two questions are posed, the first being, "When is a compact semigroup imbeddable in a compact regular semigroup?" The second is like the first, with "inverse" substituted for "regular". Answers given for the latter question include an affirmative answer for compact totally $\mathcal{K}$-quasi-ordered semigroups. Moreover, a full description of the universal compact inverse semigroup for a given compact totally $\mathcal{K}$-quasi-ordered semigroup is given. This description relies on the recent work by C. E. Clark and J. H. Carruth on the structure of compact totally $\mathcal{K}$-quasi-ordered semigroups, which has not yet been submitted for publication. (Received September 27, 1973.)

*709-A27. PAUL C. KAINEN, Case Western Reserve University, Cleveland, Ohio 44106. The chromatic number of a graph with specified skewness.

The skewness $\mu(G)$ of a graph G is the minimum number of edges whose removal makes G planar. Theorem. If $\mu(G) < \binom{k}{2}$, where $k \geq 3$ and $\binom{k}{2}$ denotes the binomial coefficient, then $\chi(G) \leq 2 + k$. (Note that the statement of the Theorem for $k = 2$ is just the four color conjecture.) The proof for $k \geq 5$ is essentially just the proof of Heawood's bound on the chromatic number of graphs with specified genus, which result is, in fact, generalized by the Theorem. For $k = 4$, the result is trivial but for $k = 3$ it is a bit more delicate. The Theorem is best possible since $\mu(K_r) = \binom{r-3}{2}$ for $r \geq 5$. (Received September 27, 1973.)

*709-A28. CATHERINE C. AUST, Georgia Institute of Technology, Atlanta, Georgia 30332. Primitive elements and one relation algebras.

Let F be a free algebra in a variety V. An element p of F is called primitive if it is contained in some free generating set for F. In 1936, J. H. C. Whitehead proved that a group with generators $g_1, \ldots, g_n$ and one relation $r = 1$ is free if and only if the relator r is primitive in the free group on $g_1, \ldots, g_n$. The question of whether there is an analogous theorem for other varieties is considered. A necessary and sufficient condition that a finitely generated, one relation algebra be free is proved for any Schreier variety of nonassociative linear algebras and for any variety defined by balanced identities. An identity $u(x_1, \ldots, x_n) = v(x_1, \ldots, x_n)$ is called balanced if each of u and v has the same length and number of occurrences of each x_i. General sufficiency conditions that a finitely generated, one relation algebra be free are given, and all of the known results analogous to the Whitehead theorem are listed and shown to be equivalent to a general necessary condition. Also an algebraic proof of Whitehead's theorem is outlined to suggest the line of argument for other varieties. (Received September 27, 1973.)

709-A29. EUGENE THOMAS BEASLEY, JR., Ohio State University, Columbus, Ohio 43210 and PETER M. GIBSON, University of Alabama, Huntsville, Alabama 35807. A relationship between characteristic values and vectors.

If A is an n-square matrix over a field F, then A is stochastic if all the row sums of A or all the row sums of A^T (the transpose of A) are 1. Clearly if A is stochastic, then 1 is a characteristic value of PA for every n-square permutation matrix P. R. A. Brualdi and H. W. Wielandt [Linear Algebra and Appl. 1(1968), 65-71] prove the converse. In this paper it is shown that the set of n! permutation matrices can be replaced by a set of n-square matrices of cardinality $n^2 - 2n + 2$. This is a corollary to the more general result. Theorem. For all nonzero n-component column vectors α and β over a field F there exists a set Γ of n-square matrices over F of cardinality $n^2 - 2n + 2$ such that, for each n-square matrix A over F, $A\alpha = \alpha$ or $A^T\beta = \beta$ if and only if 1 is a characteristic value of PA for every $P \in \Gamma$. (Received September 28, 1973.)

*709-A30. PAUL D. HILL, Florida State University, Tallahassee, Florida 32306. Commutative groups and the cardinal connection.

• First, a survey is given of the structure of commutative groups determined by certain cardinal invariants. The emphasis, of course, is on the Kaplansky-Mackey cardinals (= Ulm invariants). As is well known, these cardinals completely determine the structure of those commutative groups sufficiently restrained also by cardinals. The development begins with the results of Prufer and Ulm and continues through the author's results for third countable groups. Then various criteria for a group to be third countable are presented. In particular, the following new result is announced. Theorem. A primary group G satisfies the third axiom of countability if it is the union of a countable chain of isotype subgroups H_n each of which is third countable. The preceding theorem has some interesting applications one of which is the proof of the generalized Kulikov criterion. In connection with the third axiom of countability, three definitions are shown to be equivalent in a direct fashion, independent of the uniqueness theorem. Turning to torsion-free groups, we select freeness and $\aleph_n$-freeness as the principal topics. Criteria for freeness are given in terms of an ascending chain of free subgroups leading up to the group. The following result, due to the author, is typical. Let G be the union of an ascending smooth chain of free groups F_α, indexed by an arbitrary ordinal. If $F_{\alpha+1}/F_\alpha$ is $\aleph_1$-free for each α, then G is $\aleph_2$-free. (Received October 1, 1973.)

*709-A31. PALANIAPPAN KANNAPPAN, University of Waterloo, Waterloo, Ontario, Canada. On Moufang and extra loops.

In a loop G(•), corresponding to well-known Moufang identities, the following identical relations are considered: (1) $(yx \cdot z) \cdot \lambda x = y \cdot [x \cdot (z \cdot \lambda x)]$; (2) $xy \cdot (z \cdot \lambda x) = (x \cdot yx) \cdot \lambda x$; (3) $xy \cdot (z \cdot \lambda x) = x \cdot (yz \cdot \lambda x)$; (4) $(xy \cdot \lambda x)z = x \cdot (y \cdot (\lambda x \cdot z))$, where $\lambda : G \to G$ is any mapping. The following result regarding (1) and similar results regarding (2), (3), and (4) are proved. Theorem M. If the identity (1) holds in a loop G(•), then G(•) is Moufang and (2), (3), and (4) also hold in G. Further $\mu x \in N$ (nucleus) where $x \cdot \mu x = \lambda x$. Conversely if G(•) is Moufang and $\mu x \in N$, for all $x \in G$, then the identity (1) holds in G(•). Results similar in nature to Theorem M regarding generalized identities obtained from extra loop identities and results connecting these two sets of identical relations are also obtained. From these can be deduced necessary and sufficient conditions in order that a loop be Moufang or extra. (Received October 1, 1973.)

*709-A32. TREVOR EVANS, Emory University, Atlanta, Georgia 30322. Some finitely based varieties of rings.

It is shown that any associative ring satisfying an identity $x^n = x$, and any nonassociative nilpotent ring have finitely based identities. The proofs are elementary and are based on the universal algebraic observation that if V is a variety of algebras with the property that any finitely generated V-algebra is finitely related, then any subvariety of V which is defined by identities in some bounded number of variables is finitely based (relative to V). The above results have been obtained by different methods by H. Werner and R. Wille, "Charakterisierungen der primitiven Klassen arithmetischer Ringe", Math. Z. 115(1970), 197-200, and R. Kruse, "Identities satisfied by a finite ring", J. Algebra (to appear), respectively. It is also shown that for associative rings, the product of a nilpotent variety and a finitely based variety is finitely based. This is the analogue for rings of a group-theoretic result by G. Higman, "Some remarks on varieties of groups", Quart. J. Math. Oxford Ser. (2) 10(1959), 165-178. (Received October 1, 1973.)

*709-A33. JOEL V. BRAWLEY, Clemson University, Clemson, South Carolina 29631, LEONARD CARLITZ, Duke University, Durham, North Carolina 27701 and JACK LEVINE, North Carolina State University, Raleigh, North Carolina 27607. Power sums of matrices over a finite field.

Let $\mathcal{C}$ denote a subset of the set $F_{n \times n}$ of $n \times n$ matrices over a finite field F of q elements. For each

$A \in F_{n \times n}$, let $\sigma_i = \sigma_i(A)$ denote the ith elementary symmetric function of the roots of A $(i = 1, 2, \ldots, n)$ so that $\sigma_i(A)$ is the sum of the $i \times i$ principle subdeterminants of A. In this paper we evaluate $\sum_{A \in \mathcal{C}} \sigma_1^{\alpha_1} \sigma_2^{\alpha_2} \ldots \sigma_n^{\alpha_n} A^m$ for various classes $\mathcal{C}$, where $\alpha_1, \ldots, \alpha_n$, m are nonnegative integers, and where A^0 and $\sigma_i^0 = [\sigma_i(A)]^0$ are defined to be I and 1, respectively, for all $A \in F_{n \times n}$. One particular class we consider is $\mathcal{C} = F_{n \times n}$ and here we prove the following: Theorem. Let $n \geqq 2$. Then for all nonnegative integers $\alpha_1, \ldots, \alpha_n, m$, $\sum_{A \in F_{n \times n}} \sigma_1^{\alpha_1} \ldots \sigma_n^{\alpha_n} A^m = 0$ unless $q = n = 2$. Corollary. Let $n \geqq 2$. Then $\sum_{A \in F_{n \times n}} A^m = 0$ for all $m \geqq 0$, unless $q = n = 2$. These sums are also obtained for the special case $q = n = 2$. (Received October 1, 1973.)

*709-A34. S. H. BROWN, Auburn University, Auburn, Alabama 36830. On subdirectly irreducible rings. Preliminary report.

In recent papers, M. G. Deshpande and H. G. Moore give examples of subdirectly irreducible rings which have nonsubdirectly irreducible homomorphic images (see Deshpande's paper in Bull. Austral. Math. Soc. 4(1971), 31-34). This paper characterizes those rings which have the property that all nonzero homomorphic images are subdirectly irreducible. Theorem 1. A ring R has the property that all of its nonzero homomorphic images are subdirectly irreducible if and only if (i) R satisfies descending chain conditions on ideals and (ii) the collection of ideals of R is linearly ordered by $\subseteq$. In the event that R is a duo ring (or if the subdirect irreducibility is applied to R_R), then conditions (i) and (ii) of Theorem 1 can be replaced with the condition that R is a uniserial ring (or that R_R is a uniserial module of finite length). Theorem 2. A ring R can be written as a subdirect sum of rings which have properties (i) and (ii) of Theorem 1 if and only if there exists a collection $\mathcal{M}$ of ideals of R having the following properties: (i) $\bigcap_{I \in \mathcal{M}} I = 0$, (ii) If $I \in \mathcal{M}$, then the set of ideals which contain I is linearly ordered with respect to $\subseteq$ and satisfies the descending chain condition. (Received October 1, 1973.) (Author introduced by Professor Charles C. Lindner.)

*709-A35. DWIGHT STEEDLEY, Auburn University, Auburn, Alabama 36830. Separable quasigroups.

Let $(Q, \circ)$ be a quasigroup. By a separation of $(Q, \circ)$ is meant a partition of $(Q \times Q) \setminus \{(x,x) | x \in Q\}$ into disjoint sets Q_1 and Q_2 such that (x,y) is in Q_1 iff (y,x) is in Q_2. Denote such a partition by $S(Q_1, Q_2)$. Now let $S(Q_1, Q_2)$ be a separation of Q and define binary operations $\circ_1$ and $\circ_2$ on Q as follows: (1) $x \circ_i x = x \circ x$, all x in Q, and (2) $x \circ_i y = y \circ_i x = x \circ y$ if (x,y) is in Q_i. If the groupoids $(Q, \circ_1)$ and $(Q, \circ_2)$ are quasigroups, the quasigroup $(Q, \circ)$ is said to be separable. Let $(Q, \circ)$ be an idempotent $(x^2 = x)$ semisymmetric $(x(yx) = y)$ quasigroup of order n. If α is a cyclic automorphism of $(Q, \circ)$ of order n such that for all a, b in Q, $a\alpha^k = a \circ b$ implies $b\alpha^k = (a \circ b) \circ b$, and $b\alpha^m = a \circ b$ implies $a\alpha^m = a \circ (a \circ b)$, $0 \leqq m, k \leqq n - 1$, then we say that α is a separating automorphism of $(Q, \circ)$. Theorem. Let $(Q, \circ)$ be an idempotent semisymmetric quasigroup of order n, orthogonal to its transpose having a separating automorphism. Then $(Q, \circ)$ is separable into a pair of perpendicular Steiner quasigroups. (Received September 28, 1973.) (Author introduced by Professor Charles C. Lindner.)

*709-A36. RENU LASKAR and WILLIAM R. HARE, JR., Clemson University, Clemson, South Carolina 29631. The chromatic polynomial of a complete r-partite graph.

The complete r-partite graph with independent sets of vertices $V_1, \ldots, V_r$, where $|V_i| = p_i$, is denoted $K_{p_1, \ldots, p_r}$. In this paper it is proved that the chromatic polynomial for this graph is given by $\sum_{t_r=1}^{p_r} \ldots \sum_{t_1=1}^{p_1} \prod_{i=1}^{r} S(p_i, t_i)(t)_{t_1 + \ldots + t_r}$, where $S(p_i, t_i)$ is a Stirling number of the second kind and $(t)_{t_1 + \ldots + t_r}$ is the "falling factorial" given by $(t)(t-1)\ldots(t - t_1 - \ldots - t_r + 1)$. This directly generalizes a result of Swenson (Amer. Math. Monthly 80(1973), 797-798) and also gives the chromatic polynomial for K_r, the complete graph on r vertices, as a special case $(p_1 = \ldots = p_r = 1)$. (Received October 1, 1973.)

709-A37. KIM KI-HANG BUTLER, Pembroke State University, Pembroke, North Carolina 28372. The semigroup of Hall relations.

We claim: (0) $C_X \subset D_X \subset H_X \subset B_X$. (1) $\alpha \in H_X \Leftrightarrow \exists \beta \in R_X \ni \alpha = \gamma\beta\rho$ for $\gamma, \rho \in S_X$, where R_X is a semigroup of reflexive relations on X. (2) $|H_X| \leqq 2^{n^2}(1-2^{1-n})$. (3) $\alpha, \beta \in H_X \Rightarrow \alpha\mathcal{D}\beta \Leftrightarrow \alpha = \gamma\beta\rho$ for $\gamma, \rho \in S_X$. (4) $E(Y) = \{e \in Y : e^2 = e\}$. $\alpha \in H_X$ is a regular $\Rightarrow \exists\, \gamma, \rho \in S_X \ni \gamma\alpha\rho \in E(D_\alpha)$. (5) $V(\alpha) = \{\beta : \alpha = \alpha\beta\alpha,\ \beta = \beta\alpha\beta\}$. $\alpha \in H_X$ is a regular $\Rightarrow V(\alpha) = 1$. (6) $\alpha \in E(H_X) \Rightarrow \alpha$ is a quasi-order relation, and so $|E(H_X)| = \sum_{m=1}^{n} S(n,m)P_m$ $(= |E(R_X)|)$, where $S(n,m)$ is a Stirling number of the second kind, and P_m is the number of partial order relations on m points. (7) Let Ω_n be the set of all $n \times n$ doubly stochastic (d.s.) matrices. Then $\Omega_n \to D_X$ is a homomorphism but $E(\Omega_n) \simeq E(D_X)$. (8) $|E(\Omega_n)| = Y_n(fg_1, \ldots, fg_n)$, where Y_n is a Bell polynomial such that $f = 1$, $g_i = 1$ for every i. (9) A relation α is said to be generalized circulant (idempotent) if $\exists$ a circulant (idempotent) relation $\beta \ni \alpha = \gamma\beta\rho$ for $\gamma, \rho \in S_X$. Ω_n contains six classes of matrices; (i) circulants, (ii) generalized circulants, (iii) fully indecomposable matrices, (iv) idempotents, (v) generalized idempotents, (vi) permutation matrices. Thus, one may resolve the van der Waerden conjecture by computing the permanents of the above classes of matrices. (10) Let N_n be the set of all $n \times n$ nonnegative matrices, then $|E(N_n)| = \sum_{r=1}^{n} \sum_{k=0}^{r} (1/r!)(-1)^k \binom{r}{k} (2^{r+1} + r - 1 - k)^n$. (Received September 28, 1973.)

709-A38. PETER M. GIBSON, University of Alabama, Huntsville, Alabama 35807. Unitary and orthogonal transformations on matrices.

In this paper the question of whether a linear transformation on the real or complex matrices that preserves the Euclidean norm of all matrices of a fixed rank must be an isometry is considered. Denote the $s \times t$ complex matrices by $C(s,t)$, the $s \times t$ real matrices by $R(s,t)$, and the Euclidean norm of a matrix A by $\|A\|$. Theorem 1. If $1 \leqq r \leqq \min\{s,t\}$, and T is a linear transformation on $C(s,t)$ such that $\|TA\| = \|A\|$ for every A in $C(s,t)$ of rank r, then T is a unitary transformation. For $r = 1$ and $s = t$, this establishes a conjecture by M. Marcus [J. Res. Nat. Bur. Standards 75B(1971), 107-113]. It also contradicts a recent claim by R. Bronson [Math. Mag. 46(1973), 100] that Marcus' conjecture is false. Theorem 2. If $2 \leqq r \leqq \min\{s,t\}$, and T is a linear transformation on $R(s,t)$ such that $\|TA\| = \|A\|$ for every A in $R(s,t)$ of rank r, then T is an orthogonal transformation. If $\min\{s,t\} \geqq 2$, then there exists a nonorthogonal linear transformation T on $R(s,t)$ such that $\|TA\| = \|A\|$ for every A in $R(s,t)$ of rank 1. (Received October 1, 1973.)

*709-A39. JAPHETH HALL, JR., Institute for Services to Education, Incorporated, 2001 S Street, N.W., Washington, D. C. 20009. Geometric rings. Preliminary report.

Let R be a nontrivial ring. By definition, R is geometric if there is a left module V over R such that if $r \in R$, $s \in R$, $q \in R$, $x \in V$, and $y \in V$ such that $r(x-y) \neq s(x-y)$, then there is a $p \in R$ such that $[p(r-s)+s](x-y) = q(x-y)$. Observation. There is a geometric ring with divisors of zero. Let V be a left module over R. Let L denote the function from V^2 to 2^V such that if $x \in V$ and $y \in V$, then $L(x,y) = \{r(x-y)+y : r \in R\}$. By definition, L is geometric if the following condition is satisfied: If $x \in V$, $y \in V$, $z \in L(x,y)$, and $w \in L(x,y)$, then $L(z,w) \subseteq L(x,y)$ and $L(z,w) = L(x,y)$ if $z \neq w$. Theorem 1. R is geometric if and only if L is geometric. Let $||$ denote the binary relation on the range of L such that if $x \in V$, $y \in V$, $z \in V$, and $w \in V$, then "$L(x,y) \,||\, L(z,w)$" means that $z - w = r(x-y)$ for some $r \in R$ such that $r \neq 0$. By definition, V is locally unitary over R if for each $x \in V$, there is an $r \in R$ such that $rx = x$. Theorem 2. If V is locally unitary over R, then R is geometric if and only if $||$ is symmetric. (Received October 1, 1973.)

*709-A40. ANDREW SOBCZYK, Clemson University, Clemson, South Carolina 29631. Factorization of certain decimal integers.

Let I_c denote the decimal integer 11...1 with c digits (all c of the digits are 1's). It is proved for each odd integer q, relatively prime to 5, that for some $c = c(q) \leqq h + q$, q divides exactly into I_c. The integer h is the number of digits in q, e.g., for q = 37, c(37) = 3; for q = 9, c(q) = 9; for q = 27, c(27) = 19. In other words, for any dividend and divisor whose digital sequence is relatively prime to 2 and 5, in the ordinary algorithm for division, if 1's are added instead of 0's, ultimately the division will come out "even", i.e. the remainder will be zero. (Received October 1, 1973.)

709-A41. MONTE B. BOISEN, JR. and PHILIP B. SHELDON, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Some necessary conditions for pre-Prüfer domains. Preliminary report.

A commutative integral domain D will be called a pre-Prüfer domain if every proper homomorphic image of D is a Prüfer ring (that is, a ring in which every finitely generated regular ideal is invertible). The class of pre-Prüfer domains properly contains the class of all Prüfer domains, since it includes all one-dimensional domains as well. The following theorem shows that pre-Prüfer domains share some important properties with Prüfer domains relative to prime ideal structure and invertibility of ideals. Theorem. Let D be a pre-Prüfer domain. Then (a) the prime ideals contained in a given prime ideal of D are linearly ordered by inclusion; (b) if I is a finitely generated ideal containing an element a such that $\bigcap_{i=1}^{\infty} a^i D \neq (0)$, then I is invertible. An example is constructed to show that a domain satisfying conditions (a) and (b) above is not necessarily a pre-Prüfer domain. (Received October 1, 1973.)

709-A42. KIM KI-HANG BUTLER and JAMES RICHARD KRABILL, Pembroke State University, Pembroke, North Carolina 28372. The structure of the semigroup of Boolean circulant matrices. Preliminary report.

This research is a continuation of the authors' paper ["Abelian subsemigroups, enumeration, and universal matrices", Duke Math. J., to appear]. Let C_n be the set of n by n circulant matrices in B_n, S_n be the set of n by n permutation matrices, and $E(C_n)$ be the set of idempotents in C_n. Let $C \in C_n$ be weakly convergent if $C \notin E(C_n)$ and $C^n = E \in E(C_n) - \{Z, I, J\}$. Theorem 1. All matrices in C_n belong to one or more of these classes: (a) primitive matrices, (b) idempotent matrices, (c) weakly convergent matrices. Primitive matrices were enumerated in the authors' Duke Math. J. paper. Theorem 2. If A, $B \in C_n$, then A is $\mathcal{D}$-equivalent to B if and only if $\exists P, Q \in S_n$ such that A = PBQ. Theorem 3. $C \in C_n$ is idempotent if and only if the set of position numbers of the nonzero entries in the first row, numbered from 0 to (n - 1), is the set of all multiples modulo n of some divisor of n. The number of idempotents in C_n is one more than the number of divisors of n. Theorem 4. There are m regular $\mathcal{D}$-classes in C_n, where m is one more than the number of divisors of n. (Received October 1, 1973.)

Analysis

*709-B1. WILLIAM E. FITZGIBBON, University of Houston, Houston, Texas 77004. Nonlinear perturbation of m-accretive operators.

Let X be a reflexive Banach space. Conditions sufficient to guarantee that the sum, A + B, of two m-accretive operators, A and B, is m-accretive are provided. The basic requirements are that the operator B be bounded in some sense relative to A and that A and B be weakly closed. (Received August 14, 1973.)

709-B2. LYNE HYNER CARTER, Florida State University, Tallahassee, Florida 32306. An order topology in ordered topological vector spaces. Preliminary report.

If X is a partially-ordered space, define the closed sets for a topology $\mathcal{O}$ on X to be those which are closed with respect to (o)-convergence of nets. A setting of some interest is that in which (X, τ) is a real, sequentially complete, locally convex space ordered by the positive cone $K = \{x \in X : f_i(x) \geqq 0 \text{ for all } i\}$ of a Schauder basis $B = \{x_i, f_i\}_{i=1}^{\infty}$ for X. Theorem 1. If (X, τ) is as above and K is normal for τ, then $\tau \subseteq \mathcal{O}$. Theorem 2. Let

(X, τ) be a Fréchet space with Schauder basis B and basis-order topology $\mathcal{O}$. The following are equivalent: (i) B is unconditional, (ii) K is normal for τ and generating, (iii) $\mathcal{O} = \tau$, (iv) K is generating and $\tau \subseteq \mathcal{O}$, (v) K is normal for τ and $\mathcal{O} \subseteq \tau$. Corollary. If $(X, \tau) = C[0,1]$ with sup norm, ordered by the positive cone of Schauder's basis, we have that τ is strictly weaker than $\mathcal{O}$. (In Theorem 2, (i) $\Leftrightarrow$ (ii) $\Rightarrow$ (iii) is known (Ceĭtlin, Izv. Vysš. Učebn. Zaved. Matematika, 1966, 2(1951), 98-104); Theorem 1 is a considerable generalization of one of Ceĭtlin's results.) (Received August 20, 1973.)

*709-B3. JOHN R. HUBBARD, Columbus College, Columbus, Georgia 31907. Approximation of compact homogeneous maps.

Let E and F be Banach spaces over the same field, and let $\mathscr{K}(E,F)$ denote the set of all continuous maps from E into F which are homogeneous; i.e. for which $T(\lambda x) = \lambda Tx$ for all $x \in E$ and all scalars λ. Then $\mathscr{K}(E,F)$ itself is a Banach space with the uniform norm. Theorem. Every compact map in $\mathscr{K}(E,F)$ is a limit of finite-rank maps in $\mathscr{K}(E,F)$. The methods used to obtain this result involve the (nonlinear) metric projection of F onto a finite-dimensional subspace, the diametral dimensions of a bounded subset of F, and a sequence of approximation numbers for the given compact map which are similar to those introduced by Pietsch. (Received August 22, 1973.)

*709-B4. E. LEE MAY, JR., Salisbury State College, Salisbury, Maryland 21801. Nonlinear eigenvalues.

Denote by S the point set of a complex normed linear complete space H, by I the identity transformation on H, by T a linear or nonlinear transformation from a subset D(T) of S into S, and by λ a complex number. "λ is an eigenvalue of T" means that there are two points x and y of D(T) such that $Tx - Ty = \lambda\cdot(x-y)$. This definition, which is equivalent to the usual one if T is linear, yields the following results: (1) λ is an eigenvalue of T if and only if $\lambda I - T$ fails to be 1 - 1; (2) if H is an inner-product space, then every eigenvalue of T is in the numerical range of T; (3) if p is in S and $A = T + p$, then λ is an eigenvalue of T if and only if λ is an eigenvalue of A. In addition, examples are given to illustrate the variety possible in the eigenvalue sets of Lipschitzian, locally Lipschitzian, and analytic transformations. (Received August 30, 1973.)

*709-B5. ROGER T. LEWIS, Slippery Rock State College, Slippery Rock, Pennsylvania 16057. The discreteness of the spectrum of selfadjoint, even order, one-term, differential operators.

An open question which was asked by I. M. Glazman is answered. It is well known that the condition $\lim_{x\to\infty} x^{2n-1}\int_x^\infty r^{-1} = 0$ is sufficient for the discreteness and boundedness from below of the spectrum of selfadjoint extensions of $(-1)^n(ry^{(n)})^{(n)}$. This paper shows that the condition is also necessary. (Received September 4, 1973.)

*709-B6. C. WAYNE MASTIN, Mississippi State University, State College, Mississippi 39762. Rational approximation of extremal length for doubly connected domains.

In this report we consider the problem of approximating the extremal length of the family of curves separating the boundary components of a doubly connected domain. A method of successive approximations is developed using a basis for the Hilbert space of square integrable analytic functions on the domain. It is shown that the extremal length can be obtained from the solution of a minimization problem in this infinite dimensional space and furthermore, that this solution can be approximated by solving a minimization problem in a finite dimensional subspace. In the problem for this whole space there are an infinite number of constraints, but in the problem for the subspace there are only a finite number of constraints. (Received September 4, 1973.)

*709-B7. BERNARD A. ASNER, University of Dallas, Irving, Texas 75060 and ARISTIDE HALANAY, University of Bucharest, Bucharest, Romania. Delay-feedback using derivatives for minimal time linear control problems.

A modification of Popov's linear minimal time problem (see, V. M. Popov, O. D. E., 1971 NRL-MRC Conference, Academic Press, New York, 1972) is solved by allowing delay-feedback in both the state and its derivative. It is shown how to control the state of a linear time invariant differential equation to any nontrivial subspace of the state space and remain there for all future time. A technique is given for constructing the feedback law which allows the system's trajectory to reach the given subspace in minimum time. (Received September 4, 1973.)

*709-B8. JAGDISH CHANDRA, U. S. Army Research Office, Durham, North Carolina 27706 and PAUL WILLIAM DAVIS, Worcester Polytechnic Institute, Worcester, Massachusetts 01609. A monotone method for quasi-linear boundary-value problems.

Consider the quasi-linear boundary-value problem $(p(x)u')' = f(x,u)u' + g(x,u)$, $0 < x < 1$, $\alpha_i u(i) + \beta_i u'(i) = a^i(u(i))u'(i) + b^i(u(i))$, $i = 0, 1$, where f, g need not be monotone, bounded, convex, positive or otherwise geometrically restricted. By modifying the chord method of Courant, Keller, Sattinger et al, we establish that maximal and minimal solutions of this problem can be constructed as uniform limits of monotone sequences if f, g merely possess bounded derivatives. We describe an application of the method to a boundary-value problem involving the Reynold's equation of gas lubrication theory. (Received September 6, 1973.)

*709-B9. JOHN V. BAXLEY, Wake Forest University, Winston-Salem, North Carolina 27109. On singular perturbations of nonlinear two-point boundary value problems.

We consider nonlinear two-point boundary value problems of the form $\epsilon y'' + f(x,y,y',\epsilon) = 0$, $a_0(\epsilon)y(0) - a_1(\epsilon)y'(0) = \alpha(\epsilon)$, $b_0(\epsilon)y(1) + b_1(\epsilon)y'(1) = \beta(\epsilon)$, where $\epsilon > 0$ is a small parameter. Conditions are given which guarantee that a unique solution exists for ϵ sufficiently small and the solution $y(x,\epsilon)$ be bounded as $\epsilon \to 0$. This result is then applied to give an asymptotic expansion of the solution of a problem which arises in the theory of chemical flow reactors and has been previously studied by several people, including R. E. O'Malley, Jr. (J. Inst. Math. Appl. 6(1970), 12-20). (Received September 10, 1973.)

*709-B10. JOHN W. BAKER, La Salle College, Philadelphia, Pennsylvania 19141. A continuation theorem for a damped and forced nonlinear differential equation. Preliminary report.

Consider the damped and forced nonlinear ordinary differential equation (*) $u'' + \varphi(t,u,u')u' + p(t)f(u)g(u') = h(t,u,u')$, where $\varphi : I \times R^2 \to R$, $p : I \to R_+$, $f : R \to R$, $g : R \to R_+$, and $h : I \times R^2 \to R$ are continuous, and $I = [0,\infty)$, $R = (-\infty, \infty)$, and $R_+ = (0,\infty)$. We also assume that (H1) there is a continuous function $q : I \to I$ such that $-q(t) \leqq \varphi(t,x,y)$ for all $(t,x,y) \in I \times R^2$. (H2) $F(x) = \int_0^x f(s)\,ds \geqq 0$ for all $x \in R$. (H3) $G(y) = \int_0^y [s/g(s)]\,ds$, $\lim_{|y|\to\infty} G(y) = \infty$, and there is a positive constant M such that $y^2/g(y) \leqq MG(y)$ for all $|y| \geqq 1$. (H4) There are continuous functions $e_k : I \to I$, $k = 1, 2$, such that, for all $(t,x,y) \in I \times R^2$, $|h(t,x,y)| \leqq e_1(t) + e_2(t)|y|$. Theorem. Suppose that (H1)-(H4) hold. If $p \in CBV_{loc}(I)$, then each solution u of (*) is continuable to the right of its initial t-value $t_0 \in I$. (Received September 19, 1973.)

709-B11. K. MICHAEL DAY, University of Michigan, Ann Arbor, Michigan 48104. Toeplitz matrices generated by the Laurent expansion of an arbitrary rational function.

Let $f(z) = \sum_{-\infty}^{\infty} a_m z^m$ be the Laurent expansion of an arbitrary rational function. Define matrices $T_n(f)$ where $T_n(f) = (a_{i-j})$, $i, j = 0, \ldots, n$. Such matrices are called Toeplitz matrices and may be generated by functions which are not rational. Denote by σ_n the set of $n+1$ eigenvalues of $T_n(f)$, $\sigma_n = \{\lambda_{n0}, \lambda_{n1}, \ldots, \lambda_{nn}\}$. Let $B = \{\lambda : \lambda = \lim \lambda_m, \lambda_m \in \sigma_{i_m}\}$ where $i_1, i_2, \ldots$ is an increasing sequence of integers. A characterization of this set for complex valued functions was initiated in 1960 and is published for when f is a Laurent polynomial, $f(z) = \sum_{-k}^{h} a_m z^m$,

$h, k \geqq 1$ (see P. Schmidt and F. Spitzer, Math. Scand. 8(1960), 15-38). Let $D_n(f-\lambda) = \det(T_n(f-\lambda))$. Schmidt and Spitzer employed an identity of H. Widom's which up to a constant factor evaluate the $D_n(f-\lambda)$ when f is a Laurent polynomial. An identity is developed for $D_n(f-\lambda)$ for f an arbitrary rational function which, using the techniques of Schmidt and Spitzer, allows one to show that B is a point or consists of a finite number of nondegenerate analytic arcs. (Received September 21, 1973.)

*709-B12. DON B. HINTON, University of Tennessee, Knoxville, Tennessee 37916. Continuous spectra of an even order equation.

Let $\ell(y) = (1/w)\{(-1)^n(ry^{(n)})^{(n)} - qy\}$ where the coefficients w, r, and q are real continuous functions on $[a, \infty)$ with r and w positive. Associated with ℓ is the Hilbert space H of all complex-valued, measurable functions f satisfying $\int_a^\infty w|f|^2 dx < \infty$. The operator ℓ generates in H a certain minimal closed operator L_0 [cf. M. A. Naĭmark, "Linear differential operators, Part II: Linear differential operators in Hilbert space", Ungar, New York, 1968]. Denote by $C(L_0)$ the continuous spectrum of L_0. Theorem. If the functions w, r, and q are positive and twice continuously differentiable, then $C(L_0) = (-\infty, \infty)$ if the following conditions hold. (i) $w/q \to 0$ as $x \to \infty$, (ii) $(r/q)^{1/2n}(q/w)[|q'|/q+|r'|/r + |w'|/w] = O(1)$ as $x \to \infty$, (iii) $\int_a^\infty (r/q)^{1/2n}[(q'/q)^2 + (w'/q)^2 + (r'/r)^2 + |r''|/r + |q''|/q + |w''|/q]\, dx < \infty$. Sufficient conditions are also given for $C(L_0) = [0,\infty)$. (Received September 10, 1973.)

*709-B13. ROBERT J. KNOWLES, University of Connecticut, Waterbury, Connecticut 06710 and THURLOW A. COOK, University of Massachusetts, Amherst, Massachusetts 01002. Results on Auerbach bases for finite-dimensional normed spaces.

Let E be a finite-dimensional normed space. An Auerbach basis for E is a basis $\{x_1, \ldots, x_n\}$ such that the vectors of the basis together with the functionals of the dual basis all have norm one. The classical result is "Every finite-dimensional Banach space has such a basis" (H. Auerbach, 1932). One proof, given by A. F. Ruston (1962), proceeds by maximizing a certain determinant. The present paper shows that that approach is related to the problem of constructing a parallelepiped of minimal volume circumscribing the unit ball of the dual space. It is also shown that the Gram-Schmidt property can be interpreted for Auerbach bases, and that a space which admits this generalized Gram-Schmidt property is an inner-product space if its dimension is three or greater. For Auerbach bases of two-dimensional spaces, two criteria are presented: one uses critical points of the determinant; the second, R. C. James's theory of orthogonality. The paper concludes with several open questions. (Received September 21, 1973.)

709-B14. CURTIS C. TRAVIS, University of Tennessee, Knoxville, Tennessee 37916. Oscillation of hyperbolic equations.

The equation (1) $u_{tt} = \sum_{i,j=1}^n D_i(a_{ij}(x,t)D_ju) - c(x,t)u$ is said to be oscillatory if all solutions which vanish on the lateral boundary of the cylinder $\Omega = \{(x,t) : 0 \leqq t < \infty,\ x \in G$ a bounded domain in $E_n\}$ must have arbitrarily large zeros in the interior of the cylinder. Sufficient conditions are given for the oscillation of equation (1). (Received September 21, 1973.)

709-B15. RONALD SHONKWILER, Georgia Institute of Technology, Atlanta, Georgia 30332. Generalized multi-parameter resolvents.

Let Q^j, $1 < j < m+n$, be $m+n$ commuting resolvents, $Q_\lambda = \lambda(I-\lambda A)$, on a Hilbert space H^+ corresponding to the $m+n$ selfadjoint A^j, the last n of which are also positive. Let $H \subset H^+$ be a subspace and P^+ be the orthogonal projection onto H. We call the composition $Q^+(\lambda^1, \ldots, \lambda^{m+n}) = Q^1_{\lambda^1} \cdots Q^{m+n}_{\lambda^{m+n}}$ an (m, n)-parameter resolvent and $Q(\lambda) = P^+Q^+(\lambda)|_H$ a generalized one. We characterize the latter intrinsically by its values on a restricted subset of its λ^j domain. Let $S(u) = Q(iu^1, \ldots, u^{m+n})$ where $u \in \Gamma = R^m X(-\infty, 0]^n$. Then S has weakly continuous partial derivatives up to $(m+n)$th order, and for each subset $\{j_1, \ldots, j_p\} \subset \{1, \ldots, m+n\}$, the joint limit of $S(u)/(u^{j_1} \ldots u^{j_p})$ exists in the strong operator topology as the u^j's tend to zero. In particular $S(u)/\pi u \to i^m I$ where $\pi u = u^1 \ldots u^{m+n}$. We may therefore define a kernel K on Γ by $K(u_2, u_1) = \Delta_{u_1}^{u_2^*} S/i^m \pi(u_2^* - u_1)$ if no denominator vanishes, and by the appropriate joint limit otherwise. Here $\Delta_{w_1}^{w_2}$ is the difference $\Delta_{w_1}^{w_2} S = \sum(-1)^{\sum j} pS(w_{j_1}^1, \ldots, w_{j_{m+n}}^{m+n})$ with the summation taken over all j's set to 1 or 2 and $(u^1, \ldots, u^{m+n})^* = (-u^1, \ldots, -u^m, u^{m+1}, \ldots, u^{m+n})$. With S as above, K is of positive type $\sum\sum(K(u_p, u_q)x_q, x_p) \geqq 0$ for all finite choices $u_p \in \Gamma$ and $x_p \in H$. Conversely these conditions insure that S is the restriction of a generalized (m,n)-parameter resolvent. (Received September 24, 1973.)

*709-B16. R. E. WHITE, North Carolina State University, Raleigh, North Carolina 27607. A characterization of hypoelliptic differential operators with variable coefficients. Preliminary report.

Let P be a linear differential operator with coefficients in $C^\infty(\Omega)$ where $\Omega \subset \mathbb{R}^n$. We characterize the hypoelliptic operators on Ω in terms of $*$-hypoelliptic operators. P is $*$-hypoelliptic on Ω if and only if $u \in \mathcal{D}'_F(\Omega)$ and $Pu \in C^\infty(\Omega)$ imply $u \in C^\infty(\Omega)$. We prove P is $*$-hypoelliptic on Ω if and only if (i) for all $s \in \mathbb{R}, n, K \subset \Omega$ compact there exists $C = C(s,K,n) > 0$ and m such that $\|u\|_n \leqq C(\|Pu\|_m + \|u\|_{s-1})$, where $u \in \{u \in H^c_{s-1}(K) \mid Pu \in C_c^\infty(K)\}$, and (ii) for all $s \in \mathbb{R}$, φ_n and $\varphi \in C_c^\infty(\Omega)$, there exists φ_m and $C = C(s, \varphi_n, \varphi) > 0$ such that $\|\varphi_n[P, \varphi]u\|_n \leqq C(\|\varphi_m Pu\|_m + \|\varphi_m u\|_{s-1})$, where $u \in \{u \in H^{loc}_{s-1}(\Omega) \mid Pu \in C^\infty(\Omega)\}$ and $\varphi_n \in C_c^\infty(\Omega)$ such that support $\varphi_n \subset$ support φ_{n+1}, $\varphi_{n+1} \equiv 1$ on the support φ_n and $\bigcup_n$ support $\varphi_n = \Omega$. We prove p is hypoelliptic on Ω if and only if for $u \in \mathcal{D}'(\Omega)$ and $Pu \in C^\infty(\Omega')$ with $\Omega' \subset \Omega$, there exists for each $x_0 \in \Omega'$ a relatively compact open neighborhood Ω_{x_0} of x_0 such that P is $*$-hypoelliptic on $\Omega_{x_0} \subset \Omega'$. (Received September 24, 1973.) (Author introduced by Professor Y. W. Chen.)

*709-B17. EUTIQUIO C. YOUNG, Florida State University, Tallahassee, Florida 32306. Uniqueness theorems for a singular ultrahyperbolic equation. Preliminary report.

It is well known that a solution in a bounded region R of the hyperbolic partial differential equation $\Box u \equiv u_{tt} - \Delta u = 0$ is unique if R is bounded by a characteristic cone and by a plane t = const., and if the solution and its normal derivative assume prescribed values on the plane boundary of R. The theorem is proved by integrating the expression $2u_t \Box u$ over R. The paper presents analogues of this classical integral procedure to establish certain uniqueness theorems for the singular ultrahyperbolic equation $\sum_{i=1}^m (u_{x_i x_i} + \alpha_i u_{x_i}/x_i) - \sum_{j,k=1}^n (a_{jk} u_{y_j})_{y_k} + cu = 0$. The theorems are applicable to mixed problems having boundary conditions of the first and the second kind. (Received September 24, 1973.)

709-B18. DONALD E. RYAN, Northwestern State University of Louisiana, Natchitoches, Louisiana 71457. Concerning some subsets of functions analytic on the unit disc. Preliminary report.

A function f is said to be of finite span on [a,b] iff $\sup_n |n\int_{a_1}^{b_1} f(x)e^{inx}dx| < \infty$, $[a_1, b_1] \subseteq [a,b]$. Let $f \in F$ iff f is analytic in $|z| < 1$ and continuous and of finite span on $|z| = 1$. Using a suitable norm and convolution, $B = (F, +, *, \|\ \|)$ is a Banach algebra. Theorem. If $I_p = \{f \mid f \in F, f^{(p)}(0) = 0\}$ then I_p is a maximal modular ideal in B. Theorem. If $g \in F$ and for every integer $n\int_0^{2\pi} g(z)e^{-inz}dz \neq 2\pi$, then there exists a function $f \in F$ such that $2\pi(f+g) = \int_0^{2\pi} g(x-y)f(y)\,dy$ and g is quasi-regular. (Received September 25, 1973.)

*709-B19. THOMAS G. HALLAM, Florida State University, Tallahassee, Florida 32306. An analysis of some third order initial value problems arising in magneto hydrodynamics.

This discussion surveys recent results, obtained jointly with J. W. Heidel and D. Loper, on some third order nonlinear initial value problems that arise in an investigation of a nonlinear, resistive, hydromagnetic boundary layer in a rotating electrically conducting fluid. The significant differential equation $2s''' + 6ss'' + (s')^2 + 4s^2s' = 0$

is obtained through simplification and transformation from the Navier-Stokes equations coupled with Maxwell's equation. In the case that the boundary is a rigid electrically insulating plate, the relevant initial data are $s(0) = s''(0) = 0$, $s'(0) = \pm 2$. A geometric description of the solutions of these initial value problems is given. Other initial value problems which we can qualitatively solve are indicated. An open related problem occurring when the boundary is an electrically conducting plate is mentioned. (Received September 24, 1973.)

709-B20. M. ZUHAIR NASHED, Georgia Institute of Technology, Atlanta, Georgia 30332. Some minimax problems arising from systems.

The main purpose of this talk is to discuss certain minimax problems associated with certain physical systems and allocation problems, and to demonstrate an interplay between these problems and dual extremum principles. (Received September 27, 1973.)

*709-B21. CURTIS CLYDE TRAVIS, University of Tennessee, Knoxville, Tennessee 37916 and GLENN F. WEBB, University of Kentucky, Lexington, Kentucky 40506. Existence and stability for partial functional differential equations.

The existence and stability properties of a class of partial functional differential equations are investigated. The problem is formulated as an abstract ordinary functional differential equation of the form $(d/dt)u(t) = Au(t) + F(u_t)$, $t \geqq 0$, $u_0 = \varphi \in C$, where $u: [-r,\infty] \to X$, $u_t \in C$ is defined by $u_t(\theta) = u(t+\theta)$, $\theta \in [-r,0]$, F is Lipschitz from C to X, A is the infinitesimal generator of a strongly continuous semigroup of linear operators $T(t)$, $t \geqq 0$, on X, X is a Banach space, and $C = C([-r, 0]; X)$. The solutions are studied as a semigroup of linear or nonlinear operators on C with infinitesimal generator $A_u\varphi = \varphi'$, $D(A_u) = \{\varphi \in C: \varphi' \in C, \varphi'^{-}(0) = A\varphi(0) + F(\varphi)\}$. In the case that F is linear, stability results are obtained by analyzing the spectral properties of A_u. In the case that F is nonlinear with Lipschitz constant L and $|T(t)| \leqq e^{wt}$, then the asymptotic stability of the solutions is demonstrated when $w + L < 0$. (Received September 27, 1973.)

709-B22. PETER D. JOHNSON, JR., Emory University, Atlanta, Georgia 30322. Approximation numbers of diagonal maps from ℓ^p to ℓ^q, $1 \leqq p < q \leqq \infty$. Preliminary report.

E and F denote Banach spaces. For a nonnegative integer n and $T \in L(E,F)$, $\alpha_n(T) = \inf\{\|T - A\|; A \in L(E,F)$ and rank $A \leqq n\}$, $\beta_n(T) = \inf\{\|T|_H\|; H$ is a subspace of E of codimension $\leqq n\}$, and $\delta_n(T) = \inf\{\|Q_L \cdot T\|; L$ is a subspace of F of dimension $\leqq n\}$, with Q_L denoting the usual map of F onto F/L. The problem undertaken is to determine $\alpha_n(T)$, $\beta_n(T)$, and $\delta_n(T)$ when T is a diagonal map from ℓ^p to ℓ^q, or from ℓ^p to c_0, $1 \leqq p < q \leqq \infty$, as explicit functions of the diagonal sequence. The statements of the results obtained are too long to be given here, because of the necessity of considering different cases. The approximation numbers are determined exactly when the diagonal sequence (t_j) satisfies $|t_j| \leqq 1$ for all j, and $\lim_j \sup|t_j| = 1$, and estimated in the general case. The results obtained are sufficient to explode certain current conjectures, such as that $\delta_n(T) = \beta_n(T')$, or that $\alpha_n(T) = \alpha_n(T')$, for a continuous linear map T between normed spaces with adjoint T'. (Received September 26, 1973.)

709-B23. RICHARD D. CARMICHAEL, Wake Forest University, Winston-Salem, North Carolina 27109. Distributions of exponential growth and their Fourier transforms.

Let H denote the set of all C^∞ functions $\varphi(t)$, $t \in \mathbb{R}^n$, such that $(\exp(k|t|)\, D^\alpha\varphi(t))$ is bounded in $\mathbb{R}^n$ for each $k = 0,1,2,\ldots$ and for any n-tuple α of nonnegative integers. The space of generalized functions defined on H is the space of distributions of exponential growth Λ_∞, which was first introduced by Sebastião E. Silva [Math. Ann. 136(1958), 58-96]. Let C be an open convex cone. In this paper we show that elements of a certain set of functions which are analytic in $T^C = \mathbb{R}^n + iC$ obtain distributional boundary values in the Fourier transform space of Λ_∞ on the distinguished boundary of T^C, and we obtain representation results for these functions in terms of elements in Λ_∞. (Received September 26, 1973.)

*709-B24. SOO BONG CHAE, New College, Sarasota, Florida 33578 and VINCENT C. PECK, University of Rochester, Rochester, New York 14627. A generalization of the Steinhaus-Kemperman theorem.

In the following, let A and B be two subsets of $\mathbb{R}$ of positive Lebesgue measure and let A^* be the set of all points in A at which A is of unit density. It is well known that $m(A) = m(A^*)$. Let $A + B$ denote the set of all points $a + b$, where $a \in A$ and $b \in B$. The Steinhaus-Kemperman theorem states that $A^* + B^*$ is open (Cf. Trans. Amer. Math. Soc. 86(1957), 31). In other words, it is possible to delete sets of measure zero from A and B in such a way that $A + B$ becomes open. We announce the following generalization of the preceding theorem. Theorem. Let $f: \mathbb{R}^2 \to \mathbb{R}$ be continuously differentiable on some open subset of $\mathbb{R}^2$, and be such that the partial derivatives are nonzero on U. If A and B are subsets of $\mathbb{R}$ of positive measure such that $A \times B$ is contained in U, then $f(A^* \times B^*)$ is open. (Received September 26, 1973.)

*709-B25. TIEN-YIEN LI, University of Maryland, College Park, Maryland 20742. Existence of solutions for ordinary differential equations in Banach spaces. Preliminary report.

Let E be a real Banach space. Consider the differential equation (E) $x' = f(t,x)$, $x(0) = x_0$ where $f: R^1 \times E \to E$. It has been shown that continuity of the right-hand side is insufficient for existence of solutions of (E). There are two directions in studying the existence theorems. One direction was aimed to find compactness conditions guaranteeing the existence of solutions as in the works of Corduneanu, Ambrosetti, Szufla, Cellina, etc. The other direction was aimed to show the existence of solutions under an accretive type condition as the works of Martin, Crandall, etc. The purpose of this paper is to unify both directions by letting f satisfy some α-Lipschitz type conditions, where α is Kuratowski's set function measuring noncompactness. The results presented unify and generalize the works in the first direction and, when f is uniformly continuous, Martin's results in the second direction. (Received September 26, 1973.)

*709-B26. JAMES D. BUCKHOLTZ, University of Kentucky, Lexington, Kentucky 40506 and J. K. SHAW, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. A characterization of completely convex functions.

An infinitely differentiable function f defined on $[0,1]$ is said to be completely convex if $(-1)^n f^{(2n)}(x) \geqq 0$, $0 \leqq x \leqq 1$, $n = 0, 1, 2, \ldots$. Three end-point conditions involving the sequences $\{f^{(2n)}(1)\}_0^\infty$, $\{f^{(2n)}(0)\}_0^\infty$ and $\{f^{(2n+1)}(0)\}_0^\infty$ are shown to be necessary and sufficient for f to be completely convex. Theorem. A function $f \in C^\infty[0,1]$ is completely convex if and only if (i) $(-1)^n f^{(2n)}(1) \geqq 0$ and $(-1)^n f^{(2n)}(0) \geqq 0$, $n = 0, 1, 2, \ldots$; (ii) the series $\sum_{n=0}^\infty (-1)^n \{f^{(2n)}(1) + f^{(2n)}(0)\} \pi^{-2n}$ converges; and (iii) $\lim_{n\to\infty} (-1)^n f^{(2n+1)}(0) \pi^{-2n-1}$ exists and is nonnegative. The proof relies on Widder's characterization of minimal completely convex functions [Trans. Amer. Math. Soc. 51(1942), 387-398], Boas' extension of Widder's result to completely convex functions [Amer. J. Math. 81(1959), 709-714], and some recent results concerning Lidstone series obtained by the present authors [J. Math. Anal. Appl., to appear]. (Received September 26, 1973.)

*709-B27. THOMAS G. PROCTOR, Clemson University, Clemson, South Carolina 29631. Bounds for solutions of perturbed differential equations.

A modified form of the Aleekseev variation of constants equation is used to relate solutions of the two systems $x' = f(t,x,\lambda)$ and $y' = f(t,y,\psi(t)) + g(t,y)$. Hypotheses are placed on these systems so that bounds for one system can be calculated for the solutions of the other. Several results are given including Theorem. Let n be a positive integer, let $A(t)$ be a C' $n \times n$ matrix for $t \geqq 0$ such that for some $\alpha > 0$, $M \geqq 1$, $|e^{A(\lambda)t}| \leqq Me^{-\alpha t}$ for $t \geqq 0$, $\lambda \geqq 0$. If $y(t)$ is a solution of $y' = A(t)y$ then $|y(t)| \leqq \sigma(t)e^{-\alpha t}$, $t \geqq 0$, where σ is the solution of $\sigma'' - M^2|A'(t)|\sigma = 0$, $\sigma(0) = M|y(0)|$, $\sigma'(0) = 0$. (Received September 28, 1973.)

*709-B28. WILLIAM P. NOVINGER, Florida State University, Tallahassee, Florida 32306. Linear isometries of subspaces of spaces of continuous functions.

This is an extension of a result announced in Abstract 73T-B99, these *Notices* 20(1973), A-277.

Theorem. Let X and Y be respectively compact and locally compact Hausdorff spaces, and A be a linear subspace of C(X) which separates points and contains the constant functions. Suppose that T is a linear isometry of A into $C_0(Y)$, $B = T(A)$, and that Ch(A) and Ch(B) denote the Choquet boundary of A and B respectively. Then there are continuous functions e and h on Y such that $e: Ch(B) \to \{z : |z| = 1\}$, $h: Ch(B) \to Ch(A)$ is onto, and such that $Tf(y) = e(y)f(h(y))$ for all $f \in A$ and all $y \in Ch(B)$. (Received September 27, 1973.)

709-B29. W. ROBERT BOLAND, Clemson University, Clemson, South Carolina 29631 and PAUL NELSON, JR., Texas Tech University, Lubbock, Texas 79409. Ordinary differential equations in neutron transport theory.

A central problem associated with nuclear reactors is the (computational or experimental) determination of critical dimensions. For one-dimensional models the underlying equations are a linear first-order system of ordinary differential equations. The corresponding critical length is determined by the smallest interval over which a certain associated boundary-value problem of the second kind fails to have a unique solution. There has been considerable good computational experience in determining this length by numerically integrating the associated matrix Riccati equation to machine overflow. Nonetheless this method does not seem to be popular, presumably, at least partially, because of the lack of a supporting theory to show that assorted conceivable difficulties cannot arise. In this paper, under appropriate hypotheses, estimates are presented which constitute such a supporting theory. Illustrative numerical examples are also presented. (Received September 27, 1973.)

709-B30. WILLIAM P. McKIBBEN, Georgia Institute of Technology, Atlanta, Georgia 30332. Distributions for orthogonal polynomials whose recurrence is almost uniform. Preliminary report.

If a_n $(\neq 0)$, b_n, c_n $(n \geqq 0)$ are real and $c_n/a_n a_{n-1} > 0$ $(n \geqq 1)$, the recurrence relation $P_{n+1}(x) = (a_n x + b_n) P_n(x) - c_n P_{n-1}(x)$ $(n \geqq 0)$ (where $P_{-1}(x) = 0$, $P_0(x) = 1$) is known to generate a sequence of orthogonal polynomials; i.e., there is a real-valued, bounded, nondecreasing function α with infinitely many points of increase such that $\int_{-\infty}^{\infty} P_n P_m \, d\alpha = \delta_{mn} \Gamma_n$ $(\Gamma_n > 0$ for $n \geqq 0)$. Classical methods are used to justify a procedure for constructing α in case all but finitely many of the a_n are the same, all but finitely many of the b_n are the same, and all but finitely many of the c_n are the same. α is represented as $\alpha_1 + \alpha_2$, where α_1 is absolutely continuous and α_2 is a step function. $d\alpha_1$ has bounded support and is given by $[w(x)/S(x)]\,dx$, where $S(x)$ is a polynomial determined from the representation of $P_n(x)$ as a linear combination of elements of a sequence of classical orthogonal polynomials, and $w(x)$ is the weight function for this classical sequence. The discontinuities of α_2 may occur only outside the support of $d\alpha_1$ and are also determined from this representation. (Received September 27, 1973.)

*709-B31. SEATON D. PURDOM, Georgia Institute of Technology, Atlanta, Georgia 30332. Evolution system approximations of solutions to closed linear operator equations.

With S a linearly ordered set with the least upper bound property, with g a nonincreasing real-valued function on S, and with A a densely defined dissipative linear operator, an evolution system M is developed to solve the modified Stieltjes integral equation $M(s,t)x = x + A((L)\int_s^t dg\, M(\cdot, t)x)$. An affine version of this equation is also considered. Under the hypothesis that the evolution system associated with the linear equation is strongly (resp. weakly) asymptotically convergent, an evolution system is used to strongly (resp. weakly) approximate solutions to the closed operator equation $Ay = -z$. (Received September 27, 1973.)

*709-B32. GARY D. JONES, Murray State University, Murray, Kentucky 42071. Oscillation criteria for third order differential equations.

Several sufficient conditions for the differential equation (1) $(ry'')' + py' + qy = 0$ to have oscillatory solutions are given. Some typical theorems are as follows. Theorem 1. If $r' \geqq 0$, $2q - p' \geqq 0$, and $q - p' > 0$ with $r' + (2q - p') = 0$ only at isolated points, (1) is oscillatory if $(ry')' + (p/4r)y = 0$ is oscillatory. Theorem 2. If $r' \geqq 0$, $2q - p' \geqq 0$, $p \geqq 0$, and $q \geqq 0$ with zeros of $r' + 2q - p'$, p, and q possible only at isolated points, then (1) is oscillatory if $(ry')' + (p + m \times q)y = 0$ is oscillatory for some m, $0 < m < \frac{1}{2}$. Theorem 3. If $r \equiv 1$, $q > 0$, $p < B$ for some real number B and (1) is C_I, then (1) is oscillatory if $\int_N q - (2(-p)^{3/2}/3\sqrt{3}) + \int_{-N} q = +\infty$ where $N = \{x : p(x) \leqq 0\}$. (Received September 28, 1973.)

*709-B33. FRANCIS A. ROACH, University of Houston, Houston, Texas 77004. Convergence sets and value regions for continued fractions. Preliminary report.

Suppose that E is a subset of the complex plane. Denote by V_E the set of all complex numbers of the form $\underline{1}/\overline{b}_1 + \underline{1}/\overline{b}_2 + \ldots + \underline{1}/\overline{b}_n$ where each b_p lies in E. The set E is said to be a convergence set provided that for every sequence $b_1, b_2, b_3, \ldots$, with each term in E, $\underline{1}/\overline{b}_1 + \underline{1}/\overline{b}_2 + \underline{1}/\overline{b}_3 + \ldots$ converges. Theorem 1. If V_E is bounded and -1 is not a limit point of the set of all products xy where both x and y are in V_E, then E is a convergence set. Theorem 2. If E is a convergence set and is closed, then V_E is bounded. Remark. In view of these results and certain examples of convergence sets, one might conjecture that E is a convergence set if and only if V_E is bounded. (Received October 1, 1973.)

*709-B34. GLADYS HAYES CRATES and JOHN W. JAYNE, University of Tennessee, Chattanooga, Tennessee 37401. Zeros of some recursively generated polynomials.

A sequence $\{\varphi_n\}$ of monic polynomials generated by a recurrence relation of the form $\varphi_0 = 1$, $\varphi_1 = x + b_0$, $\varphi_{n+1} = (x + b_n)\varphi_n - c_n\varphi_{n-1}$, $n \geqq 1$, where b_n is real and $c_n > 0$, is orthogonal with respect to some distribution on the real line. The smallest interval containing all the zeros of every φ_n is called the true interval of orthogonality. If $c_n \not> 0$, the zeros are not necessarily real and the polynomials do not form a sequence orthogonal on the real line. The proofs of the following are based on a theorem of Ky Fan [Duke Math. J. 25(1958), 441-445]. Theorem 1. Let the sequences of polynomials $\{Q_n\}$, $\{\varphi_n\}$, $\{P_n\}$ be generated respectively by (i) $Q_0 = 1$, $Q_1 = z$, $Q_{n+1} = zQ_n - \gamma_n Q_{n-1}$; (ii) $\varphi_0 = 1$, $\varphi_1 = z$, $\varphi_{n+1} = z\varphi_n - c_n\varphi_{n-1}$; (iii) $P_0 = 1$, $P_1 = z$, $P_{n+1} = zP_n - \delta_n P_{n-1}$, $n \geqq 1$. Suppose the sequence $\{Q_n\}$ is orthogonal on the true interval of orthogonality $(-a, a)$ and that all the zeros of each P_n lie in $(-a, a)$. If $\gamma_n \leqq c_n \leqq \delta_n$ for $n \geqq 1$, then all the zeros of each φ_n lie in $(-a, a)$, and this is the smallest such interval for $\{\varphi_n\}$ as well as $\{P_n\}$. Theorem 2. In the recurrences (ii) and (iii) of Theorem 1, suppose $|c_n| \leqq \delta_n$, $n \geqq 1$, c_n complex; then all the zeros of each φ_n lie in the disk $|z| < a$. (Received October 1, 1973.)

709-B35. MARVIN B. SLEDD, Georgia Institute of Technology, Atlanta, Georgia 30332. Initial-value problems for some infinite systems of ordinary differential equations. Preliminary report.

Initial-value problems are considered for countably infinite systems of ordinary differential equations which are the equations of one-dimensional motion of infinite or half-infinite chains of harmonic oscillators. Arbitrary initial values and arbitrary initial first derivatives may be prescribed for any finite number of the functions to be determined, and any finite number of the equations may be nonhomogeneous owing to the presence of forcing functions. Integral representations of solutions are given in terms of orthogonal polynomials generated by three-term recurrence relations in which the coefficients are determined by the physical parameters of the chains. (Received October 1, 1973.)

*709-B36. THURLOW A. COOK, University of Massachusetts, Amherst, Massachusetts 01002. Absolute Schauder bases in $C(X)$.

Let X be a locally compact Hausdorff space, $C(X)$ the algebra of all real valued continuous functions on X, and let $C(X)$ have the compact-open topology. If $(x_n; f_n)$ is a Schauder basis for a locally convex space E, then the basis is called an absolute basis provided: for each x in E, $\sum_{n=1}^{\infty} |f_n(x)| \rho(x_n) < \infty$ for each continuous seminorm ρ on E. Theorem. If $C(X)$ has an absolute Schauder basis $(x_n; \mu_n)$ with (x_n) converging weakly to zero, then $C(X)$ is isomorphic to the countable product of real lines. (Received October 1, 1973.)

*709-B37. STEVEN C. TEFTELLER, University of Alabama, Birmingham, Alabama 35294. Oscillation of second order nonhomogeneous linear differential equations.

This paper is concerned with second order nonhomogeneous linear differential equations of the form (1) $(r(x)y')' + q(x)y = F(x)$, together with the associated homogeneous equations (2) $(r(x)u')' + q(x)u = 0$, where r, q, F are in $C[a,\infty)$. Conditions are provided for (1) to be oscillatory on the half axis $[a,\infty)$, if and only if (2) is oscillatory on $[a,\infty)$. In addition, separation theorems for equations (1) and (2) are established. These results extend those of L. P. Burton [Pacific J. Math. 2(1952), 281-289], M. S. Keener [Applicable Anal. 1(1971), 57-63], and M. E. Hammett [Ph.D. Dissertation, Auburn University, 1967]. (Received October 1, 1973.)

*709-B38. JOHN R. QUINE, JR., Florida State University, Tallahassee, Florida 32306. On univalent polynomials.

We define $V_n \subseteq \mathbb{C}^{n-1}$ to be the set of $(n-1)$-tuples $(a_2,\ldots,a_n)$ such that the polynomial $p(z) = z + a_2z^2 + \ldots + a_nz^n$ is univalent, i.e., one-to-one in $|z| < 1$. In this paper we construct a real polynomial h of degree $4(2(n-1)^2-1)(n-1)$ such that if $(a_2,\ldots,a_n)$ is in the boundary of V_n then $h(\mathrm{Re}\, a_2, \mathrm{Im}\, a_2, \ldots, \mathrm{Re}\, a_n, \mathrm{Im}\, a_n) = 0$. This shows that the boundary of V_n is a subset of an algebraic submanifold of $R^{2(n-1)}$. (Received October 1, 1973.)

*709-B39. JAMES A. RENEKE, Clemson University, Clemson, South Carolina 29631. The existence of realizations of hereditary systems.

For simplicity, we will understand an hereditary system to be a linear space G of functions from $S' = (-\infty, \infty)$ into a linear space X together with a function W from $S \times S$, where $S = [0,\infty)$, into the class of functions from G into G. W is called a transition function for the system provided $[W(v,u)f](x) = f(x)$, for each f in G, subinterval $[u,v]$ of S, and x in $(-\infty,u]$; $W(v,x)W(x,u) = W(v,u)$ for $0 \leqq u \leqq x \leqq v$; and if f is in G, h is a function from S' into X, $h(x) = f(x)$ for x in $(-\infty,0]$, and $h(x) = [W(x,0)f](x)$ for x in S, then h is in G. The problem of realization is to find an "integral" K defined on G such that f and h in G are related as above if and only if $h(x) = f(x)$ for x in $(-\infty,0]$ and $h(x) = f(x) + [K(0,x)h](x)$ for x in S. The problem of representation is to represent K in terms of familiar integrals. For systems which satisfy a Lipschitz condition, the realization problem is solved and, in some special cases, a representation is obtained for the integral. (Received October 1, 1973.)

*709-B40. RAYMOND H. ROLWING, University of Cincinnati, Cincinnati, Ohio 45221. Explicit solutions of a certain homogeneous autonomous system. Preliminary report.

We consider the system $X_i' = X_i^3 - \sum_{j=1}^{3} X_j$, $i = 1, 2, 3$. Two different substitutions are presented to find explicit solutions. The system is reduced to the single DE: $-3uu'' + 5u'^2 + 5u^3u' - u^6 + a^2u^4 = 0$, $a^2 = 27/c$ where a unique substitution $u' = p = a^3(1+\lambda\delta)\delta^{-3}$ is used to obtain the equation $3(1+\lambda\delta)d\lambda + (1-\lambda^2)d\delta = 0$. (Received October 1, 1973.)

*709-B41. COKE S. REED, Auburn University, Auburn, Alabama 36830. Weighted Franklin series. Preliminary report.

Theorem. If g is a nonconstant continuous function on $[0,1]$, then there exists a reversible number sequence $(a_1, a_2, \ldots)$ whose terms all lie in $[0,1]$ and are dense in $[0,1]$, and a strictly increasing, infinitely differentiable function W on $[0,1]$, such that if for each positive integer n, $f_n(x) = 0$ for $0 \leqq x \leqq a_n$ and $f_n(x) = x - a_n$ for $a_n \leqq x \leqq 1$ and $f_0(x) = 0$ for $0 \leqq x \leqq 1$, and $\varphi_0, \varphi_1, \varphi_2, \ldots$ is the orthonormal function sequence obtained from $f_0, f_1, \ldots$ using the Gram Schmidt process where the inner product $((h,k)) = \int_0^1 hk\,dW$, then the number sequence $\alpha_p = \sum_{i=1}^p ((\varphi_i, g))\,\varphi_i(0)$ is unbounded. (Received October 1, 1973.) (Author introduced by Professor Jack W. Rogers, Jr.)

Applied Mathematics

*709-C1. STEPAN KARAMARDIAN, University of California, Irvine, California 92664 and Clemson University, Clemson, South Carolina 29631. Complementarity problems over cones with monotone and pseudo-monotone maps. Preliminary report.

Let C be a convex closed cone in E^n with nonempty interior, C^* its polar cone, and $\geqq^C$, $\geqq^{C^*}$ the partial orderings generated by C and C^* respectively. Let F be a map from C into E^n. Consider the complementarity problem: (1) $x \geqq^C 0$, $F(x) \geqq^{C^*} 0$, $(x, F(x)) = 0$, or equivalently the variational inequality $(x-u, F(u)) \geqq 0\ \forall x \in C$. Theorem. Let F be continuous on C: (i) If F is pseudo-monotone and there exists $\bar{x} \geqq^C 0$ with $F(\bar{x}) >^{C^*} 0$, then there exists a solution to (1). (ii) If F is monotone and there exists $\bar{x} \geqq^C 0$ with $F(\bar{x}) \geqq^{C^*} 0$ and the set $S = \{x | x \geqq^C \bar{x},\ F(x) \geqq^{C^*} F(\bar{x}),\ (x-\bar{x})(F(x) - F(\bar{x})) = 0\}$ is bounded, then there exists a solution to (1). These results are then applied to certain nonlinear programming problems. (Received September 17, 1973.)

709-C2. CHRISTOPHER HUNTER, Florida State University, Tallahassee, Florida 32306. Synthesis problems for self-consistent stellar systems.

Systems of large numbers of stars can be described by a distribution function $f(\vec{x}, \vec{v})$, where $f\,d\vec{x}\,d\vec{v}$ gives the total mass of stars with position vectors in the range $\vec{x}$ to $\vec{x} + d\vec{x}$, and velocity vectors in the range $\vec{v}$ to $\vec{v} + d\vec{v}$. When close encounters between individual stars can be ignored, each star can be regarded as moving in the smooth gravitational field due to the density (a) $\rho(\vec{x}) = \iiint f\,d\vec{v}$. The triple integration here is over all velocities represented at the space point $\vec{x}$. By Jeans' theorem, f must be a function of the integrals of the differential equations of motion, and only isolating or possibly quasi-isolating integrals can be used. Equation (a) is nonlinear in ρ, but it is a linear integral equation for f when a specific density ρ is chosen. So far, only spherical, axisymmetric and some special triaxial ellipsoidal systems have been discussed. These examples show that the integral equation for f may have either an infinity of solutions, a unique solution, or even no solution. Some detailed results for rotating ellipsoidal systems will be given. (Received September 24, 1973.) (Author introduced by Professor John W. Heidel.)

*709-C3. ALAN DAVID SLOAN, Georgia Institute of Technology, Atlanta, Georgia 30332. Support maximizing operators in quantum field theory.

Let T be an operator on an L^2 space. T maximizes support if T(f) differs from zero almost everywhere for any L^2 function f, except for f identically zero almost everywhere. Let H be any one of the Hamiltonians for the following quantum field theory models defined on a Fock space, F: (i) free boson; (ii) boson in linear external source; (iii) momentum cutoff polaron with fixed total momentum; (iv) polaron of fixed total momentum without cutoffs in two space dimensions; and (v) spatially cutoff boson field with real, bounded below, even ordered polynomial self-interaction in one dimension. Models (i)–(iii) are in arbitrary space dimensions. Theorem. There is an unitary operator U such that UF is an L^2 space and such that $Ue^{-tH}U^{-1}$ maximizes support. When H is the Hamiltonian in a Q.F.T. model, e^{-tH} is often positivity preserving in the sense that, when F is viewed as an L^2 space, $e^{-tH}(f)$ is nonnegative almost everywhere whenever f is nonnegative almost everywhere. If e^{-tH} is both positivity preserving and support maximizing, then any ground state of H is nondegenerate. (Received September 24, 1973.)

*709-C4. M. ZUHAIR NASHED, Georgia Institute of Technology, Atlanta, Georgia 30332. Regularization and numerical analysis of ill-posed operator equations.

• Let T be an operator from X into Y, where X and Y are normed linear spaces. The equation (1) $Tx = y$ is said to be well-posed (relative to X and Y) if for each $y \in Y$, (1) has a unique "solution" which depends continuously on y; otherwise the equation is said to be ill-posed. (The term "solution" includes the classical notion of a solution, pseudosolutions, constrained solutions, etc.) A regularization of an ill-posed problem (IPP) means, roughly speaking, an analysis (including numerical) of the problem via consideration of a well-posed problem (WPP) or a sequence of WPP which provides a suitable approximation to the given IPP. This suggests several approaches which are, generally speaking, based on (a) a change in the concept of a solution or the spaces in question; (b) a change in the operator itself; (c) concepts of regularization operators and algorithms; (d) statistical regularization or well-posed stochastic extension of IPP; (e) none of the above, but hard to classify in this short note. This paper examines from the viewpoint of both operator theory and numerical analysis some aspects of regularization and approximation of ill-posed operator equations. Our approach unifies several regularization methods proposed by various authors during the past two decades, and provides new simultaneous regularization and approximation schemes for such problems. Particular emphasis will be placed on (i) the role of reproducing kernel Hilbert spaces and approximation theory of generalized inverses of linear operators; (ii) convergence rates of discretization, moment-discretization, and projection methods for regularization problems; and (iii) the choice of the regularization parameters. Most of the treatment will be confined to linear equations in the context of Hilbert spaces, with particular reference to integral and operator equations of the first kind. Extensions to nonlinear problems, as well as more general spaces, will be indicated briefly. The paper concludes with certain aspects of IPP that remain an unexplored territory. (Received September 24, 1973.)

709-C5. JOHAN G. F. BELINFANTE, Georgia Institute of Technology, Atlanta, Georgia 30332. Subspaces of transition probability spaces. Preliminary report.

A transition probability space consists of a set S and a symmetric function $p: S \times S \to [0, 1]$ satisfying two axioms. The first axiom is that $p(x,y) = 1$ if and only if $x = y$. Call x and y orthogonal if $p(x,y) = 0$. The second axiom is that the least upper bound for all finite sums $p(x,y_1) + \ldots + p(x,y_n)$ where $y_1, \ldots, y_n$ belong to any maximal set of pairwise orthogonal elements of S is unity for all x. A subset T of S is a subspace if the restriction of p to $T \times T$ defines a transition probability on T. Theorems. Any set of pairwise orthogonal elements is a subspace. The orthogonal complement of a subspace is a subspace. A subset T is a subspace if and only if it is contained in the second orthogonal complement of any maximal set of pairwise orthogonal elements of T. Conjectures. The orthogonal complement of any subset is a subspace. The intersection of two subspaces is a subspace. (Received September 26, 1973.)

*709-C6. G. W. REDDIEN, JR., Vanderbilt University, Nashville, Tennessee 37235. Projection methods and singular two-point boundary value problems.

Collocation methods and the method of moments are studied as applied to boundary value problems of the form $-(p(x)u'(x))' + q(x)u(x) = f(x)$ for $0 < x < 1$ with $u(1) = 0$, $p(0) = 0$, and $p(x) > 0$ on $(0,1]$. A sample result and problem is the following. Let $p(x) = x^{1/2}$, let f and q be continuous, and add the boundary condition $u(0) = 0$. Suppose a solution u_0 exists and is unique. Let $\{\Delta_n\}$ be a sequence of partitions of $[0,1]$ with mesh norm $|\Delta_n| \to 0$. Approximate solutions $u_n \in C^2(0,1] \cap C[0,1]$ that on each subinterval have the form $a_0 + a_1x^{1/2} + a_2x^{3/2} + a_3x^{5/2}$ and satisfy both the boundary conditions and the differential equation at the mesh points are shown to exist and converge to u_0. If $(pu')' \in C^2[0,1]$, then $\|u_n - u_0\|_\infty = 0(|\Delta_n|^2)$. (Received September 27, 1973.)

709-C7. STEPHEN R. BERNFELD, Memphis State University, Memphis, Tennessee 38152, R. B. BECKMAN, and M. ZATZMAN, Department of Physiology, University of Missouri, Columbia, Missouri 65201. Modeling the efficiency of the kidney. Preliminary report.

The kidney is an extremely important excretory organ of the body, eliminating the nitrogeneous wastes of the body as well as salts and water. The formation of urine is due to essentially three processes: (1) the filtration of the blood through the glomerular capula; (2) the selective reabsorption of materials by the renal tubula, and (3) the secretion by the tubules of certain substances from the blood into the tubular lumen. We attempt here to model the functioning of the kidney, in particular, the three above-mentioned processes. By injecting various substances into the body, such as creatinine and inulin and taking blood samples, we are able to determine renal clearance. The model consists of a two-dimensional linear nonhomogeneous system of scalar ordinary differential equations. Using our model, we are thus able to predict the glomerular filtration rate as well as the functional efficiency of the tubules without taking urine samples (only blood samples) in a shorter period of time than is presently done clinically. (Received September 27, 1973.)

*709-C8. DON R. WILHELMSEN, University of Georgia, Athens, Georgia 30602. A Markov inequality in several dimensions.

A. Markov's inequality for polynomials of a single variable is well known. It states that $\max_{-1\leq x\leq 1}|p'(x)| \leq k^2$ whenever p is a polynomial of degree k or less and bounded in magnitude by 1. No such inequality is known in the general case for multivariate polynomials. In this paper it is shown that $\max_{t\in T}\|\nabla p(t)\| < 4k^2/\omega_T$ whenever T is a compact, convex subset of Euclidean n-dimensional space, ω_T is a parameter describing the "width" of T, and p is a multivariate polynomial of total degree k or less and bounded in magnitude by 1. The proof is obtained by a simple application of support hyperplanes and restriction mappings. Although the inequality is not sharp, a sharpness conjecture is made. (Received October 1, 1973.)

Logic and Foundations

*709-E1. RICHARD A. SANERIB, JR., Emory University, Atlanta, Georgia 30322. Elementary types of some groups related to infinite symmetric groups. Preliminary report.

Let $\mathfrak{m}$ and $\mathfrak{n}$ be infinite cardinal numbers, $\mathrm{Sym}(\mathfrak{m})$ the full symmetric group on $\mathfrak{m}$, and $\mathrm{Sym}(\mathfrak{m},\mathfrak{n}) = \{\pi \in \mathrm{Sym}(\mathfrak{m}): \mathrm{card}\{i \in \mathfrak{m}: \pi(i) \neq i\} < \mathfrak{n}\}$. For an ultrafilter F on an infinite set I, let $\mathrm{Aut}^*(F) = \{\pi \in \mathrm{Sym}(I) : \{i \in I : \pi(i) = i\} \in F\}$. Aut*(F) is isomorphic to the automorphism group of F where F is considered as a lattice under the natural operations. We say $G_1 \equiv_{ee} G_2$ if the groups G_1 and G_2 are elementarily equivalent. Theorem 1. If F_i is an ultrafilter on $\aleph_{\alpha_i}$ for each $i \in I$ and $\mathscr{D}$ is an ultrafilter on I, then $\mathrm{Prod}_{i\in I}\mathrm{Aut}^*(F_i)/\mathscr{D}$ is a maximal subgroup of $\mathrm{Prod}_{i\in I}\mathrm{Sym}(\aleph_{\alpha_i})/\mathscr{D}$. Theorem 2. Let F be a uniform ultrafilter on $\aleph_\alpha$ and let $\aleph_\beta > \aleph_\alpha$. Then $\mathrm{Aut}^*(F) \equiv_{ee} \mathrm{Sym}(\aleph_\beta, \aleph_{\alpha+1})$. Theorem 3. Let F_1 and F_2 be uniform ultrafilters on $\aleph_\alpha$ and $\aleph_\beta$ respectively. Then $\mathrm{Sym}(\aleph_\alpha) \equiv_{ee} \mathrm{Sym}(\aleph_\beta)$ if and only if $\mathrm{Aut}^*(F_1) \equiv_{ee} \mathrm{Aut}^*(F_2)$. Theorem 4. Let $\aleph_\beta \leq \aleph_\alpha \leq \mathfrak{m}^+$ and $\aleph_\delta \leq \aleph_\gamma \leq \mathfrak{n}^+$ be given. If $\mathrm{Sym}(\mathfrak{m},\aleph_\alpha)/\mathrm{Sym}(\mathfrak{m},\aleph_\beta) \equiv_{ee} \mathrm{Sym}(\mathfrak{n},\aleph_\gamma)/\mathrm{Sym}(\mathfrak{n},\aleph_\delta)$, then $\alpha \sim \beta \equiv_{ee} \gamma \sim \delta$. Theorem 5. Let $\mathfrak{m},\mathfrak{n}$ be infinite cardinals and α an ordinal less than $\omega\cdot\omega$ with $\aleph_\alpha \leq \mathfrak{m},\mathfrak{n}$. If $\mathrm{Sym}(\mathfrak{m}) \equiv_{ee} \mathrm{Sym}(\mathfrak{n})$ then $\mathrm{Sym}(\mathfrak{m})/\mathrm{Sym}(\mathfrak{m},\aleph_\alpha) \equiv_{ee} \mathrm{Sym}(\mathfrak{n})/\mathrm{Sym}(\mathfrak{n},\aleph_\alpha)$. The above results are all obtained via ultraproducts. (Received September 24, 1973.)

Statistics and Probability

709-F1. DHANDAPANI KANNAN, University of Georgia, Athens, Georgia 30602. Potential operators for perturbed Markov processes. Preliminary report.

Let $\{S_t, t \geqq 0\}$ be a semigroup of operators on a Banach space $\mathcal{X}$ and let $\{V_\lambda, \lambda > \lambda_0 \geqq 0\}$ be its resolvent family. The potential operator V of $\{S_t\}$ is defined by $Vf = s - \lim_{\lambda \to \lambda_0} V_\lambda f$. (Here the strong limit is assumed to exist on a dense subspace of $\mathcal{X}$.) Let A be the infinitesimal generator of a strong Markov process $\{X_t\}$, and let B be a suitable closed operator. Then, as is well known, A + B generates a strong Markov process $\{Y_t\}$ with creation and death points. We study the perturbed process. Some of the results are: X_t has a potential operator iff Y_t has a potential operator. If X_t is recurrent, then so is Y_t. Under suitable restrictions, Y_t satisfies the domination principle, and Y_t is transient iff X is transient. An explicit formula for the potential operator of Y_t is obtained. (Received September 28, 1973.)

Topology

709-G1. NORMA F. LAUER, Auburn University, Auburn, Alabama 36830. Countable dense homogeneity of products of universal curves and manifolds.

The late Professor Ralph Bennett defined a countable dense homogeneous space to be a separable space X which has the following property. If M and N are two countable dense subsets of X, there is a homeomorphism h of X onto X such that h(M) = N [Fund. Math. 74(1972), 189-194]. He proved that the universal curve is countable dense homogeneous, and raised the question as to whether the product of a finite number of simple closed curves and universal curves is countable dense homogeneous. Theorem 1. If $X = X_1 \times X_2$ where X_1 is the product of finitely many universal curves and X_2 is a compact connected manifold without boundary, then X is countable dense homogeneous. Theorem 2. If X is the product of countably many universal curves then X is countable dense homogeneous. Theorem 3. If $X = X_1 \times X_2$ where X_1 is the product of finitely many universal curves and X_2 is the product of countably many connected compact manifolds with boundary, then X is countable dense homogeneous. (Received March 14, 1973.)

*709-G2. PETER FLETCHER, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061 and WILLIAM F. LINDGREN, Slippery Rock State College, Slippery Rock, Pennsylvania 16057. Generalizations of γ-spaces.

The characterizations of a γ-space given by the authors in a previous abstract suggest the following generalizations. Let (X, τ) be a T_1 topological space and let $\langle V_n \rangle$ be a sequence of relations on X such that for each positive integer n and each x in X, $x \in V_{n+1}(x) \subset V_n(x)$. The following conditions on $\langle V_n \rangle$ are investigated. (A) For each x in X, $\bigcap_{n=1}^{\infty} \overline{V_n^2(x)} = \{x\}$; (B) For each x in X, $\bigcap_{n=1}^{\infty} V_n^2(x) = \{x\}$; (C) If K is a compact subset of X, $\bigcap_{n=1}^{\infty} V_n(K) = K$; (D) If $\langle x_n \rangle$ is a sequence in X that converges to x_0 and $K = \{x_i\}_{i=0}^{\infty}$, then $\bigcap_{n=1}^{\infty} V_n(K) = K$. In any space (X, τ), $A \Rightarrow B \Rightarrow C \Rightarrow D$ and if X is first countable, $D \Rightarrow B$. Every β space that satisfies B or is regular and satisfies D is semistratifiable. For a T_2 space (X, τ), X is a γ-space iff it is a wγ-space [R. E. Hodel, Duke Math. J. 39(1972), 252-263] and satisfies (A). (Received July 25, 1973.)

*709-G3. GORDON G. JOHNSON, University of Houston, Houston, Texas 77004. An example in fixed point theory.

An example is given of a homeomorphism of R^2 onto R^2 which has no fixed point such that each of its iterates has a fixed point. (Received May 30, 1973.)

709-G4. MAURICE HUGH MILLER, JR., University of Alabama, University, Alabama 35486. Characterizations of real functions by continua. Preliminary report.

Let f denote the graph of a real function with domain the segment (a,b) of the X-axis such that each point of f is a limit point of f from the left and right. Then f is a Darboux (respectively connected, property A, property B) graph if and only if f meets every horizontal interval (respectively continuum, continuous function with domain a closed connected subset of (a,b), arc) H which meets the set of points above f and the set of points below f. An example of a property A (property B) graph which is not a property B (connected) graph is given. Garrett, Nelms, and Kellum [Jber. Deutsch. Math.-Verein. 73(1971), 131—137] define f to be Darboux from the left at the point p in (a,b) if f meets every horizontal interval K whose right end lies on $\{p\} \times R$ strictly between two limit points (p,c) and (p,d) of f from the left. Theorem. The graph f is Darboux if and only if f is Darboux from the left at each point of (a,b). Analogous theorems are obtained by replacing left by right or by replacing Darboux by connected, property A, or property B. (Received May 29, 1973.) (Author introduced by Professor Harvey Rosen.)

*709-G5. PETER R. MUELLER-ROEMER, East Carolina University, Greenville, North Carolina 27834. A unipotent, but not contractible group.

A locally compact group N is contractible provided for every compact $K \subset N$ and any 1-neighborhood W in N there exists a homeomorphic automorphism $h \in \mathrm{Aut}\, N$ with $hK \subset W$. In my article "Contracting extensions and contractible groups", Bull. Amer. Math. Soc. (to appear), I announced that a Lie group G over the reals or over the p-adic numbers is contractible if and only if its Lie algebra LG is contractible (similar definition) and $G = \exp LG$, hence, only if LG is nilpotent and G is unipotent (is a group of (upper) triangular matrices with 1's in the main diagonal). All nilpotent Lie algebras of dimension $\leqq 6$ are contractible (equivalently for unipotent groups), but there is a 7-dimensional nilpotent Lie algebra which is not contractible; it is defined by the following nonzero Lie-brackets $[x_i x_j]$ with $i < j$: $[x_1 x_j] = x_{j+1}$ for $2 \leqq j \leqq 6$, $[x_2 x_3] = x_6$, $[x_2 x_4] = x_7$, $[x_2 x_5] = -x_7$, $[x_3 x_4] = x_7$. This also provides an example of a 7-dimensional nilpotent Lie algebra whose automorphism group is unipotent. (Received May 23, 1973.)

*709-G6. NELLY KROONENBERG, Louisiana State University, Baton Rouge, Louisiana 70803. Pseudo interiors of hyperspaces.

Let 2^X be the space of all nonempty compact subsets of a metric space X and let C(X) be the space of nonempty compact connected subsets, both with the Hausdorff metric. Let $Q = \Pi_n^\infty I_n$ and $s = \Pi_n^\infty I_n^0$ where $I_n = [-1,1]$ and $I_n^0 = (-1, 1)$. It is known that s is homeomorphic to $(\cong)$ ℓ_2. A subset Y of Q such that Y can be mapped onto s by an autohomeomorphism of Q is a pseudo interior for Q. A closed subset K of a metric space X is a Z-set if for any ϵ there is a map $f: X \to X \backslash K$ such that $d(f, \mathrm{id}_X) < \epsilon$. Z-sets and pseudo interiors have played important roles in infinite dimensional topology. Schori and West (Bull. Amer. Math. Soc. 78(1972), 402-406) showed that $2^I \cong Q$. Curtis and Schori (Bull. Amer. Math. Soc., to appear) showed that $2^X \cong Q$ for X a nondegenerate Peano continuum and $C(X) \cong Q$ for X a nondegenerate Peano continuum without free arcs. The author shows: Theorem 1. For X a compact Q-manifold, the collection of (connected) Z-sets in X is a pseudo interior for 2^X (for C(X)). Corollary. For X an ℓ_2-manifold, $2^X \cong \ell_2$ and $C(X) \cong \ell_2$. Theorem 2. The collection of 0-dimensional closed subsets of the interval I and the collection of Cantorsets in I are pseudo interiors for 2^I. (Received August 29, 1973.)

*709-G7. JAMES AUSTIN FRENCH, David Lipscomb College, Nashville, Tennessee 37203. A condition equivalent to covering dimension for normal spaces.

Boundary covering dimension is denoted by bcd. Complete boundary covering dimension is denoted by Cbcd. Covering dimension is denoted by dim. Complete covering dimension is denoted by Cdim. Definition. bcd X $\leqq$ n (Cbcd X $\leqq$ n) means if H is a closed set, W is an open set, $H \subset W$, and G is a finite open cover (G is an open cover) of X, then there are an open set V and discrete collections $G_1, G_2, \ldots, G_n$ of closed sets such that $H \subset V \subset W$, $\bigcup_{j=1}^{n} G_j$ refines G, and $B(V) = \bigcup(\bigcup_{j=1}^{n} G_j)$. Definition. $\dim X \leqq n$ ($\mathrm{Cdim}\, X \leqq n$) means if G is a finite open cover (G is an open cover) of X, then there is an open cover H of X such that H refines G and $\mathrm{ord}\, H \leqq n + 1$. Theorem. If X is a normal space, then bcd X = dim X. Theorem. If X is a paracompact T_2-space, then bcd X = dim X = Cbcd X = Cdim X. (Received August 30, 1973.)

*709-G8. RICHARD DEE BARNHART, Bryan College, Dayton, Tennessee 37321. Automorphisms of handlebodies. Preliminary report.

The fundamental group of a handlebody T_n of genus n is the free group F_n on n generators. It is shown that all automorphisms of F_n are induced by homeomorphisms of T_n. (Received September 7, 1973.)

*709-G9. GEORGE M. REED, Ohio University, Athens, Ohio 45701. On continuous images of Moore spaces.

In recent papers the author has obtained several counterexamples to conjectures involving chain conditions, normality conditions, completeness conditions, and the existence of point countable bases in Moore spaces. Each of these examples was obtained by constructing, by various means, a Moore space based on another space X_0. In this paper the author unifies these techniques and states some of the relationships between the original spaces and the derived Moore spaces. In addition the author investigates the mapping properties of these constructions and obtains two new results of unexpected generality. The first shows that a well-known theorem of Stone in Proc. Amer. Math. Soc. 7(1956), 690-700 cannot be extended from metrizable spaces to Moore spaces, and the second answers a question raised by Arhangel'skiĭ in Soviet Math. Dokl. 7(1966), 249-253. Theorem 1. Each locally separable, regular, first countable T_1-space is the open countable-to-one continuous image of a locally separable Moore space. Theorem 2. Each uncountable separable metrizable space is the image of a completely regular, separable, nonmetrizable Moore space under a continuous, open, countable-to-one map by which point inverses are compact. (Received September 12, 1973.)

*709-G10. KENNETH R. KELLUM, Miles College, Birmingham, Alabama 35208. Almost continuous retracts.

Stallings ("Fixed point theorems for connectivity maps", Fund. Math. 47(1959), 249-263) has stated a question due to Borsuk which suggests the possibility of proving fixed point theorems using noncontinuous functions. The question is whether an acyclic plane continuum is an almost continuous retract of a 2-cell. An almost continuous retract of an n-cell has the fixed point property for continuous functions. Suppose $f: A \to B$. We make no distinction between f and its graph. If each open set containing f also contains a continuous function with domain A, then f is almost continuous. Suppose $B \subset A$. We say that B is an almost continuous retract of A if there exists an almost continuous function $f: A \to B$ such that $f(b) = b$ for each b in B. In this paper we obtain partial solutions to this question. We show the existence of a nonlocally connected, almost continuous retract of the unit square. On the other hand, we show that no pseudo-arc is an almost continuous retract of a Peano continuum. (Received September 17, 1973.)

*709-G11. ANDRZEJ LELEK, University of Houston, Houston, Texas 77004. A sum theorem for confluent mappings.

Suppose X, Y are compact metric spaces and $f: X \to Y$ is a continuous mapping of X onto Y. Then f is called confluent [J. J. Charatonik, Fund. Math. 56(1964), 213-220] provided each component of $f^{-1}(C)$, where C is any continuum contained in Y, is mapped by f onto C. Theorem. If $Y = Y_0 \cup Y_1 \cup Y_2 \cup \ldots$ is a decomposition of Y into closed subsets Y_i such that (1) $f|f^{-1}(Y_i)$ is a confluent mapping of $f^{-1}(Y_i)$ onto Y_i for $i = 0, 1, 2, \ldots$, (2) $Y_i \cap Y_j \subset Y_0$ for $i \neq j$ and $i, j = 1, 2, \ldots$, and (3) $C \cap Y_0$ has only a finite number of components for each continuum $C \subset Y$, then f is confluent. (Received September 17, 1973.)

*709-G12. C. WAYNE PROCTOR, Stephen F. Austin State University, Nacogdoches, Texas 75961. Jones' space.

A nonmetrizable space J which is a slight alteration of the space that was originally described by F. B. Jones at the 1965 Wisconsin Topology Seminar is shown to be a Moore space whose normality is related to subsets of linearly ordered topological spaces. The space is altered by imposing an extra property on the regions. Letting M denote the set of points in the space which are end points of the arcs which are used in the construction of the space, the set M' will denote those points in M which are in limiting levels as described by Professor Jones. A maximal subset L of M' which has the property that no two points of L lie on an arc of the space that has the point p_0 as one of its end points will be called a linear space in J. A linear order is imposed on each of the linear spaces in J as suggested by the construction of J. Theorem. The space J is normal if and only if J is normal with respect to each pair of disjoint subsets whose union is a subset of a linear space in J. The normality of J with respect to such pairs of sets is also interpreted in terms of topological properties possessed by these sets within the linear space that contains their union as a subset. (Received September 17, 1973.)

*709-G13. JOHN P. HOLMES, Auburn University, Auburn, Alabama 36830. Differentiable semigroups.

Suppose S is a topological semigroup. S is said to be differentiable if S is a Banach space manifold and the multiplication of S is continuously differentiable with respect to the manifold structure on S. If S is a differentiable semigroup which contains an idempotent e, then the maximal subgroup, H(e), of S which contains e is a differentiable topological group and is an open subset of eSe. Moreover, there is an open subsemigroup of Se containing e which is, algebraically and topologically, the product of a left trivial semigroup, L, and H(e). H(e) is a manifold and each component of L is a manifold. Dual results are given for eS. (Received September 21, 1973.)

709-G14. ETHAN M. COVEN, Wesleyan University, Middletown, Connecticut 06457 and BENJAMIN G. KLEIN, Davidson College, Davidson, North Carolina 28036. The index of periodicity of a transitive flow. Preliminary report.

Let (X,T) be a (point) transitive discrete flow. A finite, closed partition $P = [X_1, \ldots, X_n]$ of X is cyclic if $T(X_i) = X_{i+1}$. The index of periodicity of (X,T) is $I(X,T) = \sup[\mathrm{card}\, P \mid P$ is a cyclic partition of $X]$. If $I(X,T) = 1$, then (X,T) is called aperiodic. Weakly mixing flows are aperiodic; the converse is false. If $I(X,T) = n$, let $(X', T') = (X_1, T^n)$; if $I(X,T)$ is infinite, let $(X',T') = (X,T)$. Theorem. $(X,T) \cong (Y,S)$ if and only if $I(X,T) = I(Y,S)$ and $(X',T') \cong (Y',S')$. Theorem. If (Ω_A, σ) is an irreducible subshift of finite type, then $I(\Omega_A, \sigma)$ is the index of imprimitivity of A. Corollary. There is a mixing subshift of finite type $(\Omega_{A'}, \sigma)$ such that $(\Omega'_A, \sigma') \cong (\Omega_{A'}, \sigma)$. Corollary. The index of imprimitivity of an $n \times n$ matrix $A \geqq 0$ is $\gcd[k \leqq n \mid \mathrm{Tr}(A^k) \neq 0]$. (Received September 24, 1973.)

709-G15. BEN FITZPATRICK, JR., Auburn University, Auburn, Alabama 36830. Moore spaces.

• A survey of Moore spaces. Complete Moore spaces, dense metric subspaces of Moore spaces, embedding Moore spaces in complete, semicomplete, and separable Moore spaces. Metrization theory, including consistency results of Jones, Tall and others on the normal Moore space problem. Recent developments and open questions. (Received September 24, 1973.)

*709-G16. THOMAS T. BOWMAN, University of Florida, Gainesville, Florida 32601. The semilattice of left translations of a compact semilattice.

A topological semilattice X is a commutative semigroup in which every element is an idempotent, and multiplication on X is continuous as a map from $X \times X$ into X. A left translation λ is a map from X into X such that

$\lambda(xy) = (\lambda x)y$ for all $x, y \in X$. The following results are proved. Theorem 1. Every left translation of a compact semilattice is continuous. Theorem 2. If X is a compact semilattice and $\Lambda(X)$ is the set of all left translations of X, then $\Lambda(X)$ is a compact semilattice in the compact-open topology. (Received September 26, 1973.)

709-G17. JAMES C. KROPA, Judson College, Marion, Alabama 36756. Connectedness of noninvertible elements in semigroups. Preliminary report.

Let S be a topological semigroup with identity e. Let G be the maximal subgroup of S containing e. Theorem. If S is connected and S is not a group, then S - G is connected. (Received September 26, 1973.)

*709-G18. W. KUPERBERG, University of Houston, Houston, Texas 77004. Mapping arcwise connected continua onto cyclic continua.

A contravariant functor A from the category of arcwise connected metric continua with mappings as morphisms into the category of Abelian groups is introduced as follows: For any object $X \in \mathcal{C}$, the group A(X) is the subgroup of the first cohomotopy group $\pi^1(X)$ consisting of all homotopy classes of mappings f of X into the circle S^1 which "kill" the fundamental group of X, i.e. $[f] \in A(X)$ iff $f_*(\pi_1(X)) \approx 0$. For any mapping $f: X \to Y$ in the category $\mathcal{C}$, the homomorphism $f^A: A(Y) \to A(X)$ induced by f is defined by just restricting both the domain and the range of the induced cohomotopy homomorphism $f^*: \pi^1(Y) \to \pi^1(X)$. The main result is: if the mapping $f: X \to Y$ is onto, then the homomorphism f^A is a monomorphism. One of the applications: If A(X) does not contain any isomorphic copy of A(Y), then X cannot be mapped onto Y. (Received September 27, 1973.)

709-G19. KENNETH I. GROSS, University of North Carolina, Chapel Hill, North Carolina 27514. Invariants for Stiefel manifolds in Hilbert algebras.

Let M be a finite-dimensional complex Hilbert algebra, A its multiplicative group, and U the unitary subgroup of A. Denote by $\mathfrak{M}$ a real form of M, and by $z \to z'$ the $\mathbb{C}$-linear involution of M inherited from $\mathfrak{M}$. This note concerns the representation theory (finite-dimensional) of the group G of 2×2 matrices over M such that gsg' = s where $s = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$. In particular, let $\Sigma = K/K_1$, the Stiefel manifold associated to $\mathfrak{M}$, where K is the maximal compact subgroup of G, and K_1 the stability group in K of the point $1 \in M$ under the left action by $\mathbb{R}$-linear transformations of K on M. (When $\mathfrak{M}$ is simple it follows that $\mathfrak{M} = \mathbf{F}^{n \times n}$ where $\mathbf{F} = \mathbb{R}, \mathbb{C}$, or $\mathbb{H}$; in which case $\Sigma \cong SO(2n)/SO(n)$, $U(2n)/U(n)$, or $Sp(2n)/Sp(n)$ respectively.) By means of a quite explicit version of the Borel-Weil theorem for G we describe the correspondence between the dual of K (resp. holomorphic dual of G) and the dual of U (resp. holomorphic dual of A), completely determine the primary decomposition of $L^2(\Sigma)$ under the action of K, and obtain the resulting decomposition of $L^2(M)$. These results are part of a collaboration with R. A. Kunze and are ancillary to the study of generalized Bessel functions on homogeneous half-spaces, decomposition of metaplectic representations of the groups of these half-spaces, and new realizations of holomorphic discrete series and their limits. (Received September 27, 1973.)

709-G20. HARIHARAIER SUBRAMANIAN, State University of New York at Buffalo, Amherst, New York 14226 and M. RAJAGOPALAN, Memphis State University, Memphis, Tennessee 38111. On dense subgroups. Preliminary report.

Dietrich conjectured in Problem 764 of Colloq. Math. that every nondiscrete locally compact Abelian group has a proper dense subgroup. Counterexamples are given to deny this. One such example is to consider a suitable algebraic direct sum of $Z_2(\infty)$ with itself and topologize this group in a suitable manner. (Received September 28, 1973.)

709-G21. PETER W. HARLEY III, University of South Carolina, Columbia, South Carolina 29208. Metric and symmetric spaces.

In this paper we give an alternative proof, without reference to Urysohn's lemma, of the Bing-Nagata-Smirnov metrization theorem via the theory of symmetric spaces as developed by A. V. Arhangel'skiǐ and H. Martin ("Mappings and spaces", Russian Math. Surveys 21(1966), 115-162, "Metrization of symmetric spaces and regular maps", Proc. Amer. Math. Soc. 35(1972), 269-274). (Received October 1, 1973.)

*709-G22. GEORGE L. CAIN, Jr. and ROBERT H. KASRIEL, Georgia Institute of Technology, Atlanta, Georgia 30332. Fixed and periodic points of local contraction mappings on probabilistic metric spaces.

In the paper "Fixed points of contraction mappings on probabilistic metric spaces" [Math. Systems Theory 6(1972), 97-102], V. M. Sehgal and A. T. Bharucha-Reid define a contraction mapping in the setting of probabilistic metric spaces and obtain fixed point theorems for complete Menger spaces, including a generalization of the classical Banach theorem. In this paper, it is shown that the topology for a probabilistic metric space is generated by a certain collection of pseudometrics $\{d_\alpha\}$ which is suggested by the definition of the probabilistic metric. Several theorems involving fixed and periodic points for local contraction mappings on complete Menger spaces are obtained. These theorems are deduced not by working directly with probabilistic metric spaces, but by first proving appropriate fixed point type theorems for a uniform space generated by the family $\{d_\alpha\}$. (Received October 1, 1973.)

709-G23. HOWARD H. WICKE and JOHN M. WORRELL, Jr., Ohio University, Athens, Ohio 45701. Uniformly primitively complete mappings.

A concept of uniformly primitively complete mappings is defined, which, in the context of T_1 spaces, generalizes properly the concepts of uniformly λ-complete [General Topology and Appl. 1(1971), 92] and uniformly monotonically complete [Duke Math. J. 34(1967), 257] mappings, and, therefore, that of compact mapping. For definition of primitive base see Abstract 706-54-5, these *Notices* 20(1973), A-533. Theorem. A T_1-space has a primitive base if and only if it is an open continuous uniformly primitively complete image of a metrizable space. Characterizations of uniformly primitively complete open continuous T_0 pararegular images of paracompact p-spaces and of T_0 regular M-spaces are also given. (Received October 1, 1973.)

709-G24. GARY A. FEUERBACHER, University of Houston, Houston, Texas 77004. Weakly chainable circle-like continua. Preliminary report.

There has been some theoretical interest in the problem of deciding which circle-like continua are the continuous images of the pseudo-arc. This result is a characterization of such continua in terms of mapping properties. Denote by W the class of all continua Y such that if X is a continuum, and f is a map from X onto Y, then f is weakly confluent. Theorem. If C is a circle-like continuum, then C is weakly chainable if and only if either C is chainable or C is not in class W. (Received September 27, 1973.) (Author introduced by Dr. W. T. Ingram.)

*709-G25. JACK B. BROWN, Auburn University, Auburn, Alabama 36830. Baire's and Reed's convergence criteria in totally nonmeagre spaces.

Consider the following two conditions for real valued functions f defined on a topological space (X, T): (1) (Baire) if M is a perfect subset of X, then $f|M$ has a point of continuity, and (2) (Reed, Fund. Math. 67(1970), 183-193) if $\alpha > \beta$, $U \subset \{x|f(x) \geqq \alpha\}$, and $V \subset \{x|f(x) \leqq \beta\}$, then either $U \not\subset \mathrm{Cl}(V)$ or $V \not\subset \mathrm{Cl}(U)$. Both of these conditions have been related in various settings to: (3) f is the pointwise limit of a sequence of continuous functions defined on X. The relationship between (1) and (2) is given by the following Theorem. If (X, T) is a topological space, then (1) → (2) for real valued functions defined on X and (2) → (1) if and only if (X, T) is totally nonmeagre (i.e. every closed subset of X is second category in itself). A discussion of totally nonmeagre spaces can be found in Aarts and Lutzer, Proc. Amer. Math. Soc. 38(1973), 198-200. (Received October 1, 1973.)

*709-G26. RICHARD E. HODEL, Duke University, Durham, North Carolina 27706 and JERRY E. VAUGHAN, University of North Carolina, Greensboro, North Carolina 27412. A note on [a,b]-compactness.

In this note we study the relationship between [a,b]-compactness in the sense of open covers (called [a,b]-compactness), and [a,b]-compactness in the sense of complete accumulation points (called [a,b]-compactness^r). Theorem. Let a and b be uncountable cardinals with $a \leqq b$. The interval of cardinal numbers [a,b] contains a singular cardinal iff there is a space which is [a,b]-compact^r and is not [a,b]-compact. This generalizes a result of A. Miščenko that there is a space in which every uncountable set of regular cardinality has a complete accumulation point, but is not a Lindelöf space. Definition. A space satisfies property I(a) if every increasing open cover $U_0 \subset U_1 \subset \dots \subset U_\alpha \subset \dots$, where $\alpha < m$ and m is a cardinal less than a, has a closed refinement of cardinality $\leqq$ a. Theorem. Let a be a regular cardinal. If X is [a,b]-compact^r and satisfies I(a), then X is [a,b]-compact. Corollary 1 (Alexandroff and Urysohn). If $a = \aleph_0$, then [a,b]-compact and [a,b]-compact^r are equivalent. Corollary 2 (Miščenko). If a is regular and every open cover U of X has a closed refinement $\{F_u : u \text{ in } U\}$ with $F_u \subset u$ for all u in U, then [a,b]-compact and [a,b]-compact^r are equivalent. Corollary 3 (N. Howes). If $a = \aleph_1$, $b = \infty$, and X is countably metacompact, then X is [a,b]-compact iff X is [a,b]-compact^r. (Received October 1, 1973.)

*709-G27. EUGENE M. NORRIS, University of South Carolina, Columbia, South Carolina 29208. Semigroups of continuous relations. Preliminary report.

A relation R from X to Y is wide if its domain is X. The set of wide, closed relations on a compact Hausdorff space is a semigroup under relation composition; since these relations are points in the hyperspace K(X × X) of compact subsets of X × X with the Michael topology, one asks if we have a topological semigroup. Even when X is the closed unit interval, the answer is no; relation composition is discontinuous. Are there any interesting examples of topological semigroups of relations? We say that a relation R is continuous if R is point-compact and both upper and lower semicontinuous. The collection C(X) of all continuous wide relations on any space X is seen to be a semigroup, partially ordered by set containment. Let S denote the semigroup of those continuous functions on the hyperspace K(X) which preserve unions; S is pointwise ordered by declaring $f \leqq g$ if $f(A) \subseteq g(A)$ for each A in K(X). Theorem. C(X) and S are order-isomorphic. If X (hence K(X)) is locally compact and Hausdorff, S is a topological semigroup in the compact-open topology; hence C(X) is a topological semigroup. Investigation of classes of spaces for which C(X) is a topological invariant are underway; some preliminary results have been obtained. (Received October 1, 1973.)

709-G28. JEONG SHENG YANG, University of South Carolina, Columbia, South Carolina 29208. Extensions of homomorphisms in C(X, G). Preliminary report.

For a topological space X and a topological group G, let C(X, G) denote the topological group of all continuous functions of X into G with pointwise multiplication and the compact-open topology. Call a homomorphism h of C(X, G) into C(Y, G) a c-homomorphism if h maps every constant function on X into the corresponding constant function on Y. If A is a subset of X, let $I: C(X,G) \to C(A,G)$ defined by $I(f) = f \circ i$, where i is the inclusion map. Theorem 1. Let A be a closed subset of X. Then every continuous function $f: A \to G$ may be extended to all of X if and only if every c-homomorphism $h: C(G,G) \to C(A,G)$ may be extended to a c-homomorphism $H: C(G,G) \to C(X,G)$ such that $H \circ i = h$. Theorem 2. If X × G is a k-space, every homomorphism of C(X, G) into a topological group L may be extended to a homomorphism of C(X × G, G) into L. (Received October 1, 1973.)

*709-G29. KERMIT N. SIGMON, University of Florida, Gainesville, Florida 32611. A simple proof of the Künneth theorem for Alexander cohomology.

As far as is known to the author, there exists in the literature no straightforward development of the Künneth theorem for Alexander cohomology, and all proofs of such that appear to be known follow a rather long and tedious route. This paper fills this gap with a relatively direct and simple proof which is based on a recent result of Lawson's [Aequationes Math. 5(1970), 236-246] on the uniqueness of cohomology theories. Our development avoids explicit need of an Eilenberg-Zilber theorem, and avoids the need of the equivalence of Cech and Alexander cohomology. (Received October 1, 1973.)

The November Meeting in Tucson, Arizona
November 24, 1973

Algebra & Theory of Numbers

710-A1. ANDREW P. OGG, University of California, Berkeley, California 94720. Diophantine equations and modular forms.

• This talk will describe an attack on an apparently intractable problem in diophantine equations—the problem of finding all rational isogenies on elliptic curves—by various old and new methods from the theory of modular forms, especially the descent theory of B. Mazur. Occasionally, the diophantine problem in turn yields information about modular functions. This work so far has led to a few general theorems, some techniques that appear to work well for any given level (e.g. proving the nonexistence of a rational isogeny of degree 47), and some plausible conjectures. (Received September 14, 1973.)

*710-A2. R. ARTHUR KNOEBEL, New Mexico State University, Las Cruces, New Mexico 88003. A decomposition theorem for multi-sorted algebras.

Let $\mathcal{O}$ be a multi-sorted algebra (i.e., a multi-sorted relational system in which all relations are functions) of type τ. Suppose σ is another multi-sorted algebraic type, and further suppose that formulas are given for the expression of the operation symbols of type τ as polynomials of those of type σ. The result is that there is a multi-sorted algebra of type σ which simulates the operations of $\mathcal{O}$ in the specified manner if, and only if, there is a sequence of congruences on products of the carriers of $\mathcal{O}$ which the operations of $\mathcal{O}$ preserve, the specific pattern of preservation being determined by the type σ. As applications, we derive Birkhoff's theorem on subdirect products of single-sorted algebras, Ashenhurst's fundamental theorem on switching functions, and a criterion for automata to be feedback shift-registers. (Received October 1, 1973.)

*710-A3. DAVID A. GAY, New College, Sarasota, Florida 33578. The Weyl group of SU(n) on zero-weight spaces.

Let SU(n) denote the group of $n \times n$ complex unitary matrices, and S_r the symmetric group on r letters. The Weyl group of SU(n) is S_n and, if M is an SU(n)-module, the zero-weight space M_0 of M is naturally an S_n-module. (In case M is the Lie algebra of SU(n), M_0 is the Cartan subalgebra.) By Young's theory, (a) given a simple character χ of S_r, there is a simple SU(n)-module M^χ and (b) every simple finite-dimensional SU(n)-module is isomorphic to an M^χ, χ a simple character of S_r for some r. It can be shown that $(M^\chi)_0 \neq \{0\}$ iff $r = nm$ for some m. To determine the S_n-module structure of $(M^\chi)_0$, let H be a subgroup of S_{nm} formed by the direct product of n-copies of S_m, and N(H) its normalizer in S_{nm}. Then $N(H)/H \simeq S_n$. Let ψ_i $(1 \leqq i \leqq k)$ be the simple characters of S_n and $\hat{\psi}_i$ the simple characters of N(H) with kernel H. Let χ_j $(1 \leqq j \leqq p)$ be the simple characters of S_{nm}. Theorem. The character of the Weyl group S_n on $(M^{\chi_j})_0$ is $\sum_{i=1}^k m_{ij}\psi_i$ where $\sum_{j=1}^p \chi_j$ is the character of S_{mn} induced up from $\hat{\psi}_i$.

(Note. The character $\sum_{j=1}^{p} m_{ij}\chi_j$ is the symmetric group "dual" to D. E. Littlewood's plethysm of S-functions: $\{m\} \otimes \{\psi_i\}$.) (Received October 1, 1973.)

710-A4. JAMES R. CLAY, University of Arizona, Tucson, Arizona 85721. The dilitation group of a generalized affine plane. Preliminary report.

On an incidence structure $\pi = (\mathcal{P}, \mathcal{L}, I)$, define $\sim$ on the "points" of $\mathcal{P}$ by $P \sim Q$ if there are distinct "lines" $m, n \in \mathcal{L}$ such that P, QIm and P, QIn. Then π is a generalized affine plane if: (1) $P, Q \in \mathcal{P}$ implies there is an $m \in \mathcal{L}$ such that PIm and QIm; (2) if $m \in \mathcal{L}$, $P \in \mathcal{P}$, and $P\not{I}m$, then there is a unique $n \in \mathcal{L}$ such that $m \| n$ and $P|n$; (3) there are three noncollinear points in $\mathcal{P}$; (4) if $P, Q \in \mathcal{P}$ and $m \in \mathcal{L}$, then PIm and $P \sim Q$ imply QIm. It follows that $\sim$ is an equivalence relation on $\mathcal{P}$ and $\bar{\pi} = \pi/\sim$ gives an affine plane in the natural way. We will assume also that π satisfies a uniform axiom: if $P, Q \in \mathcal{P}$, then there is a translation $\sigma: \mathcal{P} \to \mathcal{P}$ such that $\sigma P = Q$. (Fundamental investigations of these concepts were done by F. Lane Hardy and Nancy Jane Armentrout.) The group of dilations D and the group of translations T have structures described in terms of wreath products. Also T has a family of normal subgroups $\{T_i\}$ such that $\mathcal{T} = (T, \bigcup T/T_i, \epsilon)$ is a generalized affine plane containing π as a subplane. The factor group T/D defines the group of dilations in $\bar{\pi}$ fixing a point $\bar{P}$ of $\bar{\pi}$. The trace preserving quasi-endomorphisms Λ of D form a near-ring, and for an appropriate ideal Σ of Λ, Λ/Σ is a skew field whose multiplicative group is isomorphic to T/D. (Received October 1, 1973.)

Analysis

710-B1. WITHDRAWN.

*710-B2. JOHN V. BAXLEY, Wake Forest University, Winston-Salem, North Carolina 27109. Variations on a theme of Coddington and Levinson.

We consider nonlinear two-point boundary value problems of the form $\epsilon y'' + f(x, y, y') = 0$, $a_0(\epsilon)y(0) - a_1(\epsilon)y'(0) = \alpha(\epsilon)$, $b_0(\epsilon)y(1) + b_1(\epsilon)y'(1) = \beta(\epsilon)$, where $\epsilon > 0$ is a small parameter. Conditions are given which guarantee that a unique solution exists for ϵ sufficiently small and that the solution $y(x, \epsilon)$ and its derivatives of various orders converge as $\epsilon \to 0$ uniformly on compact subintervals excluding the boundary layer. These conditions allow f to be nonlinear in y' and include a wide class of boundary conditions. The results generalize the now classic theorem of Coddington and Levinson (Proc. Amer. Math. Soc. 3(1952), 73-81). (Received September 10, 1973.)

710-B3. NORMAN R. LEBOVITZ and RICHARD J. SCHAAR, University of Chicago, Chicago, Illinois 60637. The singularly perturbed initial-value problem when the reduced path encounters a point of bifurcation. Preliminary report.

The singularly perturbed initial-value problem $\dot{x} = f(x,y)$, $\epsilon \mathring{y} = g(x,y)$, $x(0) = \alpha$, $y(0) = \beta$ is considered in the neighborhood of the point $(x,y) = (0,0)$ through which pass two solutions, $y = \varphi(x)$ and $y = \psi(x)$ of the reduced problem $g(x,y) = 0$. The path consisting of the curve $y = \varphi(x)$ when $x \leqq 0$ and $y = \psi(x)$ when $x > 0$ is the asymptotically stable family of solutions of the boundary-layer equation under simple and natural assumptions concerning the function g. It is argued heuristically that this path should approximate the solution of the full problem to $O(\epsilon^{1/2})$ in an $\epsilon^{1/2}$-neighborhood of the origin, and this behavior is verified rigorously except when $\varphi^1(0) = 0$, which is shown to be a genuine exception. The asymptotic solutions near the point of bifurcation are matched with those already known away from the point of bifurcation to give a complete asymptotic solution in an interval independent of ϵ. Examples are discussed in which problems having the structure of the equations above arise. (Received September 11, 1973.)

*710-B4. K. W. CHANG, University of Calgary, Calgary, Alberta, Canada. Diagonalization method in singular perturbations.

Consider the nonlinear boundary value problem $\epsilon y'' + f(t, \epsilon, y, y') = 0$, $y(0, \epsilon) = \alpha(\epsilon)$, $y(1, \epsilon) = \beta(\epsilon)$ under the assumption that there exists an approximate solution $u = u(t, \epsilon)$ in the sense that $\epsilon u'' + f(t, \epsilon, u, u') = O(\gamma) + O(\gamma\epsilon^{-1}e^{-m\psi/\epsilon})$, $u(0, \epsilon) = \alpha(\epsilon) + O(\gamma)$, $u(1, \epsilon) = \beta(\epsilon) + O(\gamma)$, where $\gamma = O(\epsilon)$, $m > 0$ and $\psi = \int_0^t f_{y'}[s, \epsilon, u(s,\epsilon), u'(s, \epsilon)]\,ds$. Under additional assumptions, earlier writers (A. Erdélyi, "Approximate solutions of a nonlinear boundary value problem", Arch. Rational Mech. Anal. 29(1968), 1-17; D. Willet, "On a nonlinear boundary value problem with a small parameter multiplying the highest derivative", Arch. Rational Mech. Anal. 23(1966), 276-287) have shown that there exists a solution $y(t, \epsilon)$ on $[0,1]$ such that the difference $y(t, \epsilon) - u(t, \epsilon)$ tends to zero as $\epsilon \to 0$. Their method of proof involves a complicated analysis of the solutions of an appropriate second order equation. The main aim here is to give a new and simpler proof. We essentially aim at transforming the appropriate second order equation into a diagonalized system of two first order equations in such a way that the proof can be based on two Volterra integral equations with simple, explicitly given kernels. (Received September 14, 1973.)

710-B5. DONALD S. COHEN, University of Arizona, Tucson, Arizona 85721 and California Institute of Technology, Pasadena, California 91109. Instabilities and relaxation oscillations in the spatial and temporal organizations of chemical systems.

Recently I. Prigogine, G. Nicolis, and their coworkers have proposed and investigated a simple model of coupled chemical reactions in order to study the spatial and temporal organization of chemical and biochemical systems. As initial and final product concentrations are varied, the system undergoes very unusual and interesting exchanges of stability patterns leading to various kinds of perturbation problems. Prigogine, Nicolis, et al have analyzed their model principally by numerical techniques very cleverly devised to handle the problem. In order to obtain a deeper understanding of the qualitative features of the system, J. Boa and D. S. Cohen have studied the problem using multi-scale and singular perturbation methods. The present talk will outline the general problems, and then we shall specifically discuss one kind of temporal organization yielding concentration waves of relaxation oscillation type. More details and an analysis of the other situations which can occur are given in the forthcoming publication of J. Boa. (Received September 14, 1973.)

710-B6. JULIAN D. COLE, University of California, Los Angeles, California 90024. Some singular perturbation problems of cell physiology. Preliminary report.

The problem of determining the electric potential φ in situations corresponding to either an unsteady or steady source of current in a single cell is considered. The capacitive-resistive boundary condition at the membrane bounding the cell is essential. In suitable dimensionless variables a small parameter appears since the conductivity of the membrane is much less than that of the interior of the cell. A perturbation theory based on this parameter is developed and several nonuniformities become apparent. For a finite or spherical cell a nonuniformity in time appears and for an infinite cylindrical cell one at infinity. It is shown how different asymptotic expansions of the solution can be constructed in different domains and matched to obtain a uniformly valid approximation. It is also shown how this procedure leads to a sequence of physically significant problems. For certain cases a comparison is made with exact eigenfunction expansions to verify the heuristic ideas of singular perturbation theory. Finally, some indication of the practical use of the results is given. (Received September 17, 1973.) (Author introduced by Professor Paul C. Fife.)

710-B7. FRANK C. HOPPENSTEADT, Courant Institute, New York University, New York, New York 10012. Asymptotic stability of singularity perturbed systems.

An important question in singular perturbation problems is: How does the presence of small parameters in a system of differential equations influence the large-time stability of the system? Most previous analyses of this question involved the construction of Liapunov functions. These were often quite tedious to apply. However, frequently it is possible to obtain much sharper results by employing the well-known perturbation method, the method of matched asymptotic expansions. This approach yields sharp estimates of the system's domain of stability, as well as "easy" approximations to the large-time state, and in the asymptotically steady case, to the steady state. (Received September 20, 1973.)

*710-B8. ROBERT R. PHELPS, University of Washington, Seattle, Washington 98195. The Choquet representation theorem in the complex case.

• The Choquet-Bishop-de Leeuw integral representation theorem can be considered as an infinite dimensional generalization of the Minkowski representation (as barycenters of extreme points) of the points of a finite dimensional compact convex set. This geometrical viewpoint, while extremely useful, has tended to focus attention on real linear spaces, and only in recent years have several authors (Hustad, Fuhr and Phelps, Choquet) formulated and proved existence and uniqueness theorems (by means of complex measures, rather than probability measures) for complex linear functionals on certain spaces. For example, let X be a compact metric space, let M be a linear subspace (which separates points and contains the constants) of C(X), the complex continuous functions on X, and let L be a continuous linear functional on M. The existence theorem asserts that there is a complex regular Borel measure μ on the Choquet boundary $\partial M \subset X$ such that $\|\mu\| = \|L\|$ and $L(f) = \int f\,d\mu$ for each $f \in M$. The uniqueness theorem gives conditions under which there is precisely one such μ for each L in M*. Since the existence theorem can be considered as a representation theorem (in terms of a complex measure on certain extreme points) for an element of the weak* compact convex unit ball of M*, the methods are still geometrical, but are complicated slightly by the presence of complex scalars. An exposition will be given of both the real and complex cases and, possibly, an outline will be presented of the modifications which make it possible to drop the hypotheses that X be metric and that M contain the constants. (Received September 26, 1973.)

*710-B9. CHARLES DOWNEY, New Mexico State University, Las Cruces, New Mexico 88003. A classification theorem for the Hilbert transform over a local field.

In this paper the essential uniqueness of the Hilbert transform over a zero dimensional, nondiscrete, locally compact field is shown. The Hilbert transform over the reals can be characterized up to a constant multiple by a few intrinsic properties. Over a local field, a cyclic group of transformations is obtained which gives an analogous classification. (Received September 27, 1973.) (Author introduced by Dr. Keith Phillips.)

*710-B10. JOSEPH A. CIMA and WARREN R. WOGEN, University of North Carolina, Chapel Hill, North Carolina 27514. On algebras generated by composition operators.

Let H^p, $1 \leqq p \leqq \infty$, denote the classical Hardy space on the unit disk Δ, and, for φ an analytic mapping of Δ into Δ, let $C\varphi$ denote the bounded linear operator on H^p which is functional composition. Theorem 1. Let f be nonconstant in H^p; the linear span of the set $\{C\varphi(f)\}$, as φ varies over all analytic automorphisms of the unit disk, is all of H^p. Theorem 2. Let A(L) be the weakly closed algebra (in the space $\beta(H^2)$) generated by the set $\{C\varphi\}$. The algebra A(L) is reflexive. (Received September 27, 1973.)

*710-B11. MOSES E. COHEN, California State University, Fresno, California 93710. Differential recurrence theorems associated with a class of polynomials.

In Abstract 702-B15, these *Notices* 20(1973), A-352, we presented a generating function for a general class of polynomials whose special cases include the Jacobi, Laguerre, Hermite, and allied polynomials. In this paper, the above result is used to prove two theorems concerning differential relations satisfied by a generalized hypergeometric polynomial. (Received September 27, 1973.)

710-B12. J. KEVORKIAN, Department of Aeronautics and Astronautics, University of Washington. Seattle, Washington 98195. Resonance in systems with slowly varying coefficients and small nonlinearities.

Many problems of physical interest can be modeled by either ordinary or partial differential equations with slowly varying coefficients and small nonlinearities. Examples from flight mechanics, celestial mechanics, and acoustics are mentioned. In such problems resonance occurs when two or more slowly varying modes of oscillation coincide, and quite often this leads to interesting anomalous behavior of the solution. This talk concerns a singular perturbation procedure for approximating the solution prior to, during, and after resonance. The representation of the solution before and after resonance requires two generalized multiple scale expansions, and the procedure for calculating these expansions is discussed in some detail as it involves certain novel features. During resonance a third multiple scale expansion is introduced and matched sequentially to the two nonresonant expansions. One can then calculate a uniformly valid representation of the entire solution. Throughout the talk, ideas will be illustrated using simple examples. (Received September 28, 1973.) (Author introduced by Professor W. M. Greenlee.)

710-B13. FREDERICK A. HOWES, University of Southern California, Los Angeles, California 90007. An application of Nagumo's lemma to some singularly perturbed systems.

The existence and asymptotic behavior as $\epsilon \to 0^+$ of periodic, almost periodic, and bounded solutions of the differential system (*) $x' = f(t,x,y,\epsilon)$, $\Omega y' = g(t,x,y,\epsilon)$ are considered, where x, f are n-vectors, y, g are m-vectors and $\Omega = \operatorname{diag}\{\epsilon^{h_1},\ldots,\epsilon^{h_m}\}$ for integral $h_i, h_1 \leq h_2 \leq \ldots \leq h_m$. Assuming that f and g are of class $C^{(1)}$ and certain reducibility conditions on the Jacobian matrix g_y, the system (*) is shown to have solutions which are uniform perturbations of order $O(\epsilon)$ of known solutions of the corresponding reduced system $x' = f(t,x,y,0)$, $0 = g(t,x,y,0)$. The principal tools are a lemma of Nagumo which allows the construction of appropriate upper and lower solutions of (*) and the asymptotic theory of singularly perturbed linear differential systems. Results are also indicated for autonomous systems and for totally degenerate systems $\epsilon x' = f(t,x,\epsilon)$. (Received October 1, 1973.)

Applied Mathematics

710-C1. ROBERT E. O'MALLEY, JR., University of Arizona, Tucson, Arizona 85721. The singular perturbation solution to problems of cheap control.

The following is an important singular problem of optimal control: $\dot{x} = Ax + Bu$, $x(0)$ given, $J(\epsilon) = 1/2 \cdot \int_0^1 (x^TQx + \epsilon^2 u^T Ru)\,dt$ to be minimized, and $\epsilon = 0$. For the nearby problem with ϵ small and positive (and Q and R positive definite), control is cheap relative to state x. In a variety of cases, singular perturbation theory for linear boundary value problems leads to asymptotic solutions for $\epsilon \to 0$. These asymptotic solutions show that limiting behavior will be impulse-like near $t = 0$ with convergence elsewhere to "singular arcs". Moreover, the regions of nonuniform convergence (or "boundary layers") become thicker for more restrictive problems. Representative solutions (due to Jameson and O'Malley) will be analyzed, and open questions involving constrained controls and other singular problems will be discussed. (Received September 24, 1973.)

*710-C2. KETILL INGÓLFSSON, Science Institute, University of Iceland, Reykjavík, Iceland. On the mathematical structure of a model converging in a space of semidefinite metric.

Let a physical situation be governed by the linear selfadjoint operators H_1 and H_2 densely defined in the separable Hilbert spaces $\mathscr{H}_1$ and $\mathscr{H}_2$ respectively. The latter space is contained in the first one and reduces H_1 to $\bar{H}_1$.

The operator V entering the decomposition of H_2 along $\bar{H}_1$ is supposed to be a trace class operator on $\mathcal{H}_2$. Let the state ψ be absolutely continuous with respect to H_2 and singular with respect to $\bar{H}_1$. Scattering problems are now solvable according to the theorem of Kato and Rosenblum: The generalized wave operators $W_{\pm}(H_2, \bar{H}_1)$ and $W_{\pm}(\bar{H}_1, H_2)$ exist and are complete on $\mathcal{H}_2$. Decay may also be calculated from infinite series in which the terms are determined by the recurrence relation $A_n(t)\psi = -i \cdot \exp(-i\bar{H}_1 t) \int_0^t dt' \exp(i\bar{H}_1 t') V A_{n-1}(t')\psi$ for $n \geqq 1$ and $A_0(t)\psi = \exp(-i\bar{H}_1 t)\psi$. The series converges for $t \to \infty$. The further procedure is aimed at pointing out a relationship of $\mathcal{H}_2$ to $\mathcal{H}_1$ which can support the basic assumption on V. Unfortunately quantum electrodynamics does not converge in the above sense on pure Fock spaces. The spaces $\mathcal{H}_1$ and $\mathcal{H}_2$ can be reached by a partially isometric mapping of certain tensor products of $\mathcal{L}^2$ spaces. The freedom in determining the physical fields in the product spaces corresponds to the semidefinite metric as it was introduced for free fields by B. L. van der Waerden. (Received October 1, 1973.) (Author introduced by Professor Henry B. Mann.)

Geometry

710-D1. VICTOR L. KLEE, JR., University of Washington, Seattle, Washington 98105. Polytope pairs and their relationship to linear programming.

An important development in the theory of (convex) polytopes was the determination by Barnette and McMullen of the minimum and maximum of v(P) (number of vertices of P) as P ranges over all simple d-polytopes with n facets. Their results are here extended to certain pairs consisting of a polytope and one of its facets. Corollaries of our main results are the determination of the minimum and maximum of v(P) as P ranges over all simple d-polyhedra with u unbounded and $n - u$ bounded facets, and of the minimum and maximum of $v(P{\sim}F)/v(F)$ as (P, F) ranges over all pairs consisting of a simple d-polytope P with n facets and a facet F intersecting all other facets of P. Such pairs, called Kirkman pairs of class (d, n), are related to several aspects of linear programming, including a recent algorithm of Mattheiss for finding all vertices of a polytope defined by a system of linear inequalities. (Received September 21, 1973.)

Statistics and Probability

*710-F1. ALI KYRALA, Arizona State University, Tempe, Arizona 85281. Integrodifferential equations of real and complex Markov chains.

A derivation of integrodifferential and partial differential equations for probability densities of physical processes representable by Markov chains is given by imposing selection rules restricting steps per transition and causality. Applications to active transport and condensation problems are mentioned. By using unitarity instead of causality, integrodifferential and partial differential equations for probability amplitudes of relativistic quantum mechanics are derived. These are developed from complex (analogs of) Markov chains. Finally by prohibiting transitions between infinitely remote states a generalization of the Dirac equation is obtained. (Received September 14, 1973.)

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MATHEMATICIAN, TEACHING AND RESEARCH OR RESEARCH. Ph.D. 1965. Age 38. Speciality: Algebraic Number Theory. Eleven articles published and some under preparation. Seven years teaching experience at all levels. Available immediately. M. Bhaskaran, 91 Carr St., Perth, Western Australia 6000.

MATHEMATICS PROFESSOR. Ph.D. 1967 University of Washington. Ring Theory. Six years teaching experience in U.S. and Iran. Have taught most courses in undergraduate curriculum, and some graduate courses. Vita and references on request. Available immediately. Denis R. Floyd, 517 E. Lancaster, #206, St. David's, Pennsylvania 10987.

MATHEMATICS TEACHING–UNDERGRADUATE, SERVICE AREAS. Ph.D. 1967, Illinois, Algebra. Male, age 33, 6 years experience. 6 papers. Co-author sophomore text, Linear Algebra and Vector Calculus, to appear fall 1974. Prefer west, midwest. J. Stephen Montague, Mathematics Department, University of Tennessee, Knoxville, Tennessee 37916.

MATHEMATICS TEACHING–UNDERGRADUATES, FUTURE, IN-SERVICE TEACHERS. Ph.D. 1961, Wisconsin, Algebra. Female, age 36, 12 years experience. 4 papers. Co-author sophomore text, Linear Algebra and Vector Calculus, to appear fall 1974. Prefer west, midwest. Patricia Tucker Montague, Mathematics Department, University of Tennessee, Knoxville, Tennessee 37916.

MATHEMATICS TEACHER AND RESEARCHER. Age 27. Ph.D., University of Southern California, 1973; in Combinatorial Mathematics. Four articles accepted for publication. One year professional experience at Louisiana State University, teaching wide range of undergraduate courses and lecturing at graduate seminars. Reference and resume on request. K. B. Gross, 336 W. Parker Blvd., #25, Baton Rouge, Louisiana 70808.

MATHEMATICS PROFESSOR, TEACHING AND RESEARCH. Ph.D. 1970. Speciality: Statistics and Probability. One publication and two under preparation. Experience in teaching of mathematics and non-mathematics majors and consulting. References and resume available upon writing. Available immediately. D. Gupta, 6 Round Hill Road, Potsdam, New York 13676.

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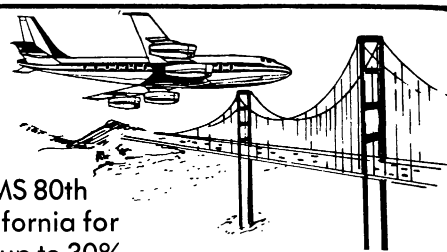

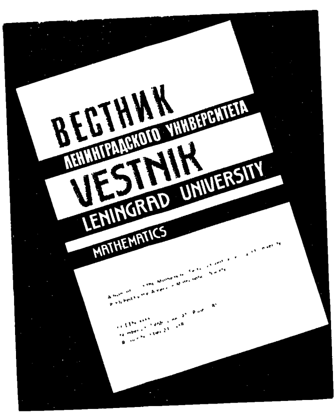
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January 15-19, 1974

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a. ______________________ b. ______________________

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signature

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Computer Short Course
January 13-14, 1974

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* For definitions of student and unemployed member, see page 332.

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11) I will arrive (date) ______________ at (hour) ______________ a.m./p.m.
I will depart (date) ______________ at (hour) ______________a.m./p.m.

12) Persons for whom this reservation is made. Please list names and type of room for each (bracket the names of those persons sharing a room). Each participant should complete separate preregistration form.

a.______________ c. ______________
b.______________ d. ______________

13) I will (will not) share a room

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