Star Clusters

Even a small telescope shows obvious local concentrations of stars scattered around the sky. These star clusters are not chance juxtapositions of unrelated stars. They are, instead, physically associated groups of stars, moving together through the Galaxy. The stars in a cluster are held together either permanently or temporarily by their mutual gravitational attraction.

The classic, and best known, example of a star cluster is the Pleiades, visible in the evening sky in early winter (from the northern hemisphere) as a group of 7–10 stars (depending on one's eyesight). More than 600 Pleiades members have been identified telescopically.

Clusters are generally distinguished as being either GALACTIC OPEN CLUSTERS or GLOBULAR CLUSTERS, corresponding to their appearance as seen through a moderate-aperture telescope. Even in a relatively small telescope, the stars in an open cluster can be individually distinguished right at the center, whereas a globular cluster typically appears to consist of a sprinkling of bright, resolved stars superposed on a continuous background of unresolved fainter stars. Although the original distinction of the two classes of clusters was based almost entirely on their visual appearance, we now know that the globulars and the open clusters are very distinct from one another in almost every respect.

At the opposite extreme of concentration from the globular clusters are the stellar associations, loose, un-bound groups of stars with particular spectral properties. Associations are normally observed as widespread regions with an excess density of spectral type O and B stars ('OB ASSOCIATIONS') or T TAURI STARS ('T associations'). In principle, there could be other types of association as well. It is not uncommon to find one or more open clusters at the core of a stellar association. The implication is that the stars in the central cluster or clusters and the surrounding association all formed more or less at the same time out of the same material. Since the association is not a gravitationally bound system, after a relatively short period of time, its stars will disperse and the association will lose its identity; for that reason, associations always consist of young stars.

The stars in an unbound association or a cluster that is evaporating will nevertheless all be moving with almost identical velocities, so at some later time, one might expect to find stars spread out in space, but still moving with similar velocities. Stars selected by velocity are sometimes called MOVING GROUPS. The best known of the moving groups is called the Hyades moving group. It consists of a number of stars, located in all directions from the Sun, which share the space motion of the Hyades star cluster. However, it is uncertain whether these stars share a common origin with the Hyades cluster.

The category of moving groups is not to be confused with moving clusters. These are clusters near enough to the Sun that all three dimensions of their motion through the Galaxy can be used to derive the cluster distance, independently of parallax measurements.

A final category of clusters has recently been identified based on observations with infrared detectors. These are the embedded clusters—star clusters still in the process of formation, and still embedded in the clouds out of which they formed. Because it is possible to see through the dust much better at infrared wavelengths than in the optical, these clusters have suddenly become observable.

Star clusters are of considerable astrophysical importance to probe models of stellar evolution and dynamics, explore the star formation process, calibrate the extragalactic distance scale and most importantly to measure the age and evolution of the Galaxy.

Definition of a star cluster—cluster catalogs

Just what *is* a cluster? How many stars does it take to make a cluster? Trumpler in 1930 defined open clusters as 'Star groupings which undoubtedly form physical systems (stars situated at the same distance and probably of the same origin) and which at the same time are sufficiently rich in stars for statistical investigation.' Since virtually all clusters have been discovered either by visual examination of the sky with a telescope, or from inspection of photographic or electronic images in the visual or infrared, an operational definition of a star cluster is suggested:

> A star cluster is defined to be an obvious concentration of several stars or more above the surrounding stellar background, apparently localized in space and identifiable on visual or infrared images covering a suitable field of view.

While this definition may seem imprecise, it conveniently includes most of those objects (in our Galaxy at least) listed in the existing cluster catalogs. At the same time, it excludes galaxy-sized systems or objects with insignificant numbers of stars. Presumably most clusters defined in this way are physical systems, although they may or may not be gravitationally bound systems. So some of the embedded clusters or slightly older, optically revealed clusters may be unbound star-forming regions in the process of dispersing. The above definition avoids the difficulty of distinguishing bound systems from unbound ones—if it looks like a cluster, it is a cluster.

From the theoretical point of view, a cluster might be better defined as a self-gravitating system of stars residing within the Galactic gravitational potential. Using this definition, unbound embedded clusters or young optically revealed clusters would not be considered to be true clusters. In addition, a star that has achieved escape velocity from a cluster but is still within the tidal radius of the cluster would be considered to be a nonmember by this theoretical definition, but would be a member by the observational definition.

Few objects have been catalogued as extensively as star clusters. The MESSIER CATALOG of 107 nonstellar objects includes 59 objects now known to be star clusters. The

first major catalog that includes clusters is the NEW GENERAL CATALOG (NGC), listing 487 clusters. The update to the NGC, the IC, contains another 30 clusters.

There have been at least a couple of dozen other more or less systematic attempts to search for and catalog additional star clusters. The entire sky has now been searched for clusters to the limiting magnitude and resolution of the wide-field Schmidt telescope photographic surveys. Unfortunately, the large number of partly redundant catalogs has generated a considerable amount of confusion about cluster names, and some clusters have almost a dozen aliases. The existing cluster catalogs are almost certainly biased in various ways, and any statistical results based on them are subject to systematic errors and selection effects of one sort or another.

As the number of catalogs has grown, and as the astrophysical importance of star clusters has been more appreciated, the number of secondary catalogs, summarizing the known properties of clusters, has also grown. Some useful catalogs are given in the Bibliography.

Summary of cluster properties

Table 1 summarizes the properties of star clusters. While there is a considerable dispersion in the properties of individual clusters, as well as a substantial uncertainty in some of the values, these numbers are representative of the cluster systems.

Table 1 shows that there is a rather sharp distinction between the globular clusters, which are representative of the Population II stars of the galaxy, and the open clusters (including the OB associations, moving groups and embedded clusters), which are Population I stars. The most massive of the open clusters are roughly the same as the least massive globulars, and the oldest of the open clusters is no older than the youngest of the globulars.

Globular clusters
The known globular clusters in our Galaxy number about 147 and, taking into account obscured regions of the Galaxy, the total Galactic population of globulars is probably about 200. Most of the 'missing' globulars are likely to be located close to the Galactic center or near the Galactic plane on the far side of the Galaxy.

The globulars are substantial stellar systems. Even the smallest of them contain some 10 000 solar masses of stars and a few, such as M22 and Omega Centauri, exceed one million solar masses.

As described later in this article (see also STELLAR DYNAMICS), the stellar distribution within a globular cluster is well represented by King models, which can be characterized by two parameters: the central surface brightness and the core radius, defined to be the radius at which the brightness drops to one-half the central value. Once these two parameters are determined, the appropriate King model will yield a tidal radius for the cluster. The globular clusters cover a substantial range in size. The median core radius is close to one parsec, but a

few clusters in the outer halo of the Galaxy have core radii in excess of 10 parsecs. The median tidal radius is about 35 parsecs, but again there are some in the outer halo with much larger tidal radii.

Open clusters
Some 1200 open clusters have been catalogued, but as many as 250 of these appear to be simply slight enhancements in the stellar density along a line of sight or small asterisms of unrelated stars. Such asterisms should not be considered as clusters by either of the definitions of clusters given above. Most of the open clusters in the Galaxy have probably not yet been found, since open clusters are found near the Galactic plane where the interstellar dust is the most opaque. There may be as many as 10^5 open clusters in the Galaxy.

The smallest of the open clusters contain fewer than a dozen stars, so the minimum mass for an open cluster is about $10\,M_\odot$. At the other extreme, some of the most massive open clusters are old ones such as NGC 6791, with more than 10 000 members, again with typical masses near one solar mass. The most populous younger clusters contain a few thousand stars, but since their most massive stars are substantially more massive than the Sun, the total mass of some of the younger clusters exceeds $10^4\,M_\odot$.

Few if any open clusters have enough members to be characterized in terms of dynamically meaningful quantities, such as the core radius or tidal radius. Instead, open cluster sizes are based on the 'angular diameter' measurement where the diameter is the apparent size of the cluster as measured on the appropriate sky survey photographs. Even when the distance to the cluster is well-known, the angular diameter does not yield a well-defined linear dimension to the cluster. Nevertheless, there is a rather small spread in measured cluster diameters, the typical open cluster being between 4 and 5 pc in diameter. A few clusters are more than twice this size, and some are only one or two parsecs in diameter, but the majority fall rather close to this value. The core radius of open clusters is probably roughly 1 pc, that is to say, about the same as the typical core radii of globular clusters. There is no strong correlation of cluster size with age, although several of the oldest clusters are somewhat larger and some of the very youngest clusters are also larger. Nor is there any correlation of cluster size with Galactic location or with the number of members.

OB associations
OB associations are areas on the sky with a higher than normal density of O and B stars, They are both large and diffuse; they are in fact sufficiently amorphous that distinct boundaries are hard to define. They typically cover many tens of parsecs and often contain one or more discrete clusters within their borders. Their diffuse nature makes it impossible to identify fainter stars as being members of associations, although there may well be large numbers of lower-mass stars that formed from the same molecular cloud.

Table 1. Summary properties of star clusters.

	Globular clusters	Open clusters	OB associations	Moving groups	Embedded clusters
Numbers of clusters:					
Catalogued	147	1200	70	< 10	∼ 25
Likely Galactic total	200	10^5	> 1000	—	—
Typical Sizes (pc):					
Core radius (median)	1.0	—	unbound	unbound	—
Tidal radius (median)	35	—	unbound	unbound	—
Apparent diameter	—	4	> 100	—	1
Masses ($M_\odot$):					
Minimum	10^4	10	1000	—	100
Maximum	10^6	10^4	10^4	—	> 1000
Ages (yr):					
Minimum	8×10^9	5×10^6	5×10^6	—	$< 10^6$
Maximum	16×10^9	9×10^9	2×10^7	—	5×10^6
Metallicities ([Fe/H]):					
Range	−2.3 to +0.2	−0.7 to 0.3	Pop I	Pop I	Pop I

Approximately 70 OB associations have been catalogued. An association may consist of as few as a dozen luminous O and B stars, but because they range in mass from about 10 $M_\odot$ to some 70 $M_\odot$, the total mass of the association may range from about 1000 $M_\odot$ to more than ten times that number.

Moving groups
Several moving groups have been identified, but because the group may be spread out over a large region of the galaxy, the membership of a group is likely to be seriously incomplete, assuming that the group has a common origin at all. No reliable estimates of the total mass of a moving group exist.

Embedded clusters
At the other extreme of size from associations are the embedded clusters. This newly identified class of object consists of groups of stars still deeply embedded in the nebulosity out of which they have just formed. They range from a few tenths of a parsec to about two parsecs in diameter. They are found only through IR imaging surveys—it is important to stress that these surveys are now sufficiently extensive that the apparent small size of the embedded clusters is not just a result of inadequate sampling. They really are rather compact objects. Although a few, like the Orion Trapezium region, are rich systems with hundreds of stars, most of them consist of small aggregates of stars.

No comprehensive catalog exists, but as many as 25 embedded clusters have been described; the total Galactic population of embedded clusters probably exceeds this number by several orders of magnitude. Since the stars in an embedded cluster have not yet reached the main sequence, even the relatively low-mass stars are rather luminous. Consequently they are relatively easily observable (in the infrared). They are compact, very dense

regions, containing hundreds to thousands of solar masses of stars.

If the embedded clusters are all smaller than the ordinary, optically visible open clusters, does the one class evolve into the other? That is to say, when the molecular clouds surrounding these embedded clusters dissipate in a few million years, will there be an apparent ordinary open cluster there, or will the stars dissipate as fast as the gas? The answer to this question is not yet known and is the subject of much discussion at the present time. Embedded clusters are sometimes called 'stellar aggregates' rather than clusters because of this ambiguity, but based on the definition of a cluster given here they should be considered as star clusters.

Cluster distances and the cosmic distance scale
Star clusters provide a critical step in the determination of the cosmic distance scale. The starting point for cluster distances, and a primary step in the distance scale, is the Hyades star cluster. As a result of the HIPPARCOS project, the distances to the individual stars in the cluster have been measured well enough that the three-dimensional structure of the cluster can be determined. The Hipparcos data plus additional support from other measurements show that the Hyades distance modulus is $(m - M)_o = 3.33 \pm 0.01$, that is, the center of the Hyades cluster is 46.3 ± 0.2 pc from the Sun.

To find the distances to more remote clusters, a standard technique is to relate the properties of the Hyades to those of the cluster in question. The primary mechanism for that comparison is the color-magnitude diagram, a form of HERTZSPRUNG–RUSSELL DIAGRAM. The Hertzsprung–Russell diagram shows the relation between the luminosities of stars and their temperatures; in a color–magnitude diagram the luminosity is presented as either the absolute or apparent magnitude and the temperature is represented by a color index. The color–magnitude

diagrams of two clusters will in general have the same shape, but will be shifted in magnitude by an amount equal to the difference in distance modulus between the two. There may also be additional shifts in magnitude and color index resulting from interstellar absorption. The general procedure is to apply the effects of absorption, if known, and then slide the diagrams of the two clusters until they match, according to some goodness-of-fit criterion.

The principal limitations of this method are the accuracy of the magnitudes and colors of the stars in both the Hyades and the target cluster, the amount of interstellar absorption to the target cluster (the interstellar absorption to the Hyades is effectively zero) and the effects of chemical composition on the location of the main sequence. The Hyades is rather unusually metal-rich (approximately [Fe/H] = +0.15) compared with most star clusters, which must be taken into account.
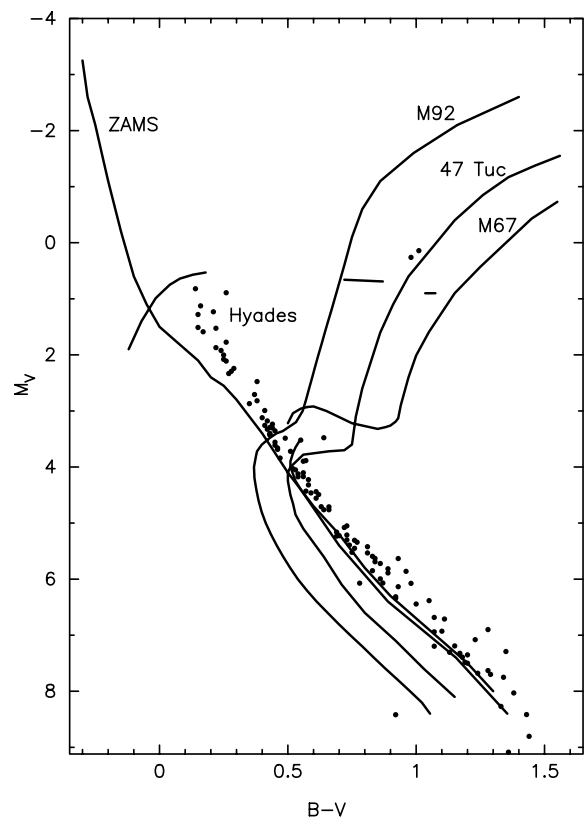
Figure 1 shows the color–magnitude diagrams of several star clusters, corrected for distance. M92 is an old, metal-poor globular cluster; 47 Tucanae is another old globular cluster, but less metal-poor; M67 is an old open cluster of solar composition, and the Hyades is a younger metal-rich open cluster. For comparison, the zero-age main sequence (ZAMS) for solar composition stars is shown as well. This diagram presents the combined effects of stellar evolution and chemical composition. The M92 main sequence lies the lowest in the diagram, as would be expected from its composition, 47 Tuc is next, then M67 and finally there are the individual Hyades stars, the most metal-rich ones in the diagram. The ZAMS line corresponds approximately to solar-compositon stars.

Figure 1 also shows that it is critical to have the right distance to a cluster. Except for the horizontal branches, the shapes of the color–magnitude diagrams for the two globular clusters are virtually identical. Unless both the distance and composition of a cluster are known very well, the age uncertainties that result are large.

Clusters and stellar evolution—cluster ages

Star clusters represent snapshots of the process of stellar evolution—they are frozen in time from a human perspective. Since all the stars in a star-forming region begin their protostellar phase within a few million years of one another, the stars in a cluster much older than a few tens of millions of years can be considered to be all the same age. Thus the color–magnitude diagram of a star cluster represents the properties of stars differing in mass, but all of the same age and chemical composition. A critical test of the validity of STELLAR EVOLUTION models is to be able to predict the observed properties of cluster stars, as seen in the color–magnitude diagram.

To compare the models with the actual stars in a cluster, a series of model evolutionary sequences is computed for stars selected to have a range of masses. An isochrone is derived by interpolating in this series of models to find the parameters for stars all the same age. The isochrone is in effect a theoretical color–magnitude diagram.

Figure 1. A composite of color–magnitude diagrams of the globular clusters M92 (from Stetson and Harris 1988) and 47 Tucanae (Hesser *et al* 1987) and the open clusters M67 (Montgomery *et al* 1993) and the Hyades (Perryman *et al* 1998). The zero-age main sequence (ZAMS) of Schmidt-Kaler (1982) is shown for comparison.

In going from the theoretical predictions of the model isochrone to the observed color–magnitude diagram, there are a number of serious complications. In addition to the various physical and numerical uncertainties of the models themselves, a major complication relates to the transformation of the theoretical parameters, effective temperature and luminosity, into the derived absolute magnitude and color index. To make this transformation, a good knowledge of the stellar flux distribution is needed (a good model atmosphere), plus a knowledge of the details of the spectral response of a typical photometric system. The latter includes atmospheric transmission, telescope plus filter spectral response and the spectral sensitivity of the detector, all of these integrated over the flux distribution of the model star.

The problem of measuring the ages of star clusters appears to be a simple one. As figure 1 shows, the main sequence turn-off of a star cluster—usually defined as bluest point (i.e., the smallest value of the color index) along the cluster sequence—is a function of the age of the cluster. This follows naturally from the basic fact of stellar evolution that the massive, more luminous,

stars evolve away from the main sequence before the less massive stars have significantly changed. But at a given mass the actual age when this occurs depends on the chemical composition of the star, and possibly other factors as well, such as rotation. There are a number of theoretical uncertainties in addition to the observational uncertainties. Furthermore, as can be seen in figure 1, the shapes of the main-sequence/turn-off/red-giant region of old cluster color–magnitude diagrams are all roughly the same. So errors in the distance, interstellar reddening or composition can lead to a cluster being matched to a stellar model of the wrong age.

The most important of these difficulties is the distance. For example, if a cluster is measured to be too close by, say, 10%, then the measured luminosities of its stars will be too small by 21%. But the luminosity of a star is just a measure of the rate at which it is burning its nuclear fuel. So in this case, the distance error would mean that the nuclear-burning rate will be underestimated, and the cluster age would be overestimated by that same 21%. Very few clusters have their distances known to 10%, so while there are additional constraints that can be put on the age calculation, the uncertainties are still substantial.

The distance problem enters in two ways. First, the distances to individual clusters have random errors relating to the particular data for that cluster, but of more significance to astrophysics is the problem of the distance scale itself. The Hipparcos satellite has helped the situation considerably, but there is still considerable controversy over the ages of globular clusters.

Galactic distributions of clusters

Globular clusters

The globular clusters, containing the oldest stars in the Galaxy, are located in a nearly spherical distribution about the Galactic center. They make up the most visible component of the Galactic halo, and as a practical matter they define the stellar halo. Because they can be identified so readily and their distances measured reliably, Harlow Shapley was able to determine the direction and distance to the Galactic center in the 1920s. The globular clusters were later shown to define the Population II stars of the Galaxy.

Close examination of the statistics of the globular cluster properties shows that they can be divided into three subpopulations on the basis of cluster composition and the shape of the horizontal branch (see HORIZONTAL-BRANCH STARS). The 'old halo' clusters have horizontal branches consistent with a great age; the 'young halo' clusters have horizontal branches suggesting a somewhat younger age; and the 'disk globulars' are rather more metal-rich (with [Fe/H] > −0.8) than the other populations and are distributed in a thick disk shape.

The three subpopulations of clusters may have originated in two or three distinct processes. The old halo clusters probably formed in the initial collapse of the Galaxy and so their stars are the oldest. The disk globular clusters appear to be members of the thick disk population

and may be significantly younger in addition to being more metal-rich. The young halo clusters may actually have been formed outside our Galaxy in nearby dwarf spheriodal galaxies that subsequently merged with our own Galaxy. The young halo clusters show no evidence for a correlation between Galactic position and composition, consistent with their having been injected into the Galaxy in random events. The old halo and disk subsystems do show correlations between galactocentric radius and composition, and there is a possibility that the old halo and thick disk populations may form a single system. If indeed this is true, then there is a regular, systematic trend between composition and position in the old halo plus thick disk subsystems, as well as between composition and age.

Open clusters

The open clusters, including associations, moving groups and embedded clusters, belong to the disk population of the Galaxy (the Population I stars). The vast majority of them are located close to the Galactic plane. Consequently, most known open clusters are within a couple of kpc of the Sun.

Assuming a distribution perpendicular to the Galactic plane of the form

$$N \propto e^{z/z_0}$$

the scale length, z_0, for clusters with ages < 700 million years is about 55 pc, whereas the older clusters have a scale height of 350 pc. There is otherwise no correlation between composition and age or distance from the Galactic plane. There is, however, a distinct correlation between [Fe/H] and galactocentric radius, with the most metal-poor stars located in the far outer reaches of the Galactic disk.

The open clusters show no systematic trend of metallicity with age. At each age, there is a substantial range in composition, but on the average the oldest ones are approximately the same composition as the youngest ones.

Cluster dynamics

Star clusters are ideal laboratories for studies of stellar dynamics—they are simple systems consisting of what are essentially point masses, moving freely under the action of a single force, gravity. Consequently it is possible to model in some detail the dynamical evolution of star clusters. It is important to note, however, that the formalism of classical celestial mechanics is not directly applicable to star clusters. In celestial mcahanics, one usually considers the motion of one body in the potential field of another body (the two-body problem) or perhaps two other bodies (the three-body problem). However, a cluster consists of several (i.e., more than three) more or less equal masses moving in their mutual potential field, perhaps also perturbed by external (Galactic) forces.

Most dynamical models begin with the assumption that they are collisionless systems—that is, the forces acting on individual particles (stars) are long-range forces only, resulting from the gravitational potential field of the

aggregate of all the stars in the system. The occasional close encounters between pairs of stars, if they are included at all, are dealt with separately. More distant encounters may, however, be common in a stellar system. These encounters will gradually change the motion of an individual star, so that after enough such encounters, the star will have changed in velocity by an amount equal to the velocity. The time it takes for such a complete exchange of energy to take place is the relaxation time.

King models

Star clusters are approximately equivalent to isothermal spheres, that is to say, the velocities of stars in the cluster have a distribution that is independent, or nearly so, of the position of the star in the cluster. However, a singular isothermal sphere has infinite density at the origin (i.e, the center) and therefore infinite mass. To represent an isothermal sphere in a well-behaved manner at the origin, it is possible to define dimensionless variables, where the radius variable

$$\tilde{r} \equiv \frac{r}{r_0}$$

is defined by the King radius,

$$r_0 \equiv \sqrt{\frac{9\sigma^2}{4\pi G\rho_0}}.$$

At the radius r_0, the density within the system has fallen to 0.5013 of its central density. Since the core radius, r_c, as defined observationally is the radius at which the density has fallen to just half the central density, r_0 is often called the core radius.

For a system with gravitational potential Φ, where a single particle has velocity v and total energy E, the relative potential Ψ and relative energy $\mathcal{E}$ of the particle can be defined as

$$\Psi \equiv -\Phi + \Phi_0 \quad \text{and} \quad \mathcal{E} \equiv -E + \Phi_0 = \Phi - \tfrac{1}{2}v^2$$

where Φ_0 is some constant. A modified form of an isothermal sphere is the King model, which resembles the isothermal sphere at small radii, where most of the particles (stars) have large values of $\mathcal{E}$, but gradually becomes less dense than the isothermal sphere at larger radii where $\mathcal{E}$ tends to be lower. By suitably defining the constant Φ_0 so that at some critical relative energy $\mathcal{E}_0 = 0$, the phase-space distribution function of the King model can be defined so that for $\mathcal{E} \gg 0$ the King model and the isothermal sphere are identical, and for $\mathcal{E} \leq 0$ the King model distribution function is zero.

This leads to a family of models for the density distribution within the system that are defined by the central relative potential, $\Psi(0)$, where the radius is in units of the King radius, r_0. At the radius where Ψ falls to zero, the density also falls to zero. This is the tidal radius, r_t. The log of the ratio of the tidal radius to the King radius defines the concentration of the cluster,

$$c \equiv \log_{10}(r_t/r_0).$$

King models with c between 0.75 and 1.75 fit globular clusters well.

Observationally, the central surface brightness, Σ_K, and the core radius, r_c, can be used in place of $\Psi(0)$ and r_0.

Bibliography
Much of the information from older cluster catalogs and other sources is now online in the SIMBAD database.

A rather complete catalog of information on the known globular clusters is maintained by W E Harris on the website:

http://physun.physics.mcmaster.ca/Globular.html

This catalog is also discussed in

Harris W E 1996 A catalogue of parameters for globular clusters in the Milky Way *Astron. J.* **112** 1487

The ages of the globular clusters are discussed in

Chaboyer B, Demarque P, Kernan P J, Krauss L M 1998 The age of globular clusters in light of Hipparcos: resolving the age problem? *Astrophys. J.* **494** 96

The dynamics of the globular clusters are described in detail in

Binney J and Tremaine S 1987 *Galactic Dynamics* (Princeton: Princeton University Press)

Open cluster data can be found in the SIMBAD database, which includes the *Catalogue of Open Cluster Data, Fifth Edition* (1987) compiled by G Lynga (Lund Observatory).

A database of photometric and spectroscopic information on open clusters called WEBDA is maintained at the Geneva Observatory at:

http://obswww.unige.ch

Stellar associations have been reviewed by

Garmany C D 1994 OB Associations: massive stars in context *Publ. Astron. Soc. Pacific* **106** 25

Some key references for individual clusters mentioned in the article are the following:

Hesser J E, Harris W E, VandenBerg D A, Allwright J W B, Shott P and Stetson P B 1987 *Publ. Astron. Soc. Pacific* **99** 739

Montgomery K A, Marschall L A and Janes K A 1993 *Astron. J.* **106** 181

Perryman M A C, Brown A G A, Lebreton Y, Gomez A, Turon C, Cayrel de Strobel G, Mermilliod J C, Robichon N, Kovalesky J and Crifo C 1998 *Astron. Astrophys.* **331** 81

Stetson P B and Harris W E 1988 *Astron. J.* **96** 909

Finally, data for the ZAMS can be found in

Schmidt-Kaler T 1982 *Landolt-Börnstein VI* vol 2b (Berlin: Springer) p 1

Kenneth Janes