Three typefaces for mathematics

The development of Times 4-line Mathematics Series 569, AMS Euler, and Cambria Math

by Daniel Rhatigan

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Typeset with Arno Pro (Adobe) and Whitney (Hoefler & Frere-Jones).

All figures shown at 100% scale unless indicated otherwise.

Daniel Rhatigan,
September 2007

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Abstract

This paper examines the issues involved in the design of typefaces for mathematics. After a brief discussion of some of the typographic and technical requirements of maths composition, three case studies in the development of maths types are presented: Times 4-line Mathematics Series 569, a complement to the Times New Roman text types as set with Monotype equipment; AMS Euler, an experimental design intended to contrast against non-mathematical typefaces set with TEX; and Cambria Math, designed in concert with a new text face to take advantage of new Microsoft solutions for screen display and maths composition.

In all three cases, the typefaces were created to show the capabilities of new technological solutions for setting maths. The technical advances inherent in each font are shown to be as central to its function as its visual characteristics.

By looking at each typeface and technology in turn, and then comparing and contrasting the issues that are addressed in each case, it becomes apparent that even though certain challenges are overcome with technical advances, the need to consider the specific behaviours of type in a maths setting remains constant .

Contents

Introduction

The difficulty of printing mathematical notation has always been a challenge for printers, who have urged mathematicians to use notation that is easier to print,[1] as well as mathematicians, who have sought new means of expression with the type and techniques available.[2] It is not enough to talk about simply choosing a typeface for setting maths when the subject matter has such complex, specific requirements.
The most effective typefaces for maths are those that have been created or adapted especially for it, anticipating both the typographic and the technical issues involved. This dissertation will examine the design and the function of alphabetic characters of three typefaces created specifically for mathematics — Times 4-line Mathematics Series 569, AMS Euler, and Cambria Math.

It will be helpful to look closely at the development of typefaces that shed light on different aspects of the issues involved in setting maths, and see how their designs evolved in response to the available means of composing mathematical work. Each typeface represents a distinct period of technical development, each in some sense reacting to the typefaces and technologies preceding it.

Times 4-line Mathematics Series 569 was created by the Monotype Corporation in the United Kingdom for use with its hot-metal composition equipment. This typeface, a variation of the popular Times New Roman, was specifically developed for use with Monotype's 4-line system, used to mechanise many aspects of maths composition without sacrificing the quality of the finished product.

AMS Euler was designed by typographer and calligrapher Hermann Zapf in collaboration with Donald E Knuth, a mathematician and computer scientist who developed pioneering software for creating digital typefaces and composing text and mathematics with computers. Euler was a test of the capabilities of these new tools, both technically and aesthetically.

Cambria Math was designed by Jelle Bosma to take advantage of two major technologies from Microsoft: one a new way to improve the appearance of text displayed on screens, the other a sophisticated new method of creating and typesetting equations.

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1 'Mathematical work: It's not just a case of 'follow copy''. *The British & Colonial Printer.* 152 (26) (1953) p. 782–785

2 Richard Lawrence. 'Maths = typography?' *TUGboat.* 24 (2) (2003) p. 165–167

The first section presents an overview of the typographic and technical issues that make maths composition so challenging. The second consists of an analysis of each of typeface in turn, including the its history, the details of its design, any features specific to its use for mathematics, and how it addresses the issues presented by the typesetting technologies for which it was developed. The third section will compare and contrast the three typefaces in terms of their design and their context.

Figure 1
A sample of text set with 4-line maths shows the the variety of characters, styles, and sizes which are often used together in maths. Characters in equations must stand alone in strings of symbols, while the characters in the text surrounding the equations are grouped into words.

Information sheet no. 156: 4-line mathematics. London: The Monotype Corporation, Ltd. (1959) p. 4

Heterogeneous Methods for Calculating Reactors 319

Most important is the result of calculating $\sum_k^{\infty} f(r_k)$ for a lattice with a cell in the form of a parallelogram the sides of which are a_1 and a_2, the angle between them being ω.

When computing the sums of f the integral representation of $K_0(x)$ may be used.

$$2\pi L^2\, \Sigma_c f(r) = K_0\left(\frac{r}{L}\right) = \int_0^{\infty} e^{-\left(z+\frac{r^2}{4L^2 z}\right)}\frac{dz}{2z}. \qquad (11.2)$$

Then summing, according to the Poisson rules gives[5]

$$\sum_k^{\infty}{}' K_0\left(\frac{\left|\mathbf{r}-\mathbf{r}_k\right|}{L}\right) = \frac{2\pi L^2}{\sigma}\sum_{\nu_1,\,\nu_2}^{\infty}\frac{e^{2\pi i\left(\frac{\nu_1}{a_1}x+\frac{\nu_2}{a_2}y\right)}}{\left[1+\frac{4\pi^2 L^2}{\sin^2\omega}\left(\frac{\nu_1^2}{a_1^2}+\frac{\nu_2^2}{a_2^2}-\frac{2\nu_1\nu_2}{a_1 a_2}\cos\omega\right)\right]}. \qquad (12.2)$$

If the conditions $\dfrac{4\pi L^2}{a_1 a_2 \sin^2\omega} \gg 1$ are fulfilled, then

$$\sum_k^{\infty}{}' K_0\left(\frac{\left|\mathbf{r}-\mathbf{r}_k\right|}{L}\right) = \frac{2\pi L^2}{\sigma}\left\{1+\frac{\sin^2\omega}{4\pi^2 L^2}\sum_{\nu_1,\,\nu_2}{}'\frac{e^{2\pi i\left(\frac{\nu_1}{a_1}x+\frac{\nu_2}{a_2}y\right)}}{\left(\frac{\nu_1^2}{a_1^2}+\frac{\nu_2^2}{a_2^2}-\frac{2\nu_1\nu_2}{a_1 a_2}\cos\omega\right)}\right\} \qquad (13.2)$$

σ is the area of the lattice-cell; a prime means that the term with $\nu_1 = \nu_2 = 0$ must be dropped.

Thermal utilization θ of an infinite lattice where $K_\infty = 1$, in the absence of

$$y_2(x) = y_1(x)\int^x \frac{\exp\left(-\int_a^{x_2}\sum_{i=-1}^{\infty} p_i x_1^i\, dx_1\right)}{x_2^{2\alpha}\left(\sum_{\lambda=0}^{\infty} a_\lambda x_2^\lambda\right)^2}\, dx_2$$

$$y_2(x) = y_1(x)\int^x \frac{\exp\left(-\int_a^{x_2}\sum_{i=-1}^{\infty} p_i x_1^i\, dx_1\right)}{x_2^{2\alpha}\left(\sum_{\lambda=0}^{\infty} a_\lambda x_2^\lambda\right)^2}\, dx_2$$

Figure 2
The equation above (set with Cambria Math) is shown at full size and magnified 3×. It could be difficult to discern *a* (shown in orange) from *alpha* (shown in blue) without a clear difference in their shapes, since they are not set within words that provide additional context, such as *mathematical* or *μαθηματική*.

John Hudson and Ross Mills, eds. *Mathematical typesetting: Mathematical and scientific typesetting solutions from Microsoft*. Redmond, WA: The Microsoft Corporation (2007) p. 54

1 The difficulty of typesetting mathematics

1.1 Typeface requirements

Mathematics is a field of study that employs its own vocabulary and conventions, and in many ways it has a language and writing system of its own. Although its notation uses many familiar characters, it uses them as symbols rather than words.[3] However, maths is set alongside the words that convey its meaning, and publishing maths requires the ability to mix the symbolic and the verbal languages with one another.

Setting complex mathematics requires the use of a wide array of characters that must work in harmony. Numbers are mixed with alphabetic characters of Latin and Greek origin in a variety of styles — italic, bold, fraktur, serif and serif forms — each with a specific semantic function. All these characters are then mixed with mathematical operators and other symbols that often conflict with the scale or texture of the other characters. Setting maths, then, requires access to a vast set of unique characters, preferably ones that have been designed to work with one another. *(See figure 1.)*

Setting the material so that it can be read and understood easily requires a delicate balance between characters that blend with one another yet can also be easily distinguished. A reader's ability to recognise words by their overall shape makes it easier to compensate for individual characters whose shape may be unusual or ambiguous. In mathematics, however, each letter, number, or symbol — as well as each of its stylistic variants — must be perfectly legible in isolation, since these are read not as parts of words but as discrete signifiers of an equation's meaning.[4] *(See figure 2.)* If the Greek letter α (alpha) is incorrectly set within the word *mathematics* (like so: *mathematics*) the word's meaning is not obscured by the error. In maths, where letters are used to represent numerical values, confusing the similar letter shapes within $x = a + 4$ and $\chi = \alpha + 4$ changes the result of the calculation. A reliable typeface for mathematics must offer enough visual cues to help the reader avoid such confusion in the event that the text surrounding the equations does not.

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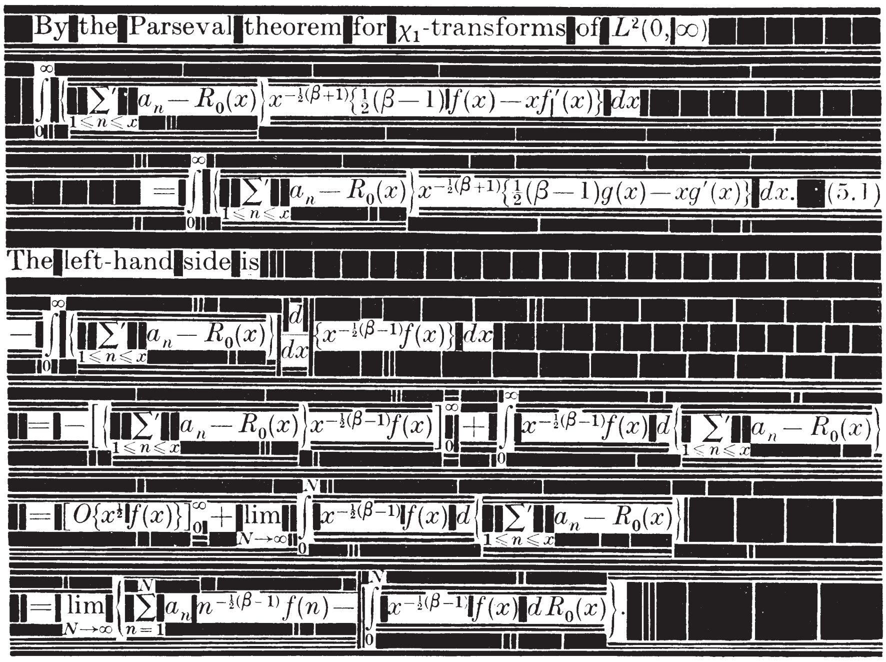
3 Arthur Phillips. 'Setting mathematics', *The Monotype Recorder.* 40 (4) (1957) p. 3

4 John Hudson and Ross Mills, eds. *Mathematical typesetting: Mathematical and scientific typesetting solutions from Microsoft.* Redmond, WA: The Microsoft Corporation (2007) p. 13

Figure 3

This diagram shows the complex arrangement of characters and spaces required to compose mathematics with metal type. Not only are numerous type styles and non-standard symbols mixed together, but characters are set on a wide variety of body sizes. The black rectangles indicate the many sizes of spaces that must be arranged to hold the characters in place. Before the introduction of the 4-line system, expressions like these were composed entirely by hand out of characters cast in multiple batches, requiring the time and skill of a highly trained compositor.

T W Chaundy, P R Barrett, and Charles Batey. *The printing of mathematics*. London: Oxford University Press (1954) p. 4



1.2 Technical concerns

A suitable typeface for maths is only a partial solution without a way to compose the material properly. Most typesetting solutions have difficulty with the spatial arrangement required for maths. Equations often work in two dimensions, with frequent use of stacked components, oversize symbols, and multiple levels of superior and inferior positioning. With so much variation along a vertical axis and with individual characters rather than words as the basic unit of meaning, horizontal spacing that has been designed for text use is rarely helpful.

Compositors working with metal type often tried to ease the process by urging authors to use alternate forms of notation that could be set more easily, but the clarity of the subject matter often depended on notation that was difficult to set. Even if there were room in the matrix case for all the symbols needed at one time, the frequent use of complex positioning and various sizes of symbols require type set on alternate body sizes and fitted together like a puzzle. This wide variety of type styles and sizes made it costly to set text with even moderately complex mathematics, since so much time and effort went into composing the material by hand at the make-up stage. *(See figure 3.)*

Filmsetting and photocomposition systems made it easier to position characters in equations since they were not constrained by the very physical problem of fitting unyielding pieces of lead together, but they still presented the problems of making the full set of necessary characters available at once. In practice, these methods of typesetting still required their own version of hand-setting, except pieces of film or photographic paper were assembled rather than pieces of metal type.[5]

Digital typesetting tools remove the physical limitations of fitting pieces of type together, but a piece of software that sets text quickly and efficiently may still have difficulties with maths. Word processing and desktop publishing applications set type in a manner that mimics physical typesetting methods: characters are set in a line, and those lines are set one after another. If the software is not written to arrange equations as easily it can arrange text, a user can only set maths by resorting to a digital equivalent of hand-setting: placing characters one by one to build an equation slowly and consciously. As with hand-setting methods for metal or film, this is slow and ultimately expensive.

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5 J E Poole. *Mathematical formulae*. London: The Monotype Corporation, Ltd. (1971) p. 6–8

CHARACTERS CUT TO DATE IN SERIES 569—10 POINT

FEBRUARY 1959

Group 1 10 pt. Latin Light: Roman and Italic

ABCDEFGHIJKLMNOPQRSTUVWXYZ
abcdefghijklmnopqrstuvwxyzfiflffiffiffl ff

ABCDEFGHIJKLMNOPQRSTUVWXYZ
abcdefghijklmnopqrstuvwxyz

A 1901A   O 15840   Q 304Q   R 573R   X 247X
Y 493Y   *Ā* 1925A   *B̄* 336B   *D* 658D   *K* 370K
N 626N   *O* 15660   *Q* 303Q   *U* 977U   *Ū* 993U
X 246X   *Z̄* 573Z   *f* 448F   *f̄* 465F   *g* 82G
q̄ 308Q   *t̄* 700T   *v* 135V   *ẍ* 42X   *x̄* 251X
ȳ 498Y   *z̄* 572Z   ∂ 297D   ∃ 945E   ℵ 1884A
℘ 275C

1234567890   ¼½¾⅜⅝⅓⅔⅙ F1379   .,:;-!?‘’()*†.–…—£$% ::

(S12003   ) S12004   [S12005   ] S12006   ⟦ S12007
⟧ S12008   *⟦* S12009   *⟧* S12010   { S12011   } S12012
(S12013   ) S12014   [S12015   ] S12016   ⟦ S12017
⟧ S12018   *⟦* S12019   *⟧* S12020   + S3460   ± S3462
∓ S12044   ± S12045   ⊕ S12046   − S3461   ⊖ S12047
× S3463   ⊗ S12048   ÷ S3464   = S3465   ≑ S5515
≒ S5864   ≐ S12066   ≠ S3526   ⧣ S3535   ⋕ S12073
≡ S3525   ≢ S10477   ≢ S12074   < S3466   > S3467
≮ S4862   ≯ S4863   ≷ S11736   ≶ S11737   ≦ S3523
≧ S3524   ≱ S12065   ⋚ S12063   ⋛ S12064   ≪ S4596
≫ S4597   ⋙ S7656   ∼ S5119   ≁ S5120   ≃ S5121
⊀ S12055   ≂ S9569   ≄ S12057   ≅ S6654   ≿ S9567
≾ S9568   ≈ S7490   ⊂ S6032   ⊃ S6033   ⊄ S6938
⊅ S6939   ⊆ S5769   ⊇ S6031   ⊈ S12095   ⊉ S12096
→ S12075   ↦ S12077   ← S12078   ↔ S12079   ↮ S12080
⇌ S12053   ⇋ S12054   ∶ S12034   ∴ S3696   ∵ S12033
⋯ S12097   ∷ S12035   ∞ S12038   ∝ S12039   ∧ S12081
≻ S12058   □ S12082   √ S12094   √ S12301   ∊ S12067
∍ S12068   ∩ S12114   ∪ S12115   ⟨ S12036   ⟩ S12037
∈ S12071   ∉ S12072   ′ S12027   ″ S12028   ‴ S12029
⁗ S12030   ‵ S12031   ° S12032   · S12040   · S12113
∣ S12001   ‖ S12002   ∣ S12021   ‖ S12022   / S120
⊢ S12069   ∤ S12103

Group 2 10 pt. Latin Bold: Roman and Italic

ABCDEFGHIJKLMNOPQRSTUVWXYZ
abcdefhijklmnopqrstuwxy

D 659D   **G** 622G   **N** 627N   **U** 987U
f 448F   ***g*** 82G   ***r̈*** 173R   ***v*** 135V   ***z*** 531Z

Group 3 10 pt. Greek Light: Upright

ΓΔΘΛΞΠΣΦΨΩ   ∏ 624P   ∑ 906S   ∇ 48D
Ϝ 651D   Θ 683T   Λ 547L   Φ 628P   Ω 15670

Group 4 10 pt. Greek Light: Inclined

αβγδεζηθκλμνξπρστυφχψωϑς
ϖ̄ 16060   *ϖ* 616P   *ϱ* 566R   *θ̂* 191T

Group 5 10 pt. Greek Bold: Upright

ΓΔΘΛΞΠΣΦΨΩ
αβγδεζηθκλμνξπρστυφχψωϑ   **ϖ** 616P   **ϱ** 566R

Group 7*
First-order Superiors or Inferiors Latin Light: Roman and Italic

ABCDEFGHIJKLMNOPQRSTUVWXYZ   i   *abcdefhijklmnopqrstuwxyz*
g 82G   v 135V   1234567890   ¼½ F1380   .,:!()[]*...—

\+ S12041   ± S12043   ‾ S12042   ×˙ S12049   ÷ S12050
= S12051   ≠ S12052   ≡ S12070   < S12059   > S12060
≮ S12061   ≯ S12062   ~ S12056   ∞ S12099   ′ S12098
″ S12112   / S12023   ∣ S12024   ‖ S12025   crit S12107
exp S12108   lim S12104   log S12105   max S12106   min S12109
mod S12110   opt S12111   cos S12397   → S12076   ← S12583

Group 8 First-order Superiors or Inferiors Greek Light: Upright

ΓΔΘΛΞΠΣΦΨΩ

Group 9 First-order Superiors or Inferiors Greek Light: Inclined

αβγδεζηθκλμνξπρστυφχψω

Group 10 Second-order Superiors Latin Light: Roman and Italic

abcdefhijklmnopqrstuwxyz   g 82G   v 135V   1234567890   ½ F1381   ()[]

\+ S12100   ‾ S12101   = S12102   > S12092   < S12093
/ S12023   ∣ S12026

Group 11 Second-order Inferiors Latin Light: Roman and Italic

abcdefhijklmnopqrstuwxyz   g 82G   v 135V   1234567890   ½ F1381   ()[]

\+ S12100   ‾ S12101   = S12102   > S12092   < S12093
/ S12023   ∣ S12026

Group 12 Second-order Superiors Greek Light: Inclined

αβγδεζηθκλμνξπρστυφχψω

Group 13 Second-order Inferiors Greek Light: Inclined

αβγδεζηθκλμνξπρστυφχψω

Group 16 10 pt. Latin Light: Italic Capitals kerned one or two units

CFHIJKMNOPRSUX
O 15660   *N* 626N   *U* 977U   *X* 246X

Group 18 10 pt. Latin Light: Script Capitals

𝒜ℬℰℱℐ𝒫ℛ

Group 19
First-order Superiors or Inferiors Latin Light: Script Capitals

𝒜

26 pt. Display

∫ S12083   (S12088   [S12089   { S12090   √ S12091
∮ S12084   ∯ S12085   ∲ S12086   ∳ S12087

** Missing group numbers are those of groups of characters that are being added to the range*

Figure 4
A specimen sheet of the matrices available for Times 4-Line Mathematics Series 569 shortly after its release. New pattern drawings were made for all the glyphs in the series to ensure consistency within the series, for both aesthetic and technical reasons.

This specimen was printed with galleys of metal type made with a Monotype composition caster fitted with 4-line maths attachments.

Information sheet no. 156, p. 6

2 The Typefaces

2.1 Times 4-line Mathematics Series 569

After World War II, an explosion of scientific and technical publishing placed a strain on traditional methods for composing mathematics. The printers specialising in mathematics wanted to increase the efficiency and reduce the cost of their operations, and the rest wanted to try and meet some of this new demand. The Monotype Corporation, Ltd., a prominent manufacturer of machinery for setting metal type, was able to adapt their equipment to mechanise many aspects of maths setting, but this advance in typesetting required a major redevelopment of a typeface to work with the new system. Monotype chose to adapt Times New Roman, one of its most successful type families.[6]

Monotype identified the type families created for its metal and film equipment with a series number. In the case of Times, the basic text face was officially named Times New Roman Series 327, but other variations were given other designations. Although the font introduced for 4-line maths was still a Times New Roman design, it was officially named Times 4-line Mathematics Series 569. *(See figure 4.)*

The design for Times New Roman was licensed to many other companies over the years, and adapted for a wide variety of typesetting methods. This analysis focuses only on Series 569, Monotype's own adaptation of the design for use in maths setting.

2.1.1 Monotype 4-line Mathematics In 1957, Monotype introduced its 4-line system for mathematics, a technique for composing mathematical formulae with metal type that would reduce the amount of time-consuming, costly hand composition needed for maths. With this new method — inspired by the 'Patton Method', a similar development at Lanston Monotype, the company's counterpart in the United States — Monotype hoped to automate some of this process without sacrificing any of the quality that had only been possible with hand-setting until that time.[7]

Monotype equipment was already used by many printers of mathematics, since it allowed them not only to set text mechanically but

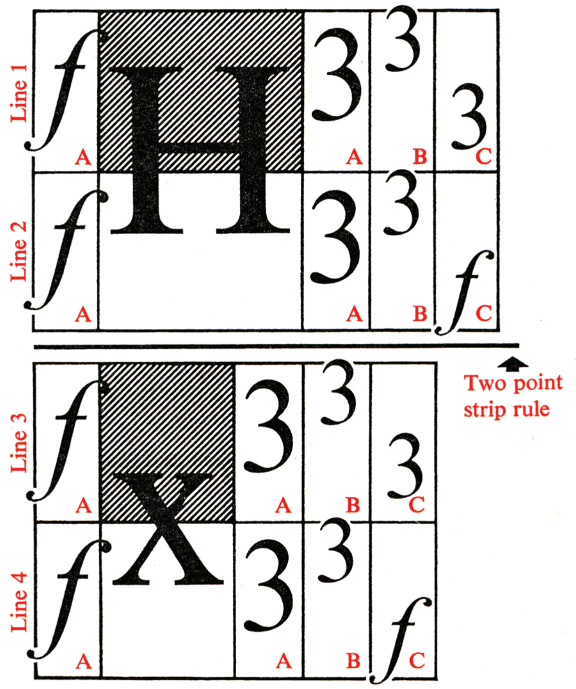
6 Lawrence W Wallis. 'Monotype time check', *The Monotype Recorder*. New series (10) (1997) p. 50

7 J E Poole. 'Mechanising mathematics', *Monotype Newsletter*. 81 (1967) p. 19–22

Figure 5

This diagram shows how characters of different sizes can be arranged within a 4-line equation. Character details that extend past the edge of any rectangle indicate kerns that must be supported by shoulder-high spaces, strip rules, or the shoulders of adjacent characters.

J E Poole. *Mathematical formulae.* London: The Monotype Corporation, Ltd. (1971) p. 2



Key to symbols
Division of top and bottom of equation each into two half-bodies allows:

A	B	C
One matrix to provide 1st order superiors and inferiors	One matrix to provide 2nd order superiors to both 1st order superiors and/or inferiors	One matrix to provide 2nd order inferiors to both 1st order superiors and/or inferiors

also to create individual pieces of type that could be composed by hand and inserted into text at a later stage. However, the skill and experience required for any maths composition confined it to specialised printers who concentrated on the subject.[8]

The 4-line system was a combination of type matrices, attachments for Monotype's keyboard and composition caster, and procedures for making up galleys after casting. Equations were set separate from the body text and inserted later, as with traditional hand-setting of maths, but the system allowed complex equations to be cast with minimal manual intervention, speeding up the overall process of composing maths considerably.

Rather than casting type on a body size that matched its point size, 4-line equations were planned as four rows of characters set on a half-size body and then stacked together. *(See figure 5.)* Characters were set at 10-point size on 6-point bodies, with overhanging details supported by spaces of the same width set in the line above. Full-size characters therefore took up two rows of the equation, while inferiors and superiors (which would barely hang over the 6-point body, if at all) could be placed on either of those rows as needed. Afterwards, a compositor could insert strip rules or oversize symbols, or arrange any other details of the equation that could not fit within the basic arrangement of four lines.[9]

New matrices for casting the type were needed as well, and Monotype devoted its resources to developing a single series that could take full advantage of the new system. The company chose the increasingly popular Times New Roman Series 327, which only had a few matching maths symbols at the time. Times 4-line Mathematics Series 569, as the new series became known, contained many new glyphs that were drawn to reduce the need for kerning and enable, where appropriate, the reuse of matrices for both superior and inferior characters. Greek characters, operators, and other mathematical symbols previously cut for other series were added to Series 569 and redrawn to harmonise with its overall design. New matrices were also made for oversized fence characters (brackets, braces, parentheses, etc.) in sizes up to 72 point.

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8 T W Chaundy, P R Barrett, and Charles Batey. *The printing of mathematics.* London: Oxford University Press (1954) p. 11–16

9 *Information sheet no. 156: 4-line mathematics.* London: The Monotype Corporation, Ltd. (1959) p. 1–2

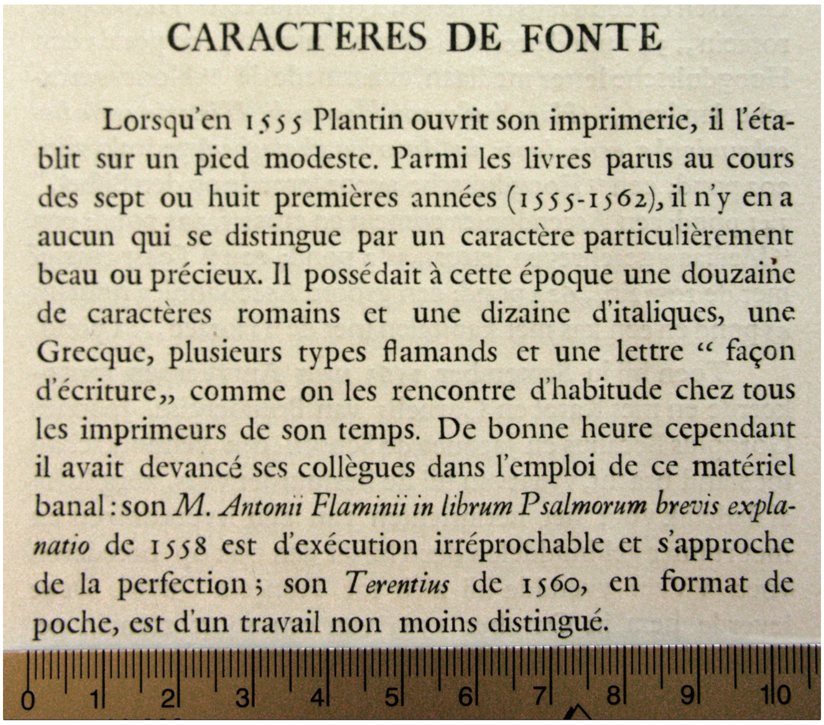
Figure 6

A Granjon's 'Gros Cicero', used in the preface of the Plantin-Moretus Museum's *Index characterum* and considered the model for Times New Roman.

B Monotype Times New Roman Series 327, 12 point.

Max Rooses. *Index Characterum Architypographiae Plantinianae*. Antwerp: Plantin-Moretus Museum (1905)

Monotype specimen sheet (1962)



CARACTERES DE FONTE

Lorsqu'en 1555 Plantin ouvrit son imprimerie, il l'établit sur un pied modeste. Parmi les livres parus au cours des sept ou huit premières années (1555-1562), il n'y en a aucun qui se distingue par un caractère particulièrement beau ou précieux. Il possédait à cette époque une douzaine de caractères romains et une dizaine d'italiques, une Grecque, plusieurs types flamands et une lettre " façon d'écriture„ comme on les rencontre d'habitude chez tous les imprimeurs de son temps. De bonne heure cependant il avait devancé ses collègues dans l'emploi de ce matériel banal : son *M. Antonii Flaminii in librum Psalmorum brevis explanatio* de 1558 est d'exécution irréprochable et s'approche de la perfection ; son *Terentius* de 1560, en format de poche, est d'un travail non moins distingué.

A

WHEN JOBS HAVE TYPE SIZES FIXED QUICKLY MARGINS
When jobs have type sizes fixed quickly margins of error widen unless the determining calculations are based upon factual rather than hypothetical figures
When jobs have type sizes fixed quickly margins of error widen unless the determining calculations are
ABCDEFGHJKLMNOPQRSTUVWXYZ *ABCDEF*

B

$$\frac{\sin^2 \omega}{4\pi^2 L^2} \sum_{\nu_1, \nu_2}{}'$$

A

$$\frac{\sin^2 \omega}{4\pi^2 L^2} \sum_{\nu_1, \nu_2}{}'$$

B

Figure 7

A Modern Series 7 composed by hand.

B Series 569 set with the 4-line system.

These details (magnified 2×) from a Monotype sample show the improvements offered with Series 569. Overall the text is slightly narrower, and features less contrast. Numerals have a lighter color and more open counters that improve their clarity at small sizes. Greek characters are drawn with a compatible x-height to that of the roman. Since the design of the series anticipates combinations found in maths, characters such as the italic L have kerns for better positioning of smaller characters.

Information sheet no. 156, p. 4

2.1.2 Development of Times New Roman and Series 569 The complete story of Times New Roman's development is outside the scope of this discussion, but the essential details are that the Times New Roman family was first developed by Monotype in 1931 for use by *The Times* newspaper of London at the recommendation of Stanley Morison, and then later released publicly and licensed out to many other foundries and type vendors as years passed.

After examining of a number of trial settings showing different faces, *The Times* asked Morison and Monotype to develop a modernised version of the typeface Plantin. In his own writings Morison claims to have shown a sheet of trial drawings to Victor Lardent, a production artist for *The Times*, who then made a set of finished drawings based on those ideas and continued to refine the design under Morison's direction. Many years later, however, Lardent claimed that Morison showed him either a specimen sheet or a photograph of a page set with one of the types on which Monotype's Plantin was based. The prevailing idea is that Lardent worked from a reproduction of a page of text from the Plantin-Moretus Museum's *Index characterum*, the preface of which was set with the 'Gros Cicero' cut by Robert Granjon circa 1568. *(See figure 6.)*

The basic alphabet design, including italic and bold styles, was expanded to a full set of characters in a range of sizes by Frank Hinman Pierpont's staff at Monotype, and finally released in 1931.[10]

Before the introduction of the 4-line system, guides to mathematical composition consistently recommended Monotype's Modern Series 7 for text featuring maths. *(See figure 7.)* However, a growing number of requests for additional matrices for Series 327 convinced Monotype that Times New Roman was becoming more popular for technical publications, and would be a strategic choice to bring to market at that time.[11,12]

Although its customers might have preferred more typographic choices, the lengthy process of creating new matrices for the vast number of characters needed for mathematics would have been too

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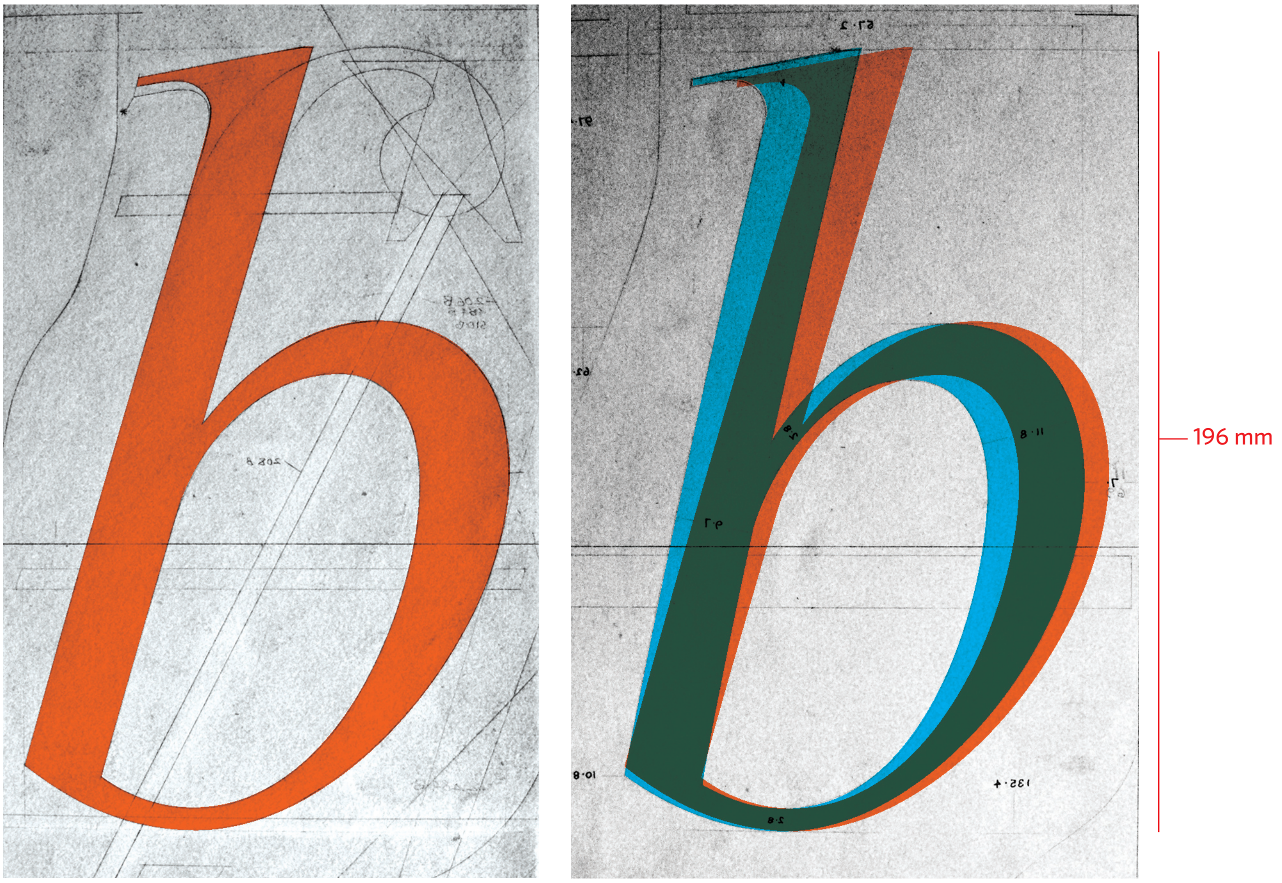
10 John Dreyfus. 'The evolution of Times New Roman', *The Penrose Annual 66*, edited by Herbert Spencer. London: Lund Humphries (1973) p. 167

11 Poole, *Mathematical formulae*, p. 2

12 Dreyfus, 'The evolution of Times New Roman', p. 172

As Dreyfus notes: 'When war-time restrictions on paper supplies impelled a more economic use of materials, the exceptional space-saving qualities of Times New Roman brought it into far wider use for every kind of printing.'



A B

Figure 8

Comparison of master drawings for italic *b* in Series 327 and 569. (Each drawing is shown in reverse to show the final orientation of the character.)

A Series 327 has a slant of 16°. The shaded area is the outline of the base character.

B Series 569 (shaded in blue) has a slant of 12°.

Archives of Monotype Imaging, Ltd. Salfords, Redhill, Surrey (photographed by the author, 3 September 2007) costly to repeat for multiple series, or even for additional point sizes within a single series.[13]

Series 569 was planned as a consolidation of various glyphs that existed within existing versions of Times New Roman and maths symbols throughout Monotype's library, all adjusted to work together within the constraints of the 4-line system. Certain aspects of Times New Roman, particularly the italics, had to be redrawn, but the intricate interaction of such a large character set eventually required that all the necessary characters be drawn and fine-tuned. Even the basic roman characters were drawn again, since there were no master drawings for 10-point Times New Roman: the 10-point matrices were created from the same drawings used for the 12-point.[14]

Customer feedback to early trials of the 4-line system led to changes in many characters used in Series 569. It is presumable that the specialist printers saw Monotype's investment in this new system — and its willingness to consult with them — as an opportunity not just to match the quality of what was capable with hand-set type, but to exceed it in terms of typographic consistency and clarity.

2.1.3 Characteristics of the design Times New Roman is a serif typeface with quite a lot of stroke contrast and very sharp, bracketed serifs (particularly in its original hot-metal version). Despite having some structural features of an oldstyle type, the overall effect is very crisp, upright, and compact. The bold weight shares its character widths with the roman, resulting in glyphs that feel quite bit narrower, with even more contrast.

One of the most obvious differences between the text and the mathematical versions of Times New Roman is the overall angle of slant in the italic faces. Series 327's italic features a dramatic 16° slant, but this angle was reduced to approximately 12° in Series 569 to minimise the need for kerning, especially since so many characters are set superior and inferior to the full-size letters. *(See figure 8.)*

The superior and inferior characters themselves are markedly different from Series 327 as well. Typically, superiors or inferiors would

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13 *A pocket picture book of 'Monotype' machines*. London: The Monotype Corporation, Ltd. (n.d., ca. 1965) p. 23

From the creation of working drawings to the final stage of quality control for a matrix, Monotype went through 82 steps to create each new character in any of its series.

14 Robin Nicholas. Interview with author. Monotype Imaging, Ltd. Salfords, Redhill, Surrey (3 September 2007)

Figure 9

Monotype’s still offers an electronic version of the Series 569 design, but the typeface is no longer identified as having a relationship to Times New Roman.

Search results for the term ‘math’ at http://fonts.com (viewed 4 September 2007) be cast separately using matrices of another font size (such as 5.5-point characters used with 10 point text) and then inserted by hand after the primary text was cast. Series 569 included full sets of alphabetic and numeric characters specifically drawn for use as superiors and inferiors. These are bolder yet narrower than the 5.5-point glyphs of Series 327, and include separate drawings with slight size differences for the first- and second-order glyphs.

Series 569 includes Greek, fraktur, and script alphabets; alternate versions of many glyphs; numerous versions with attached accent marks or maths symbols; and a vast selection of maths symbols. Somewhere between 700 and 800 different matrices were prepared for the initial release, but by 1971 this number had grown to over 8,000, with up to five new matrices still being added each week. Considering that many glyphs could be positioned as either superiors or inferiors, this actually reflected over 11,000 characters that were available for 4-line maths.[15]

2.1.4 Updates for newer technologies Although unable to automate maths composition entirely, the 4-line system nevertheless transformed the printing of mathematics. Not long after its introduction, sources described Monotype's method as the standard technique for such material.[16] Series 569 and the principles of the 4-line layout were adapted for Monophoto — Monotype's filmsetting products[17] — and their relevance carried on until the digital era demanded other solutions for setting mathematics.

The maths characters were digitised along with the other versions of Times New Roman, but Monotype had always offered Series 569 in concert with its typesetting equipment and not promoted it on its own. As Monotype turned from being an equipment manufacturer to a digital font vendor, Series 569 was no longer even identified as a member of the Times New Roman family. The fonts are still available for sale, but only offered as generic sets of symbols for mathematical usage. *(See figure 9.)*

15 Poole, *Mathematical formulae*, p. 2

16 Karel Wick. *Rules for type-setting mathematics*. Prague: Publishing House of the Czeckoslovak Academy of Sciences (1965) p. 12

17 J E Poole. 'Filmsetter mathematics', *Monotype Newsletter*. 82 (1967) p. 2
Monotype's own literature continued to assert the usefulness of 4-line metal composition over filmsetting for some kinds of notation.

Figure 10

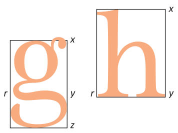
A complete specimen of the glyphs produced for the first release of AMS Euler in September 1985.

The type for this specimen was generated by METAFONT as bitmaps with a resolution of 1446 lines per inch and then output as camera-ready copy from an APS digital typesetter.

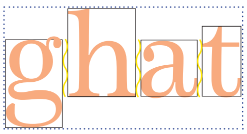
David Siegel. *The Euler Project at Stanford.* Stanford, CA: Stanford University Department of Computer Science (1985) p. 25–31

2.2 AMS Euler

The next significant breakthrough in mathematical typesetting after the 4-line system was the creation of software and fonts for digital composition. In particular, the work of mathematician and computer scientist Donald E Knuth has been responsible for giving authors tools for composing complex maths on their own. Knuth's collaboration with accomplished designer Hermann Zapf on AMS Euler *(see figure 10)*, commissioned and made freely available by the American Mathematical Society, was intended from the beginning to be a typeface for maths — a design experiment as much as a technical experiment.



Typographic elements in TeX; are defined as boxes. the x-y distance is the glyph height, the y-z is its depth. Where the two meet is the reference point (r).



TeX character boxes are 'glued' together to form word boxes (dotted line).

TeX word boxes are glued together to form line boxes (double-line) and line boxes are glued together to form paragraph boxes. The 'glue' between any of these boxes can be stretched or compressed to alter character spacing, word spacing etc. and to aid in paragraph justification.

Figure 11

TeX composes text and pages by dealing with the elements within as a series of boxes within boxes. At each level — from the spaces between letters to the spaces between paragraphs — space is treated as a flexible 'glue' that binds the boxes together in their assigned positions.

Hudson and Mills, *Mathematical Typesetting*, p. 22

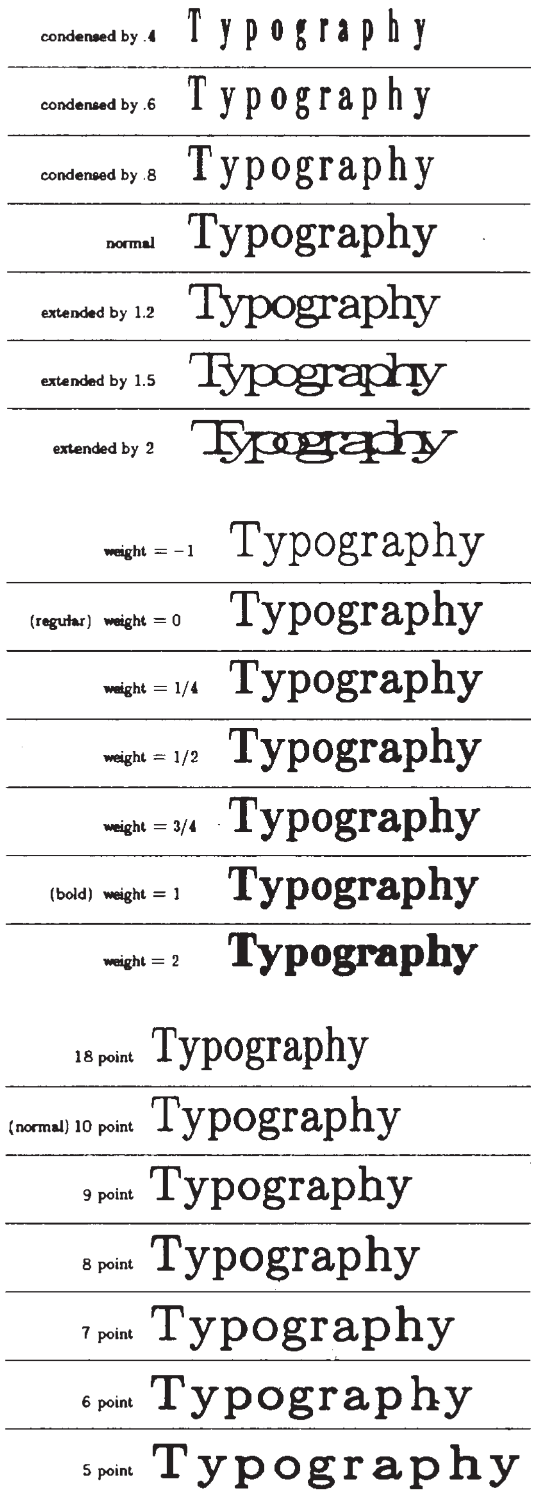
2.2.1 METAFONT and TeX Donald E Knuth, a professor of mathematics and computer science at Stanford University, began researching typographic issues in 1977, observing that 'mathematics books and journals do not look as beautiful as they used to.'[18] Frustrated by the decline in quality as publishers sought ways to publish mathematics that were cheaper than printing with metal type, Knuth began work on TeX (Tau Epsilon Chi), a system for composing text and maths. TeX is both a command language keyed along with the text and the programs that process those commands. Knuth's goal was to create a way for authors to integrate composition with their writing process, giving them control of the process to reduce costs and improve accuracy.

TeX treats every element within a composition as a box, and arranges these boxes within other boxes. The most basic element is the individual glyph, whose box is determined by its vertical and horizontal dimensions as defined in a font. Glyphs are arranged into words or equations, which are arranged into lines, which are arranged into paragraphs, which are arranged into pages. At every level, TeX provides controls to adjust positioning and spacing, the 'glue' used to fit each group of boxes within a larger box. *(See figure 11.)* The instructions for the entire arrangement are integrated with the text itself, so that each document contains complete information about how it should look. That way, the author can write and compose a document that will be printed elsewhere without needing to be created again in another medium.[19]

18 Donald E Knuth. 'Mathematical Typography', *TeX and METAFONT: New directions in typesetting*. Providence, RI: The American Mathematical Society (1979) p. 1
19 Knuth. 'Mathematical Typography', p. 11–12

Figure 12
Fonts with the same basic framework are manipulated according to different parameters with METAFONT: width, stroke contrast, and features of optical sizes (weight, width, spacing all adjusted together).

Donald E Knuth. 'Lessons learned from METAFONT.' *Digital typography*. Stanford, CA: CSLI Publications (1999) p. 317 (magnified 2×)



Knuth's research into typography also led to METAFONT, a tool for designing digital typefaces. He defined a meta-font as 'a schematic description of how to draw a family of fonts'[20] — guidelines for specifying the basic structure of letters so that distinct font styles can be generated by manipulating properties of shape, proportion, weight, slant, and more. In METAFONT, a character is described by plotting key positions along its form, and then specifying how these points should be connected. Lines connecting the points can be manipulated as if they were drawn by pens of different sizes, or held at different angles. Once a relationship between the points is defined, the overall shape can be modified by changing one or more parameters that affect the overall shape of the letter, and the strokes that produce it.[21] *(See figure 12.)*

After setting all these properties, METAFONT generates the glyphs as a bitmap font targeted for use at a specific size and output resolution.

METAFONT's dynamic font creation and TEX's powerful typesetting capabilities made it possible to circumvent the equipment and typefaces available from professional compositors, yet still produce material as difficult to typeset as mathematics. The software requires that decisions be made by a knowledgeable user, but the user can control every aspect of the work, from the content to its final layout.

2.2.2 Development of AMS Euler In 1979 the American Mathematical Society formed a committee to plan its use of emerging technologies for publishing, primarily TEX and METAFONT. The committee invited calligrapher and type designer Hermann Zapf to work with Knuth on a new typeface for publications set with TEX. The two men used this opportunity to explore a number of ideas about how maths content related to text and about the forms that suited mathematics best, resulting in a set of unconventional alphabets collectively known as AMS Euler, in honour of 18th Century mathematician Leonhard Euler.

Curiously, Knuth and Zapf's early correspondence about the project reveals an intent to develop something fairly neutral and rooted in tradition, but as each contributed his own innovations they arrived at a design that departs radically from any standard conventions for mathematical type. Zapf suggested a design with no italic slant to make it easier to combine alphabetic and numeric characters with the special

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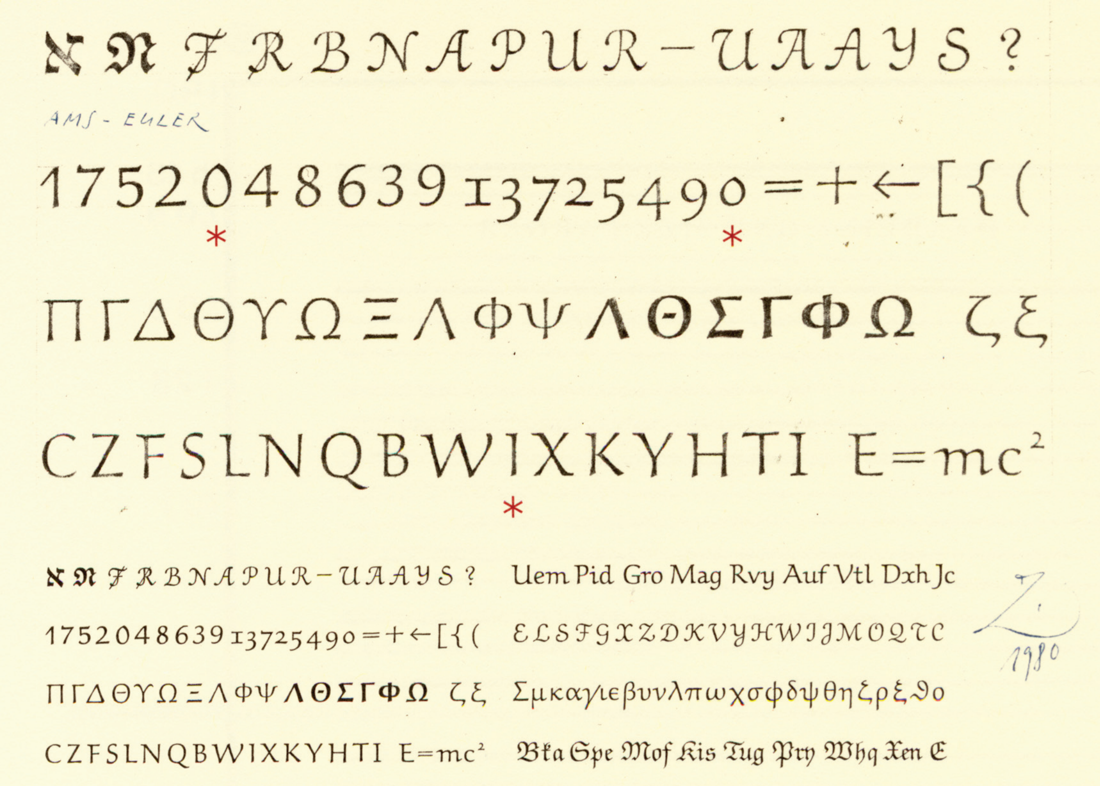
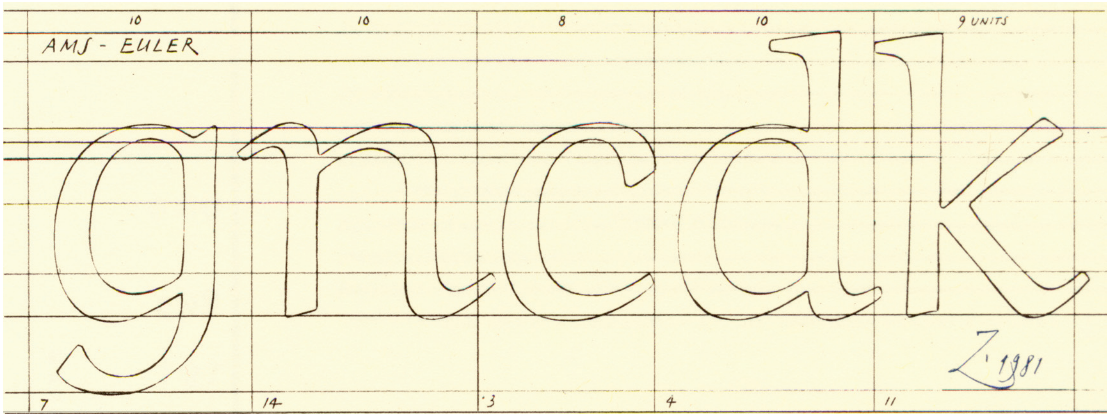
20 Donald E. Knuth. 'The concept of a meta-font', *Digital typography*. Stanford, CA: CSLI Publications (1999) p. 291

21 Donald E. Knuth. 'Lessons learned from METAFONT', *Digital typography*. Stanford, CA: CSLI Publications (1999) p. 315–319



A



B

Figure 13

A Sketches presented to Knuth and the AMS committee for feedback.

B Final drawings used for digitising the glyphs.

Hermann Zapf. *Hermann Zapf & his design philosophy*. Chicago: The Society of Typographic Arts (1987) p. 238

marks and symbols used frequently in maths. Knuth in turn suggested that a calligraphic design would resonate with mathematicians, who developed their equations in writing. Starting with these core concepts, the pair developed the idea for an alphabet of upright italic letters that stands apart from surrounding text rather than blending in with it.

Although Knuth and Zapf both contributed to the overall concept, Zapf was the primary designer, providing Knuth and the AMS committee with proposal sketches. Knuth offered feedback, also collecting and filtering feedback from the AMS committee, after which Zapf would produce detailed master drawings. *(See figure 13.)* Overall, the committee was extremely pleased with the direction of Euler, offering comments that helped Zapf bridge the gap between calligraphic conventions and forms for certain letters that felt more familiar to mathematicians.

The first phase of the project, during which Zapf created and refined a set of alphabets for Euler — two weights each of roman, Greek, fraktur, and script letters, as well as lining and non-lining figures and a few other symbols — lasted approximately two years, but the project was far from complete: the Euler fonts were not completed until 1985.[22]

Upon approval by the committee, Zapf's drawings were to be digitised and built into TEX-compatible fonts using METAFONT. Knuth and a team of Stanford students including David Fuchs, John Hobby, Scott Kim, Dan Mills, Lynn Ruggles, David Siegel, and Carol Twombly spent the next few years trying to develop the Euler drawings into working fonts. The Euler team actually rewrote the METAFONT software itself — as well as a number of software tools to support the production process — as they worked on the fonts, pushing against the limitations of the software and even the computer equipment at their disposal to expand the possibilities of the electronic medium.

Zapf's design defied some of the basic principles of METAFONT. His letters were based on calligraphy, but were subtler in form than Knuth's imagined combination of predictable pen strokes applied to essential skeletal shapes. Reproducing his drawings required the team to plot the inner and outer contours of each glyph rather than building outward from a central gesture. Once they had captured the essence of the glyphs as single programs, they had to define parameters to maintain a consistent weight for the glyphs in each font when the outlines were

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22 Donald E Knuth and Hermann Zapf. 'AMS Euler — a new typeface for mathematics.' *Scholarly publishing.* 20 (3) (1989) p. 132–153

Figure 14

This spacing test for Euler highlights the difficulties of achieving consistent stem weights when outline data was output to bitmap fonts. The *k* in particular shows the tendency of the details to fill in when contours were converted to pixels.

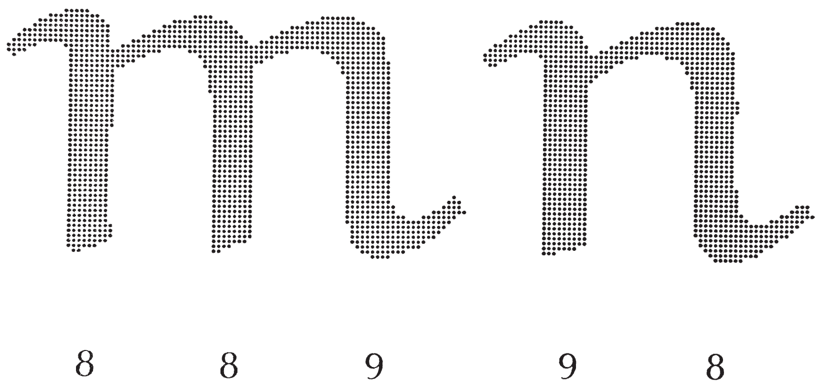
Siegel, *The Euler Project at Stanford*, p. 24

aaabacadaeafagahaiajakalamanaoapaqarasatauavawaxayaza
babbbcbdbebfbgbhbibjbkblbmbnbobpbqbrbsbtbubvbwbxbybzb
cacbcccdcecfcgchcicjckclcmcncocpcqcrcsctcucvcwcxcyczc
dadbdcdddedfdgdhdidjdkdldmdndodpdqdrdsdtdudvdwdxdydzd
eaebecedeeefegeheiejekelemeneoepeqereseteuevewexeyeze
fafbfcfdfefffgfhfifjfkflfmfnfofpfqfrfsftfufvfwfxfyfzf
gagbgcgdgegfggghgigjgkglgmgngogpgqgrgsgtgugvgwgxgygzg
hahbhchdhehfhghhhihjhkhlhmhnhohphqhrhshthuhvhwhxhyhzh
iaibicidieifigihiiijikiliminioipiqirisitiuiviwixiyizi
jajbjcjdjejfjgjhjijjjkjljmjnjojpjqjrjsjtjujvjwjxjyjzj
kakbkckdkekfkgkhkikjkkklkmknkokpkqkrksktkukvkwkxkykzk
lalblcldlelflglhliljlklllmlnlolplqlrlsltlulvlwlxlylzl

Figure 15

Without extra programming to compensate for the effects of placing forms on different pixel grids, scaling glyphs in METAFONT could result in uneven stroke widths, even though the basic shapes are constructed with identical widths.

Siegel, *The Euler Project at Stanford*, p. 22



Bitmaps of 10 point characters at 720 lines/inch.

Figure 16

AMS Euler's Fraktur glyphs (magnified 3×) include some alternate versions.

Siegel, *The Euler Project at Stanford*, p. 31

generated as bitmap fonts, another challenge that exposed the subtleties of Euler compared to earlier METAFONT projects.[23] *(See figures 14 and 15.)*

The first large project to use AMS Euler was *Concrete mathematics*, a textbook co-written by Knuth. The non-mathematical text in the book was set with Concrete Roman, a typeface built with METAFONT to blend well with Euler's relatively dark colour and narrow proportions.[24] After this successful trial, AMS Euler was added to the default fonts that were distributed with TEX.

2.2.3 Characteristics of the design The most striking feature of AMS Euler is the upright cursive design of its Latin alphabets. The uppercase letters do not have as many cursive details, but they feature angled terminals, slightly tapering strokes, and only minimal use of serifs to achieve the same informal effect.The figures share the same calligraphic origins, especially visible in the peaked top of the zero.

The Greek characters have the same cursive quality as the Latin, which gives them an extremely similar texture when characters of the two scripts are both used within an equation. The fonts include only those Greek characters which could not be confused for Latin ones, though. Characters such as the lowercase omicron or uppercase alpha, beta, epsilon, etc. were not drawn at all, since maths authors avoid ambiguity by not using these.

The Fraktur and script alphabets are also somewhat simplified compared to traditional styles, despite their more detailed forms. The AMS committee working with Zapf felt it was particularly important that the design include Fraktur glyphs that could be clearly identified, since there a were few suitable fonts available for this.[25] *(See figure 16.)*

The Euler fonts do not include glyphs specially drawn for superiors and inferiors, but they were created in a range of sizes with slight modifications so that the smallest font (5 point) could be used for this. The fonts only contain a few basic operators and symbols as well. For the most part, TEX's ability to easily combine multiple typefaces made it possible for Euler to draw symbols as needed from other fonts.

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23 David Siegel. *The Euler Project at Stanford*. Stanford, CA: Stanford University Department of Computer Science (1985) p 14–26

24 Donald E Knuth. 'Typesetting concrete mathematics', *Digital typography*. Stanford, CA: CSLI Publications (1999) p. 369–370

25 Knuth and Zapf, 'AMS Euler — a new typeface for mathematics', p. 153

2.2.4 Updates for newer technologies As Knuth himself points out, the final version of Euler was not really a meta-font. Instead, it is an outlined digitisation built with METAFONT tools. Since the difficulties of producing Euler led to a much more sophisticated version of METAFONT, he expressed a hope that others would use it to solve the problem.[26] However, the next major development for Euler was its conversion to the PostScript format,[27] also constructed with outlines, so this potential still has not been realised.

Like TEX and METAFONT, the Euler fonts are distributed at no cost, and users are free to modify them as long as they save the altered versions with a different name before sharing them with others, so there is still potential for Knuth's original plans for Euler to be realized using current design tools.[28]

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26 Ibid., p. 154–155

27 Erik-Jan Vens. 'Conversion of the Euler Metafonts Into the Postscript Type 1 Font Language.' *Proceedings of the Ninth European TEX Conference.* MAPS Special Editions (1995) p. 425–30.

28 'Computer Modern and AMSFonts in Type 1 (PostScript) Form.' http://www.ams.org/tex/type1-fonts.html (viewed 10 September 2007)

ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz

ΑΒΓΔΕΖΗΘΙΚΛΜΝΞΟΠΡΣΤΥΦΧΨΩαββγδεζηθϑικλμνξοπρσςτυφϕχψω

ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz

ΑΒΓΔΕΖΗΘΙΚΛΜΝΞΟΠΡϴΣΤΥΦΧΨΩ∇αβγδεζηθικλμνξοπρσςτυφχψω

ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz

ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz

ΑΒΓΔΕΖΗΘΙΚΛΜΝΞΟΠΡϴΣΤΥΦΧΨΩ∇αβγδεζηθικλμνξοπρσςτυφχψω

ABCDEFGHIJKLMNOPQRSTUVWXYZabcdfhijkmnpqrstuvwxyz

ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz

ABDEFGJKLMNOPQSTUVWXYabcdefghijklmnopqrstuvwxyz

ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz

ABDEFGIJKLMOSTUVWXYabcdefghijklmnopqrstuvwxyz

ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz

ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz

ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz

ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz

ΑΒΓΔΕΖΗΘΙΚΛΜΝΞΟΠΡϴΣΤΥΦΧΨΩ∇αβγδεζηθικλμνξοπρσςτυφχψω

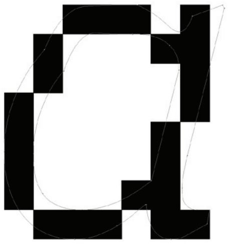
0123456789 0123456789 0123456789 0123456789

123456789 0123456789 0123456789 **0123456789** 0123456789

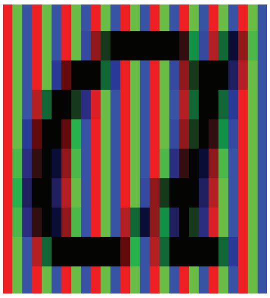
Figure 17

The various glyphs for the Latin and Greek alphabets and the numbers within Cambria Math, set at 12 point. These are not styles applied to the basic characters: each glyph shown is a unique character for mathematics with its own Unicode value. Note that there are no italic versions of the numbers, since upright forms are always used in maths.

Cambria Math contains 4683 glyphs, including variants for many characters that are optimised for use at different sizes, as well as a great number of mathematical symbols.



An italic letter hinted for black & white display. The outline is distorted by hint instructions to turn on or off specific pixels.



ClearType rendering using individual subpixels results in smoother curves and more natural diagonal strokes.

Figure 18
Comparison of how a character outline is translated for screen display with single-tone bitmaps and with ClearType. ClearType takes advantage of how the red, green, and blue subpixels will merge together when seen at a normal size.

Hudson and Mills, *Mathematical Typesetting,* p. 37

2.3 Cambria Math

Cambria is part of a collection of typefaces released by the Microsoft Corporation to take advantage of ClearType, a software-based technology that increases the readability of text displayed on computer screens. The typefaces in the ClearType collection have also been designed to take advantage of the OpenType font format, which can include many more characters in a single font than older formats, as well as a number of instructions about how those characters might be used by different software applications.

To take advantage of these and a number of other technologies—some proprietary, some publicly available—Microsoft developed Cambria Math, a member of the Cambria family specifically designed for typesetting complex mathematics. Cambria Math contains a vast set of glyphs for use with maths *(see figure 17)*, as well as detailed instructions about spacing, positioning, and choosing the correct glyphs for certain situations. This combination of visible and encoded features establishes yet another new possibility for mathematical fonts as complex pieces of software in their own right.

2.3.1 ClearType and the Microsoft maths engine In 1998, while studying issues related to electronic books, Bert Keely and Bill Hill of Microsoft invented ClearType, a way to use software to enhance certain properties of text displayed on a liquid crystal display (LCD) to approximate the effect of text displayed at a higher resolution, if not text printed on paper.[29]

Typically, fonts for screen use rely on hinting, instructions built into the design of each glyph that control how pixels should be arranged to display the font at different sizes. ClearType improves the legibility of text displayed on screen by using each of the three colour channels that comprise each pixel of an LCD monitor—red, green, and blue—as a separate unit (a 'subpixel') for rendering the shape of a glyph. *(See figure 18.)* Since the colour elements within each pixel are arranged in vertical strips, resolution is enhanced along the horizontal axis only. At the same time, anti-aliasing techniques (using shades of grey to give the illusion of smoother contours) improve the apparent quality along the vertical axis. The overall effect is smooth, clear forms that makes it easier and more comfortable to read text on a screen. Since ClearType's

29 John D Berry. *Now read this: The Microsoft ClearType font collection*. Redmond, WA: The Microsoft Corporation (2004) p. 4

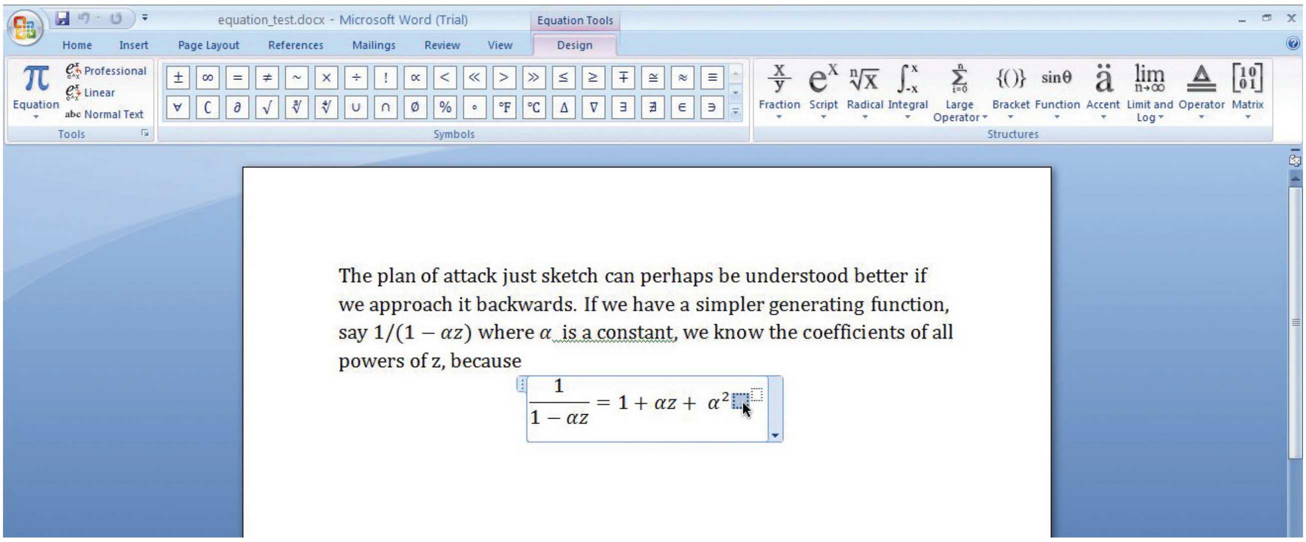


Figure 19

Microsoft Word 2007, running within Windows XP. Text is set in Cambria and Cambria Math.

The new maths engine uses its own interface within other applications. When you write or edit an equation, as shown above, you type within an embedded window that activates the functions and toolbars related to maths.

Text passage: Ronald L Graham, Donald E Knuth, and Oren Patashnik. *Concrete mathematics: A foundation for computer science*. Reading, MA: Addison-Wesley Professional (1988) p. 298

effects on text are based on software settings (as long the display device is an LCD), they can be improved with software updates, as well as adjusted to suit individual preferences.[30]

Microsoft protects ClearType with a number of software patents, but in other areas of its business it has been developing ways of integrating open standards into its products.

One outcome of this approach is a new engine for processing and displaying maths that Microsoft has begun to integrate into some of its products such as Word 2007, RichEdit, and Math Calculator. While some aspects of the maths engine are Microsoft's own — an interface to assist the creation and modification of equations, for instance *(see figure 19)* — it builds upon other tools that have been developed and implemented by others and made publicly available. The engine uses TEX and tools based upon it to format equations. Letters, figures, and symbols are identified with values defined by Unicode, an international standard for encoding text that defines thousands of unique characters. MathML, a standard for describing the content of equations rather than just their appearance, allows the material to be shared with other applications that may use methods of their own to format and display maths. Microsoft has also devised a new set of OpenType font features related to maths composition. (These have not yet been added to the OpenType standard, but they follow the model of other features in anticipation of eventual inclusion.)[31] All these developments, however, would be ineffective without a typeface built to take advantage of them, so Microsoft introduced Cambria Math to do just that.

2.3.2 Development of the Cambria family Microsoft solicited proposals for new typefaces that would take full advantage of the ClearType technology. Jelle Bosma, a designer at Agfa Monotype (a descendant of the Monotype Corporation, Ltd.), proposed a serif font for business documents — a possible replacement for the digital version of Times New Roman that had long been a default setting in Microsoft products.[32]

Bosma's goal was to create a typeface suited to ClearType's rendering capabilities for easy reading on-screen while also functioning for

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30 Berry, *Now read this,* p. 7–14

31 Murray Sargent. 'High-quality editing and display of mathematical text in Office 2007.' http://blogs.msdn.com/murrays/archive/2006/09/13/752206.aspx (13 September 2006, viewed 3 April 2007)

32 Berry, *Now read this,* p. 30–32

printed text set at small sizes. Drawing on his extensive background working with display issues for fonts with bitmaps and hinting for the TrueType format, Bosma began by taking a close look at how different kinds of shapes were rendered with ClearType. He also studied the fonts that had been released with Microsoft's electronic book, the Microsoft Reader, since these had already been hinted for use with ClearType. He found that at smaller sizes the specific shape details were less important than good hinting, but as sizes passed a certain threshold some types of shape were more accurately rendered with ClearType than others: curves that quickly turned away from the horizontal direction, strong vertical forms with very regular spacing.

Bosma's concern for clarity at small sizes on both screen and paper led to a sturdy, evenly coloured design than could work for a variety of everyday uses, and early in its development it was chosen to succeed Times New Roman as the default face for Microsoft's next major revision to its Office applications. All the new ClearType fonts included extensive character sets for Latin, Greek, and Cyrillic scripts, but Cambria's designated role as the new 'workhorse' serif typeface made it a sensible choice for further development as the typeface to showcase Microsoft's new mathematical tools.[33]

Although Cambria had been designated as the maths font, it took some time before a final specification for the necessary character set and technical features was completed. Bosma concentrated on Cambria's use for text, but in anticipation of other uses he created an initial set of maths characters (including additional superiors and subscripts, arrows and operators, and the alternate styles of the alphabetic and numeric characters shown *[see figure 17]*) with detailing that complemented the design of the main typographic forms in Cambria.[34]

Once the maths specification was completed, Cambria was adapted and extended by Ross Mills. New characters were required for complete maths coverage, and many of the maths symbols designed by Bosma produced a good texture when set within text, but not within equations. Rather than replacing the glyphs in Cambria, Mills created a separate maths font for the Cambria family. Cambria Math was based on Bosma's original design, but included alternate forms for many of the maths symbols, as well as the OpenType features needed to take advantage of Microsoft's new maths engine.

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33 Ross Mills. Email to the author (29 August 2007) *(See Appendix.)*

34 Jelle Bosma. Email to the author (5 September 2007) *(See Appendix.)*

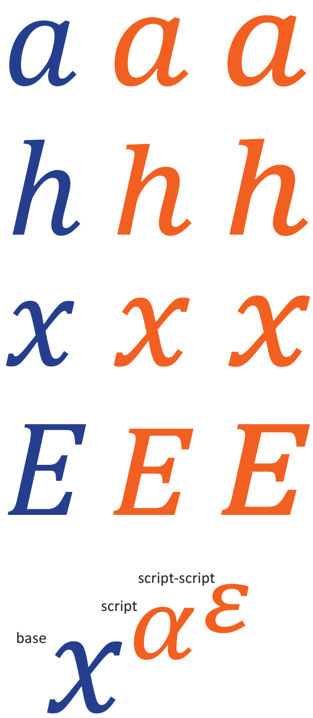
Figure 00
The glyphs of Cambria Italic are designed to blend well with the roman design and combine with one another to form discreet word shapes. The italic glyphs of Cambria Math have rounder details and wider spacing so they can be more easily discerned as individual elements within an equation.

Hudson and Mills, *Mathematical typesetting,* p. 40 (magnified 2×)

Figure 00
Compared to the Greek glyphs of Cambria Italic (shown in orange), those in Cambria Math (shown in blue) have more weight in the light descender strokes. Also, the angled strokes of delta (λ) and chi (χ) are more upright.

Figure 00
The three optical sizes within Cambria Math are not scaled in a linear way. The glyphs for 1st order superiors and inferiors are slightly heavier and have short capitals and ascenders to eliminate the need for extra line spacing as much as possible. The 2nd order glyphs, however, are heavier still but have taller capitals and ascenders for increased distinction at very small sizes.

Hudson and Mills, *Mathematical Typesetting,* p. 29



2.3.3 Characteristics of the design The overall design for the Cambria family shows an extremely regular rhythm of strong vertical strokes. Even round shapes such as those of *O*, *d*, or *g* have a clear verticality. There is some contrast, but the thin strokes are sturdy enough to avoid becoming too delicate at small sizes. Horizontal serifs are very slight, but vertical ones such as those of *S* or *f* are much heavier and slightly wedge-shaped. In Cambria Bold, weight is mostly added to the thick strokes only, increasing the overall contrast dramatically. Cambria Italic has a modest slant and very few cursive features, for an overall texture that blends very smoothly with the roman.

On the whole, the spacing of Cambria Math is quite loose, so that individual characters will separate more easily than they will combine into word shapes. Also, many aspects of Cambria's italic design required modification for use with maths (as it had been with Times New Roman). The most obvious difference is that the lowercase italic letters have much more cursive strokes than in the Italic font, making the difference between roman and italic much more explicit in an equation. *(See figure 20.)* The slanted Greek characters have also been adjusted, so that those in Cambria Math have more upright diagonal strokes, and those that taper off beneath the baseline are slightly thicker and more pronounced.[35] *(See figure 21.)*

Cambria Math includes inferior and superior variants, but these are treated differently from other maths fonts. One set of glyphs is drawn at a smaller size relative to the full-size characters, but there are also alternate forms which are substituted by the maths engine according to rules defined in the OpenType tables. Each letter and number has a variant for use as a 1st- or 2nd-order superior or inferior, with some adjustments made to enhance legibility at these smaller sizes. These are drawn at full-size to allow the maths engine to determine how much they will be reduced in scale and how far they will move from the baseline of an equation. *(See figure 22.)*

There are a number of other aspects of the design like this — capabilities defined with OpenType features but not immediately obvious from looking at the glyphs themselves. One of the most radical is how horizontal spacing adjustments are defined. Like many maths fonts used in TeX, some glyphs in Cambria Math use an italic correction value to position superiors and inferiors beside slanted glyphs. *(See figure 23.)* Taking this idea even further, the OpenType math

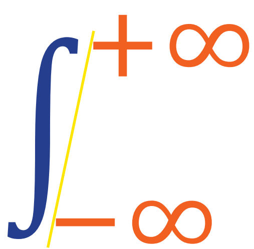


Figure 23
For asymmetrical glyphs such as the integral that are typically used at a different scale than others in an equation, italic correction determines where items should be placed beside both the top and the bottom.

Hudson and Mills, *Mathematical Typesetting,* p. 35

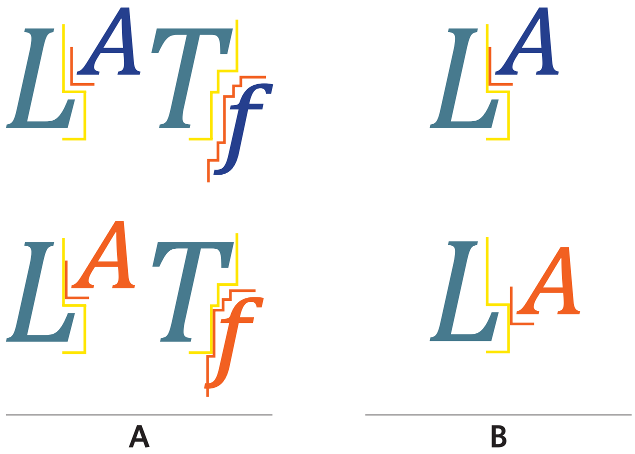
35 Jelle Bosma. *(See Appendix.)*

Figure 24
One of the new OpenType features allow the font designer to define ‘cut-ins’ on either side of a glyph. These allow glyphs to fit themselves together as their vertical positions change within an equation.

A The letters above spaced according to their normal widths. Below, fitted according to the interaction of their ‘cut-in’ boundaries.

B As the vertical position of the *A* changes, its horizontal relation to the *L* changes accordingly.

Hudson and Mills, *Mathematical typesetting,* p. 33–34



features allow the font developer to specify 'cut-ins': changes to the character width at different positions along the vertical axis. Cut-ins are specified for each glyph individually, effectively allowing the designer to define complex contours around each glyph. Since equations may include a number of scaling and vertical adjustments for which automatic kerning cannot be used, cut-ins allow glyphs to space themselves properly. As glyphs are moved up and down in an equation, the interaction of these contours — rather than overall width of each character and predefined kerning pairs — determines the distance between the glyphs.[36] *(See figure 24.)*

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36 Hudson and Mills, *Mathematical typesetting*, p. 33-35

3 Comparing the Typefaces

3.1 Visual characteristics

Substantial differences can be seen in these three typefaces, even without considering their technical backgrounds. Each typeface features different ways of addressing the typographic needs of mathematics.

3.1.1 Design of italics Italic letters are frequently used in maths, and each of these typefaces displays a different approach to the design of italics for maths compared to italics designed for text. In the case of Times Series 569, there are few differences other than the angle of slant. This adjustment eliminates many kerning and spacing problems for maths, but on the whole it is a barely discernable change from the style of Series 327. Euler employs a much more dramatic solution for the same kerning issues by eliminating the slant altogether. Since Euler makes no distinction between roman and italic styles, it relies on an overall change in form to set itself apart from any text typefaces used with it — a strikingly different solution compared to the subtle changes made for Times. Cambria Math, better able to handle kerning and spacing issues than any type set in metal, falls between these two extremes: its italic blends very well with the text italic in terms of angle, proportion, and weight, but its cursive character has been exaggerated for the sake of legibility within a maths setting.

The different approaches to the italic design suggest different priorities in each case: Monotype had little reason to change the design for the popular Times New Roman, but features of that design were not suited for maths use, and had to be corrected without changing the overall effect. The Cambria family was a new design, but one intended for a similar role as Times New Roman's — that is, a reliable and slightly conservative typeface appropriate for a wide variety of uses. However, Microsoft wanted to show that its new maths engine could set equations more skilfully as well as more easily than before, so there was a justification for making noticeable changes to the italic as long as they improved the quality of the maths. Zapf and Knuth — not only designing a new typeface, but defining practices for a new medium as well — were free to explore more radical ideas, especially since they had decided that there was little reason for a maths typeface to match the type used for the surrounding text.

Figure 25

A page from *Concrete mathematics*, the first book set using AMS Euler with Concrete Roman, another typeface Knuth created using METAFONT. Euler relies on its informal, calligraphic shapes to stand out amidst the more rigid glyphs of Concrete Roman.

Graham, Knuth, and Patashnik, *Concrete mathematics,* p. 298

We have now boiled down all the information in the Fibonacci sequence to a simple (although unrecognizable) expression $z/(1-z-z^2)$. This, believe it or not, is progress, because we can factor the denominator and then use partial fractions to achieve a formula that we can easily expand in power series. The coefficients in this power series will be a closed form for the Fibonacci numbers.

The plan of attack just sketched can perhaps be understood better if we approach it backwards. If we have a simpler generating function, say $1/(1-\alpha z)$ where α is a constant, we know the coefficients of all powers of z, because

$$\frac{1}{1-\alpha z} = 1 + \alpha z + \alpha^2 z^2 + \alpha^3 z^3 + \cdots .$$

Similarly, if we have a generating function of the form $A/(1-\alpha z)+B/(1-\beta z)$, the coefficients are easily determined, because

$$\begin{aligned}\frac{A}{1-\alpha z} + \frac{B}{1-\beta z} &= A\sum_{n\geqslant 0}(\alpha z)^n + B\sum_{n\geqslant 0}(\beta z)^n \\ &= \sum_{n\geqslant 0}(A\alpha^n + B\beta^n)z^n . \qquad (6.118)\end{aligned}$$

Therefore all we have to do is find constants A, B, α, and β such that

$$\frac{A}{1-\alpha z} + \frac{B}{1-\beta z} = \frac{z}{1-z-z^2},$$

and we will have found a closed form $A\alpha^n + B\beta^n$ for the coefficient F_n of z^n in $F(z)$. The left-hand side can be rewritten

$$\frac{A}{1-\alpha z} + \frac{B}{1-\beta z} = \frac{A - A\beta z + B - B\alpha z}{(1-\alpha z)(1-\beta z)},$$

so the four constants we seek are the solutions to two polynomial equations:

$$(1-\alpha z)(1-\beta z) = 1 - z - z^2; \qquad (6.119)$$

$$(A+B) - (A\beta + B\alpha)z = z. \qquad (6.120)$$

We want to factor the denominator of $F(z)$ into the form $(1-\alpha z)(1-\beta z)$; then we will be able to express $F(z)$ as the sum of two fractions in which the factors $(1-\alpha z)$ and $(1-\beta z)$ are conveniently separated from each other.

Notice that the denominator factors in (6.119) have been written in the form $(1-\alpha z)(1-\beta z)$, instead of the more usual form $c(z-\rho_1)(z-\rho_2)$ where ρ_1 and ρ_2 are the roots. The reason is that $(1-\alpha z)(1-\beta z)$ leads to nicer expansions in power series.

3.1.2 Relation of maths to text Monotype's decision to match the design of Times Series 569 to that of Series 327 had very practical considerations. Various aspects of mechanical type composition made it extremely difficult mix typefaces unless the text set in each face were cast separately and then later combined by hand — exactly the slow, costly situation the 4-line system was intended to minimise. Whenever possible, then, equations set within lines of text would use glyphs contained within the text font, just as any words used within maths would use glyphs contained within the maths font. Matching the overall look of the maths to the text allowed intermingling of the two to be typeset as efficiently as possible.

With METAFONT and TEX, Knuth pioneered the use of digital technology to work with type as a set of plastic rather than fixed forms.[37] His software allowed new ways of mixing typefaces and styles within text as a means of clarifying content, so his use of mathematical content as a contrasting element within a publication can be seen as both an example of his software's capabilities and an example of his desire to highlight the core material of mathematical texts. *(See figure 25.)*

Microsoft's handling of the Cambria family follows the Times New Roman model of seamless visual integration of the maths and text fonts. Whereas Euler is intended to clearly stand out from surrounding text, Cambria Math is, like Times, intended to blend in as well as possible, producing a common effect despite the numerous difference in detail between the text and maths font styles. *(See figure 26.)* Cambria is a digital font, though, without the physical constraints faced by Monotype with the Times New Roman metal fonts. In Cambria's case, the harmony of forms between the text and maths reflect conventions of use, suggesting that Knuth's heavily contrasted typographic articulation has not become the norm in maths composition.

Since the maths engine automatically changes to Cambria Math without any action from or notification to the user, in fact, it may prove to be even less noticeable as a separate typeface than Series 569 was, as long as Cambria is the font used for other material in the document. Currently, though, Cambria Math is the only typeface available for Microsoft's maths engine, so it will be used for equations regardless of the font in use when an equation is inserted. Because of this, equations are actually more easily distinguished unless Cambria is the font surrounding the equations in a document. If more typefaces are developed

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37 Knuth, 'The concept of a meta-font' p. 289

Then summing, according to the Poisson rules gives[5]

$$\sum_{k}^{\infty}{}' K_0\left(\frac{|r-r_k|}{L}\right)=\frac{2\pi L^2}{\sigma}\sum_{\nu_1,\,\nu_2}^{\infty}\frac{e^{2\pi i\left(\frac{\nu_1}{a_1}x+\frac{\nu_2}{a_2}y\right)}}{\left[1+\frac{4\pi^2L^2}{\sin^2\omega}\left(\frac{\nu_1^2}{a_1^2}+\frac{\nu_2^2}{a_2^2}-\frac{2\nu_1\nu_2}{a_1a_2}\cos\omega\right)\right]}. \qquad (12.2)$$

If the conditions $\dfrac{4\pi L^2}{a_1a_2\sin^2\omega}\gg 1$ are fulfilled, then

$$\sum_{k}^{\infty}{}' K_0\left(\frac{|r-r_k|}{L}\right)=\frac{2\pi L^2}{\sigma}\left\{1+\frac{\sin^2\omega}{4\pi^2L^2}\sum_{\nu_1,\,\nu_2}{}'\frac{e^{2\pi i\left(\frac{\nu_1}{a_1}x+\frac{\nu_2}{a_2}y\right)}}{\left(\frac{\nu_1^2}{a_1^2}+\frac{\nu_2^2}{a_2^2}-\frac{2\nu_1\nu_2}{a_1a_2}\cos\omega\right)}\right\} \qquad (13.2)$$

A

9.27 We may assume that $\alpha \neq -1$. Let $f(x) = x^\alpha$; the answer is

$$\sum_{k=1}^{n} k^\alpha = C_\alpha + \frac{n^{\alpha+1}}{\alpha+1} + \frac{n^\alpha}{2} + \sum_{k=1}^{m}\frac{B_{2k}}{2k}\binom{\alpha}{2k-1}n^{\alpha-2k+1} + O(n^{\alpha-2m-1}).$$

B

$$y_2(x) = y_1(x)\int^x \frac{\exp\left(-\int_a^{x_2}\sum_{i=-1}^{\infty}p_i x_1^i\,dx_1\right)}{x_2^{2\alpha}\left(\sum_{\lambda=0}^{\infty}a_\lambda x_2^\lambda\right)^2}dx_2,$$

where the solutions y_1 and y_2 have been normalized so that the Wronskian, $W(a) = 1$. Tackling the exponential factor first, we have

$$\int_a^{x_2}\sum_{i=-1}^{\infty}p_i x_1^i\,dx_1 = p_{-1}\ln x_2 + \sum_{k=0}^{\infty}\frac{p_k}{k+1}x_2^{k+1} + f(a).$$

C

Figure 26

Samples of equations mixed with text.

A Text set with Times New Roman Series 327, and equations set with Times 4-line Mathematics Series 569.

B Text set with Concrete Roman, and equations set with AMS Euler.

C Text set with Cambria Regular, and equations set with Cambria Math.

Monotype specimen sheet (1956)

Graham, Knuth, and Patashnik, *Concrete mathematics,* p. 594

Hudson and Mills, *Mathematical typesetting,* p. 35

according to the specifications developed for Cambria Math, this situation will surely change.

3.1.3 Range of character styles Another feature common to these typefaces is the inclusion of a broader range of related styles than ordinary text fonts, such as script and fraktur alphabets. In Series 569 these appear to have been pulled directly from other fonts without being redrawn to relate to the design of Times New Roman, but both Euler and Cambria Math feature original designs for these styles that harmonize with the weight and proportions of the rest of the typeface. These styles function much like the maths italic: they need to be clear and distinct as single characters, work alongside other glyphs found in an equation. They are drawn to refer to more traditional forms yet serve as members of the maths family.

Each typeface design also places an emphasis on the use of properly drawn glyphs for superior and inferior characters, underscoring the importance of having legible versions of these for maths. Both Series 569 and Cambria Math have separate glyphs drawn for the small optical sizes, and not just the figures found in many text typefaces, but a full assortment of Latin and Greek letters, too, in roman and italic styles. Separate glyphs for superiors and inferiors were not drawn for Euler, but the team producing the fonts at Stanford included a 5-point font with the necessary adjustments to weight and width for it to function as the superiors and inferiors for maths set at typical text sizes.

Figure 27
As part of his typographic research, Knuth compared volumes of the *Transactions of the American Mathematical Society* to see how their maths composition had changed over the years. His examples (magnified 1.5×) cite the use of Times New Roman, but fail to note that they show different typefaces based on the original design.

A A 1962 volume using 4-line maths and Series 569. The slant angle is 12°, and both 1st- and 2nd-order inferiors are used. (Highlighted in red.) The use of a 2nd-order glyph appears to be an error, but it shows that the font used contained separate matrices for the each level.

B This 1966 volume would most likely have used 4-line maths, but Knuth's example only shows text with some maths terms. Text would have been set with Series 327 (note the steeper angle of slant in the italics), even if the 4-line system and Series 569 were used for display maths.

C A digital version of Times was used in 1977. The more upright italic style of Series 569 is no longer used, and inferior and superior forms are scaled versions of the full-size characters. (Compare the inferior *k* to those in the other examples.)

Donald E Knuth. 'Mathematical typography', *TEX and METAFONT: New directions in typesetting*. Providence, RI: The American Mathematical Society (1979) p. 3–5

$$z = e^{i\theta} z^0 \equiv (e^{i\theta} z_1^0, \ldots, e^{i\theta} z_n^0), \qquad 0 \leqq \theta \leqq 2\pi,$$

$\subset C^n$ is called a Reinhardt circular set if along w
$\in E$ also the set

$$\{z \,|\, |z_k| = |z_k^0|, \quad k = 1,2,\ldots,n\}$$

bounded closed subset of C^n, unisolvent with respec
The function $b(z)$ being defined and lower semic

$$h^{(\nu)} = \{h_1^{(\nu)}, \ldots, h_\nu^{(\nu)}\}, \qquad \nu_0 = C_{\nu+n-1,n-1}$$

A

The set N_1 is nowhere dense in Z_1 and thus $N=\rho$
For each $\zeta \in Y-N$ we must provc that f_ζ satisfi
be the unique projection in $\{P_d \mid d \in D\}$ such that
the algebra $(E\mathscr{A}E)\cdot P_0$ is finite and homogeneo
onal abelian projections $E_1, E_2, \ldots, E_n$ such that
$(1 \leqq j, k \leqq n)$ be partial isometric operators in $(E\mathscr{A}$
(1) $U_{jk}U_{lm} = \delta_{mj}U_{lk}$, where δ is the Kronecker de
(2) $U_{jk}^* = U_{kj}$; and
(3) $U_{jj} = E_j$,
for all $1 \leqq j, k, l, m \leqq n$. For each A in $(E\mathscr{A}E)\cdot P_0$, t
in $\mathscr{Z}_1 P_0$ such that

B

onverges pathwise to X^λ, and uniformly for $t \in$
for which X_i is the (last) minimum of Y^λ, let Y_i^λ,
lues of Y^λ, and T_i^λ the interjump times for Y^λ.
st an i such that $Y_i^\lambda = T_i^\lambda = \infty$. Notice that Y_Q^λ is
d that as $\varepsilon \to 0$, Y_Q^λ converges to $I^\lambda = \inf_s X_s^\lambda$. Le
ts of $(-\infty, \infty)$. Then, for example, if $i \geqslant 1$

$${}_{-i} \in B, Y_{Q+k}^\lambda - Y_Q^\lambda \in C, T_{Q+k}^\lambda \in D, N > Q > i\}$$

$$\in A, T_{l-i}^\lambda \in B, Y_{l+k}^\lambda - Y_l^\lambda \in C, T_{l+k}^\lambda \in D, N > Q$$

C

3.2 The influence of Times New Roman

Times New Roman is cited as an influence in the design of both AMS Euler and the Cambria family, reinforcing the sense of its legacy as a maths typeface, which is exaggerated by the many variations of it — for all uses — that have appeared in the decades since it was introduced.

Euler was designed as a reaction against the ubiquity of Times in mathematical publishing and its perceived shortcomings. In discussing previous efforts to create fonts specifically for maths, Knuth cites knowledge of only two projects — an aborted effort by Jan van Krimpen for the Enschedé foundry in the Netherlands, and a set of fonts commissioned by AMS in 1962.[38] Notably absent from his examples is Monotype's Series 569, which would have included over 8,000 glyphs produced specifically for maths by the time of his writing.

Knuth's comments about Times, though, suggest that he was not actually aware of a distinction between Times as a text face and Series 569 as a separate design with specific features for mathematics. He mentions that he and Zapf looked at *The printing of mathematics* and found some details of the type used for it problematic,[39] but that book, though extremely influential in the field of maths composition, was published three years before the introduction of Series 569. Also, Knuth's analysis of the AMS publications being produced when he began working on TEX cites various instances of Times New Roman in use, but his examples describe different typesetting methods that would have each used a different version of Times.[40] Even when he refers to books which must have been produced using 4-line maths (a technique of which he was aware[41]), he fails to discern the concurrent use of two versions of the typeface.[42] *(See figure 27.)*

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38 Knuth and Zapf, 'AMS Euler — a New Typeface for Mathematics', p. 134.

39 Knuth and Zapf, 'AMS Euler — a New Typeface for Mathematics', p. 135.
Knuth refers to: T W Chaundy, P R Barrett, and Charles Batey. *The printing of mathematics*. London: Oxford University Press (1954).

40 Barbara Beeton. Email to author (1 September 2007) *(See Appendix.)*
In 1979, when Knuth published his analysis of their typesetting over the years, the AMS was using versions of Times adapted for the IBM Selectric typewriter, Varityper machines, Photon 200 and Photon 713 phototypesetters, and a digital system from Science Typographers, Inc.

41 Donald E Knuth. 'Digital typography', *Digital typography*. Stanford, CA: CSLI Publications (1999) p. 4

42 Donald E. Knuth. 'Mathematical typography' *TEX and METAFONT: New Directions in Typesetting*. Providence, RI: The American Mathematical Society (1979) p. 4–7

Knuth may have lacked awareness of Series 569 as a separate typeface, but this is less a shortcoming of his otherwise keen and thorough analysis of the state of mathematical works than it is an indicator of a recurring characteristic of Series 569: its deference to people's greater awareness of Times New Roman Series 327, the text family. The 4-line system and Series 569 — a comprehensive set of alphabets and symbols that dominated technical publishing for decades — were developed specifically for use with Monotype's typesetting equipment, and as such were marketed to members of the typesetting trade as accessories to Monotype products.[43] The literature released about 4-line maths heralded the typesetting techniques, but devoted little attention to the specifics of the typographic design that actually allowed Series 569 to perform so well. Monotype stressed the complementary relationship to Times New Roman, generally neglecting to clarify the differences between the two series. Except to those actually ordering matrices and typesetting the maths, there was little distinction between the versions of Times New Roman used for text and for equations.

Knuth's dislike of Times may have led to the use of other styles of typeface with TEX and its offshoots, but the inclusion of various versions of Times with most personal computers reinforced its ubiquity in more general kinds of documents produced with other programs.[44] Microsoft, in positioning Cambria as a replacement for Times in its software products, is not trying to escape influence of Times New Roman the way Knuth was. Instead, it acknowledges the general usefulness of Times and offers Cambria as alternative with a similar feel that can be used for similar work, including the kinds of technical work that featured Times Series 569 for many years.[45]

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43 *Information sheet no. 156*, p. 1–6

44 Mary Beth Henry. 'Times New Roman', *Revival of the fittest: Digital versions of classic typefaces*, edited by Philip B Meggs and Roy McKelvey. New York: RC Publications (2000) p. 169

45 Ross Mills. *(See Appendix.)*

3.3 Composing the page

Monotype's 4-line system was subject to the inherent limitations of metal type: characters were cast on rectangular pieces of lead which needed to fit together in a grid of solid material. The system was a technique for bypassing the restrictions of the grid by using sorts whose faces usually extended past the rectangular body on which they sat, but the overhanging forms still required another rectangular piece of lead to support them. Effectively this was a grid made up of smaller units, but it was a grid nonetheless. The implementation of the 4-line system for phototypesetting made it possible to ignore the physical boundaries of each character, but the characters were still conceived as rectangular units pieced together one by one, even when overlapping.

Knuth's concept for TEX composition — a series of boxed elements fitting together within ever larger boxes — is essentially a continuation of the constraints that governed the composition of metal type. Like photocomposition, electronic composition allowed much greater freedom to combine and place these boxes by making the spaces between them more flexible. The rectangular box, however, continued to be the essential unit, except the box was now defined by the programmed width and height of each character. The software assembles boxes of a certain width, unaware of the shapes contained within those boxes. Kerning adjustments between specific characters may be anticipated ahead of time or made afterward by the typesetter, but the software still responds to these as changes to the space between rectangles.

Cambria's OpenType features for maths present a new model that allows the boundaries of each glyph to be described as a shape more complex than a simple rectangle. The designer's ability to specify 'cut-ins' around the bounding box of each glyph allows software to adapt more easily to the spatial arrangements of mathematics. This method circumvents the rigidity of the box model, even within digital space, where a glyph's bounding box is more of a guide than a fixed limitation.

3.4 Typesetting roles

Each of the three typefaces discussed have been used to show the capabilities of a related technical advance that in some way redistributed the responsibility of certain aspects of composition.

4-line maths spread the work involved in maths typesetting among the keyboard and caster operators as well as the hand-compositor, who bore the bulk of the responsibility earlier. Monotype's was definitely a patented, controlled business model. It was intended to make maths setting easier and more profitable for its customers — professional compositors and printers.

TEX and METAFONT, however, were intended to democratise maths, giving authors and publishers direct control of high-quality composition and layout. Knuth intended for his software and related typefaces to be freely available, and hopefully improved upon by others, but this also required them to learn how to use that software and make good decisions about setting type well.

The fact that Cambria Math's advanced features are built into the font at the outset takes a lot of the composition responsibility away from the typesetter or author. It shows Microsoft's desire to bring the ability to set maths to even the most basic user. However, it places a much greater burden on the font developer, which may limit the number of fonts properly developed for maths.

By defining new functional possibilities with OpenType tables within the fonts themselves, Microsoft has made them publicly available for integration with other fonts, available for any composition software that can be written to take advantage of the additional information. At present, however, there are few tools that support either their creation (Microsoft used an as-yet-unreleased tool of their own, and the open-source FontForge offers limited support[46]) or their use, even within Microsoft's own product line.

Each technical advance also suggests a different relationship between the author and the mathematical work produced, with each font becoming more closely tied to the author's experience of the material. 4-line maths assumed the author would be many levels removed from the printed product, reviewing the results but entrusting the details to editors, compositors, and printers, and Monotype expressed this by offering Series 569, a barely detectable complement to a type-

46 George Williams. 'FontForge: Math typesetting information.' http://fontforge.sourceforge.net/math.html (4 September 2007)

This page demonstrates a variety of symbols and was prepared without any regard to mathematical sense.

$$S = -2 \sum_{s_c>0} C_s \cos 5x_1.$$

Superscripts^{a}, subscripts$_{b}$, and their modifiers$^{a^{x}}_{b_{y}}$ are preferably typed and positioned by a turn of the roller. No further marking for position is required unless the spread of the expression is considerable, as in $H^{(0)}_{K,qm+1,k-2} - M^{1/2}$, when marking defines the limits of the subscript.

ell

$$K = \frac{\chi}{(2k)^{k}} \int_0^{2\chi} \cdots \int_{\mathcal{O}}^{2v} F_{o,0}\ (24).\ \prod_{n^2\mu<a_n} d\varphi_n.$$

cap oh

Handwritten symbols, as $\cap$, $\wedge$, $>$, are inserted in spaces left while typing.

Greek is preferably underlined, as κ, o, ρ, χ; ordinary letter symbols, as k, o, p, x, which we will always set in italic, left unmarked. Signs for summation $\sum$, product $\prod$, and partial differentiations ∂ are not marked.

$$\int_0^1 r^2\, dn\, \frac{\gamma}{2} + \nu^2 = \int_0^1 XZ'(z_2)\, dx \cup \chi^1$$
$$= \psi\ 4U(V) - \int_0^t U^2 \vee \mathfrak{H}''\ \varepsilon\ (U)\ d\epsilon \in \mathcal{C}^1_4.$$

H = German
C = script

Abbreviations, normally set in roman, need not be marked if they are common forms such as lim, ln, cos, exp; however, a separate list of abbreviations (and symbols) attached to the manuscript is helpful to editors and printers.

$$\alpha - a_0 = u \int_{\nu}^{\mu} (1 - v^2 \sin^2 \eta)^{-1/2}\, dn.$$

Script and German letters may be typed and underlined (or circled) in a special color with marginal notation.

Figure 28

A 1954 guide to manuscript preparation shows some of the difficulties of working on a maths publication before authors had the means to typeset their own work. Knuth and Zapf developed AMS Euler based on the idea that handwriting was not just convenient, but also integral to mathematicians' conception of their work.

Mathematics in type. Richmond, VA: The William Bird Press (1954) p.28

face gaining prominence in standard book publishing. Working with TEX and METAFONT, however, empowers the author to have a direct hand in presenting the work as he or she sees fit, with AMS Euler suggesting that the author's primary relationship to the subject matter is through handwriting, with presentation as a secondary stage. *(See figure 28.)* Cambria — designed for on-screen reading and relatively automated composition of mathematical material — allows the author to work directly in an electronic medium from the outset. Print production with Cambria is secondary to its role as a working component of the author's writing tools.

Conclusion

Typefaces for mathematics must meet certain requirements that differ significantly from ordinary passages of text. In maths, each character used represents a value or a function of its own, and therefore must be legible as a distinct form, no matter where it appears within an equation, at what size, or in combination with any other character. Complicating matters further, mathematical notation is often built up vertically as well as horizontally, a practice that defies the organizational principals of typesetting methods that are most efficient when setting horizontal lines of text.

There is certainly latitude when it comes to designing the typeforms of maths fonts, as seen in the barely visible adjustments made for Times 4-line Mathematics Series 569, the significant but ultimately harmonious adaptations made for Cambria Math, to the unconventional cursive forms of AMS Euler. In any style, though, certain issues must be addressed in order for a typeface to perform well in a maths setting. Italic or cursive forms are used often and must be distinct, and they must allow upright, inferior, and superior forms to combine with them without too many spacing difficulties. The frequent use of small forms as superiors and inferiors demands alternate forms designed for clarity at reduced sizes. The concurrent use of scripts and styles such as Latin, Greek, fraktur, and script letters requires some thought as to how alike or distinct these forms should be compared to one another.

Regardless of the aesthetic decisions that are made as these and other problems are considered, typefaces for maths require additional attention paid to technical challenges of composition. The use of numerous exotic characters and the complex spatial arrangements of maths call for typesetting techniques all their own, and these specialised techniques may require fonts specifically adapted to the tools used.

In the three cases discussed, the typefaces were created to show the capabilities of new technical solutions for composing maths. Each design responded to the possibilities of its related technology as well as its constraints. Each case underscores the need to approach the design of fonts for mathematics not as an extra set of symbols tacked onto an existing design, but as typographic solutions for a script or a language with its own behaviors and needs that must be considered and — hopefully — supported.

Appendix: Correspondence

Correspondence with several individuals involved with these design projects proved to be invaluable sources of detailed information. Relevant passages are quoted here to supplement the published material cited throughout this work.

Questions from the author are quoted in italics.

Barbara Beeton, *American Mathematical Society*

1 September 2007

In his essay 'AMS Euler — a new typeface for mathematics,' Don Knuth includes a letter to Hermann Zapf from 1979 in which he mentions that AMS had been setting work with Times New Roman … at that time. I am hoping you might be able to tell me a little more about how your publications were being composed at that time, namely:

1) Was AMS using an outside composition house at the time? If so, do you know who it would have been?

In 1979, AMS was using, in-house, a program developed by Science Typographers, Inc. (Science Typographers did do some composition work themselves. They also leased their software to other publishers.) AMS had been working, under contracts with the National Science Foundation, for many years — at least since the early 1960s — with various software contractors to develop a computer-based typesetting system for math. The reports to NSF may still be available from the National Technical Information Service (NTIS), an agency of the U.S. government. When I started working at AMS, I was reporting to the person in charge of coordinating this work, and was involved peripherally in preparing the reports (typing and drafting diagrams), so I have some memory of what was involved. I was also a key player in introducing the STI system into production here, part of the team that developed an input method based on mnemonic representation of math (which was, in turn, based on the earlier work done under the NSF contract), principal trainer of the first input staff, and local maintenance person for the software. The font used by the STI system was indeed Times Roman.

2) Do you know if the math at the time was still being composed with metal type (which seems unlikely in 1979), or was film-based or even electronic type being used?

I can't think of any AMS publication by 1979 that was composed with metal type. Most journals and books were prepared using the STI system, although I think a manually operated Photon 200 (which shot images through a glass disk on which glyphs had been placed photographically) may still have been in use. Also, there may still have been several Varityper machines and IBM selectric composers in use, producing copy on paper — 'cold type'; I'm not sure when they were all retired. The font used by those machines was also a simulacrum of Times.

The STI system mostly output code to drive a CRT-based Harris Fototronic. For material with minimal, in-line only, math, STI output was converted to paper tape to be input to a Photon 713, which held the master glyphs on 35mm film negatives arrayed around a drum; a few indexes were produced by this method. Times again.

4 September 2007

Are there licensing terms that govern the distribution or modification of AMS Euler? It seems as if it's freely offered, but I can't seem to locate any guidelines. What's AMS' policy?

It doesn't seem to be stated explicitly, but the license is the same as that for the Computer Modern fonts — if you modify, change the name, otherwise they're freely usable and redistributable.

Stating this formally is something we need to do, and will try to do so next time we are working on the AMS TeX-related distributions.

Jelle Bosma, *Monotype Imaging, Ltd.*

5 September 2007

1) What was your original concept for the overall design of Cambria? (That is, if you care to elaborate on what has been noted in Now read this*?)*

I don't think I can add anything significant to *Now read this*…

2) To what extent was it conceived specifically for the ClearType project, as opposed to an existing idea that seemed appropriate to propose for the project?

…which means it was conceived for the ClearType project and didn't exist before. Some have pointed out similarities with my design Forlane, in which case it might be said I am building on previous work. But to me the differences seem more significant as the similarities.

3) At what stage in Cambria's development did Cambria Math become part of the overall brief?

It was the intention to add math support to Cambria from the start. However initially it wasn't clearly defined what that meant. One might say this has been defined in three stages.

On the outset we worked out a character set with:

— additional superiors and subscripts for Latin, Greek, numerals and basic math symbols
— combining marks
— additional (historic) Greek
— additional punctuation used for Math expressions
— the Letterlike Symbols
— arrows
— a choice of math operators
— Mathematical Alphanumeric Symbols: Bold Latin, Italic Latin, Italic Greek, Bold Italic Latin, Bold Italic Greek, Script symbols (uppercase) and a fair amount of superscripts and subscripts thereof

Later on more Mathematical Alphanumeric Symbols were added: Script Lowercase, Script Symbols Bold, Fraktur, Double Struck, Fraktur Bold, Sans Serif regular, Sans Serif bold, Sans Serif italic, Sans Serif bold italic, Greek Bold, and 4 sets of numbers. I was a bit stressed when these were added, because we were quite close to wrapping up, when the character set was suddenly expanded with what can be described as the equivalent of 10 typefaces.

But in the end I delivered a font with many math characters, not a 'math font'. I had no involvement with the third phase and got to see the result much later. In the final stage characters have been added or reworked to build equations. This means contextual forms for braces, integrals and the like. Also quite a few of the math operators have been re-designed. For the most part this meant making my drawings larger, taller or wider, where needed. The 'normal' Cambria has the standard math operators (= > < + et cetera) on figure width and the additional operators were based on that. For the math version the standard operators have been replaced with enlarged versions and most of the rest followed that.

4) What factors contributed to that decision to develop Cambria for mathematics use?

Presumably because of the two serifed designs the Cambria was selected to be used for MS Office, rather than Constantia.

5) Did the requirements of the mathematical features (either visually or technically) lead to any changes in the design of the rest of the Cambria family? That is, to what extent were Cambria and Cambria Math developed in tandem?

It would have made no difference. I worked on the assumption that Cambria will be used without math most of the time.

6) Were there any issues with the design of the math characters where the traditional handling of those symbols conflicted with Cambria's design as a text family? How were these resolved?

The non-alphabetic symbols have been drawn as typographic symbols matching the alphabetic symbols as much as possible. Most of the symbols that I have seen in other fonts, are 'compass and ruler' constructions: circles and squares being circles and squares, x-stems and y-stems of equal weight. I applied all the usual visual adaptations as one would do for letter forms: circles are taller than wider and curves rounder as a constructed circle. X-stems are heavier as y stems throughout. Connections of rounds to straights are smooth. Take for example the set operators (such as U+223C), which usually are stems slammed on half a circle.

Most of the conflicts between the text characters and their use as math symbols could be avoided because only the regular uppercase and lowercase might be used both for text and as symbol. The most important text characters used for symbols are the Latin Italic and Greek Inclined. But these are part of the math font as part of the Unicode range Mathematical Alphanumeric Symbols. In older fonts or traditional typesetting these would be separate typefaces. For many symbols these are duplicate drawings, but re-spaced to be part of an upright font. But in the lowercase the serifs have been replaced by diagonal strokes. In general the Greek math symbols have different drawings: for example the italic lambda as symbol has a larger inclination than the lambda as text character (to give it the same angle as the rest), the descenders of zeta, xi and sigma are firmer, to make sure the characters stand out on their own.

For the script section of the Mathematical Alphanumeric Symbols the problem was to make the lowercase clearly different from the italic symbols. Adding a 'Commercial Script' style font, which is a often used, would have given the problem that it would have been very unclear at screen display. If you beef it up using hints, many character would be indistinguishable from the italic that is already there. So within the proportions of the normal text lowercase I attempted a script with each individual character trying to assert its 'scriptness' as much as possible. Don't try to make words with these characters. The connection strokes have been drawn to be noticeable, not to connect!

Ross Mills, *Tiro Typeworks*

29 August 2007

1) Was the character set for Cambria Math always specified by Microsoft, or did the development of the typeface design inspire any additions to the available math characters?

There was some thought earlier in Cambria's development that it would be suitable for maths and scientific texts — perhaps more in line with the premise that it was an office workhorse to supplant Times in some respects. Although one can argue the suitability of Times for maths, nonetheless it has seen much use and if Cambria were to become the new default in Office applications, then it makes sense that it would fill the role that Times has. In the end, Cambria did not become the default, which is a bit strange. So while there was some thought that it would serve as a maths and scientific font, it was first spec'd the same as the other ClearType fonts, with the math project (and requisite extensions to Cambria) coming some time after the first version of the C* fonts were completed.

2) Did the requirements of the mathematical features (either visually or technically) lead to any changes in the design of the rest of the Cambria family? That is, to what extent were Cambria and Cambria Math developed in tandem?

The short answer is that they were not designed in tandem, except as mentioned above that there was some expectation that Cambria *might* at some point be extended to suit. So, the core Cambria (that is, the original C* character/glyph complement of approximately 1000 glyphs) was designed before the Math project went ahead.

3) Were there any issues with the design of the math characters where the traditional handling of those symbols conflicted with Cambria's design as a text family? How were these resolved?

There were some issues related to how Jelle designed his glyphs and how that might conflict with how many math users expect things to look. As I mentioned before, Cambria is a workhorse, and so has characteristics more in line with a Percheron then with an Arab. Some of the more common math characters, specifically basic operators and fences were deemed less-then ideal. The parenthesis, for instance, were too close to the shape of brackets (rather square in form) and so I made variants that were more round and open. The operators were somewhat stubby — which was OK for text settings, but lacked the sort of semantic 'weight' that math operators, in situ, should probably have, so I made variants that were larger and spaced differently. There were quite a few instances such as this, but I tried to minimize the impact on the core set. It is fairly difficult to work on someone else's typeface and I wanted to respect the decisions of the original designer but at the same time fulfill my mandate to make suitable alterations to make Cambria work better for maths. I do wish that the whole process began from the get-go to be a maths font, but it didn't turn out that way.

So, there are quite a few glyphs contained as encoded characters in the core C* set that differ from the math font. I wanted to retain the sanctity of the core set, so if there was no reason to change glyphs there, I didn't want to do it. Instead, the original glyphs are encoded in the Cambria Regular font and new math variants are encoded in the Cambria Math font, so when you are in text mode you get the glyphs from Cambria Regular and when you're in Math mode, you get them from the Math font (e.g., in regular text you will get the smaller 'plus' and squarer parens, but in Math mode you will get the larger variant plus [and other operators] and the rounder parens). The Cambria Regular and Cambria Math fonts have identical glyphs, but different CMAPS.

4) How did the technical requirements and/or opportunities of Vista's handling of math influence the design of the glyphs in Cambria Math?

I don't know that I could say that the Math engine influenced the design directly. Of course, there are technical requirements that influences basic structure of some glyphs (such as growing glyphs with several components which need to join and stretch in the right way) but I would say that a well designed math engine and font spec do not presume to dictate design beyond establishing rules — based on typographic traditions (in this case greatly modeled on TEX) — into which many fonts could be adapted. Indeed, what form a math font may take can engender all sorts of discussion and possible variation, just as typographic tradition allows and a text handler — in this case a math text handler and associated bits — should not inhibit such variation as allowed by the rules of math typesetting and fonts used for same. Many of the existing technologies have not allowed the level of flexibility that exist in other paradigms. The complexity of the problem of course necessitates such a closed system (i.e., a particular font travels with a particular text engine and adapting many other fonts for that system is not a trivial task). This is not to say the new technology will suddenly open the floodgates to many new math fonts. Even though it has more capability then other systems, it is still a major undertaking to develop fonts for it, or any other robust typesetting system. It is further hampered by currently existing within a framework that is not terribly appealing to professional users and typesetters of math (i.e., Microsoft Word). Which leads to a need for slight clarification: the Math engine (as such) is not directly a Vista product. It is currently incorporated in MS Word, but exists in its own right as an extension to RichEdit, so theoretically could be plugged into other applications that wanted to leverage it.

5) What has been added to Cambria Math since its original release with Vista? Have the additions been requested by users who have tried it, or were they another phase of development that had been planned?

There were a set of lower priority characters added that were spec'd for some time but weren't completed for the Office release. These are mostly extra Unicode blocks or subsets that weren't deemed as important (such as bold sans-serif Greek and monospaced) or were not specifically math-related (e.g., scientific sets like dentistry symbols and the like and altogether too many arrows). Although the current shipping version doesn't have all of these, you may assume some future version will have all the specified Unicode math-related and scientific characters as well as more relevant variants as required (e.g., growing variants etc.)

I have seen no feedback from users yet. I imagine that feedback will have some influence on future revisions and additions, but there is no specific plan at the moment.

30 August 2007

Cambria Math…seems to work as a proof-of-concept for the capabilities of the math engine. The engine itself does not really dictate a certain approach to the visual design of a typeface overall, just how it's engineered and encoded to work with the engine. Does that sound right?

Yes. Although semantically the word 'encoded' of course has particular meaning for fonts, so another term may be more appropriate if you mean something different. There are of course also encoding issues, but the requirements are more dictated by what you are typesetting; e.g., if another font for the system is made, the maker would have to determine their own minimum character set requirements for the material they are publishing—alternates are enumerated in the MATH table, and if an alternate is not listed or not present then secondary process will take over (so if there is no script-variant for a particular character, then it will scale the base character rather then the script variant. [Aside: SSTY variants are full-sized relative to the base glyph and are scaled by a percentage determined by the designer in the MATH table. Superscript and subscript characters are not used for this purpose unless explicitly entered by the typesetter.])

There were a set of lower priority characters added that were spec'd for some time but weren't completed for the Office release. These are mostly extra Unicode blocks or subsets that weren't deemed as important (such as bold sans-serif Greek and monospaced) or were not specifically math-related (e.g., scientific sets like dentistry symbols and the like and altogether too many arrows).

This touches on another issue: does Cambria Math use any characters not currently included within Unicode? Is there a framework for accessing any characters that users might demand before they appear in Unicode, or is official encoding a mandatory criterion for adding a character to the font? (And I'm using 'character' very intentionally, rather than 'glyph,' since it's clear that there are various glyphs available for some characters.)

There are a smattering of characters that at the time were not yet fully integrated, but were accepted for inclusion in principal. Its not mandatory, but not a good idea to use arbitrary encodings. One could add a new math (or other) character to say, the private use area, but you would have to use your own mechanism to enter it (i.e., the Unicode value) or write your own additional autocorrect sequence. You'd have to have some way to deal with documents using the PUA character once (and if) the new character does get accepted into Unicode.

Works cited

Barbara Beeton
Emails to author (1 and 4 September 2007)

John D Berry
Now read this: The Microsoft ClearType font collection. Redmond, WA: The Microsoft Corporation (2004)

Jelle Bosma
Email to author (5 September 2007)

T W Chaundy, P R Barrett, and Charles Batey
The printing of mathematics. London: Oxford University Press (1954)

'Computer Modern and AMSFonts in Type 1 (PostScript) form'.
http://www.ams.org/tex/type1-fonts.html (viewed 10 September 2007)

John Dreyfus
'The evolution of Times New Roman', *The Penrose Annual 66*, edited by Herbert Spencer. London: Lund Humphries (1973) 165–174

Ronald L Graham, Donald E Knuth, and Oren Patashnik
Concrete mathematics: A foundation for computer science. Reading, MA: Addison-Wesley Professional (1988)

John Hudson and Ross Mills, eds.
Mathematical typesetting: Mathematical and scientific typesetting solutions from Microsoft. Redmond, WA: The Microsoft Corporation (2007)

Information sheet no. 156: 4-line mathematics. London: The Monotype Corporation, Ltd. (1959)

Donald E Knuth
Digital typography. Stanford, CA: CSLI Publications (1999)

Donald E Knuth
TEX and METAFONT: New directions in typesetting. Providence, RI: The American Mathematical Society (1979)

Donald E Knuth and Hermann Zapf
'AMS Euler — a new typeface for mathematics,' *Scholarly Publishing.* 20 (3) (1989) 131–57

Mathematics in type. Richmond, VA: The William Bird Press (1954)

Ross Mills
Emails to author (29, 30 August 2007)

Robin Nicholas
Interviews with author. Monotype Imaging, Ltd. Salfords, Redhill, Surrey (31 August and 3 September 2007)

Arthur Phillips
'Setting mathematics', *The Monotype Recorder.* 40 (4) (1957)

A pocket picture book of 'Monotype' machines. London: The Monotype Corporation, Ltd. (n.d., ca. 1965)

J E Poole
'Filmsetter mathematics', *Monotype Newsletter.* 82 (1967) 2–4

J E Poole
Mathematical formulae. London: The Monotype Corporation, Ltd. (1971)

J E Poole
'Mechanising mathematics', *Monotype Newsletter.* 81 (1967) 19–22

Max Rooses
Index characterum architypographiae plantinianae. Antwerp: Plantin-Moretus Museum (1905)

Murray Sargent
'High-quality editing and display of mathematical text in Office 2007'. http://blogs.msdn.com/murrays/archive/2006/09/13/752206.aspx (13 September 2006, viewed 3 April 2007)

David Siegel
The Euler Project at Stanford. Stanford, CA: Stanford University Department of Computer Science (1985)

Erik-Jan Vens
'Conversion of the Euler metafonts into the PostScript Type 1 font language', *Proceedings of the Ninth European TEX Conference*, MAPS special editions (1995) 425–430

George Williams
'FontForge: Math typesetting information'. http://fontforge.sourceforge.net/math.html (viewed 4 September 2007)

Hermann Zapf
'The designer in the world of METAFONT', *Hermann Zapf & his design philosophy.* Chicago: The Society of Typographic Arts (1987) 93–99

Works consulted

Nicolas Barker
Stanley Morison. London: Macmillan (1972)

Book of information, 2nd edn. London: The Monotype Corporation, Ltd. (1970)

Thierry Bouche
'Diversity in math fonts' *TUGboat*. 19 (2) (1998) 121–135

Andrei Burago
'The MATH table and OpenType features for math processing'. (2006)

Sebastian Carter
Twentieth Century Type Designers. (1995)

Mary Beth Henry
'Times New Roman', *Revival of the fittest: Digital versions of classic typefaces*, edited by Philip B Meggs and Roy McKelvey. New York: RC Publications (2000) 164–169

Berthold K Horn
'Where are the math fonts?' *TUGboat*. 14 (3) (1993) 282–284

Allen Hutt
'Times Roman: A Re-Assessment.' *The Journal of Typographic Research*. IV (3) (1970) 259–270

Donald E Knuth
The METAFONTbook. Vol. C, Computers & typesetting. Reading, MA: Addison-Wesley (1986)

Steven G Krantz
Handbook of typography for the mathematical sciences. Boca Raton, FL: Chapman & Hall/CRC (2001)

Richard Lawrence
'Maths = typography?' *TUGboat*. 24 (2) (2003) 165–168

'Mathematical setting: Letterpress printer must give high standard to avoid the diverting of orders, says Arthur Phillips', *Printing World* (23 January 1958)

'Mathematical work: It's not just a case of 'follow copy''. *The British & Colonial Printer*. 152 (26) (1953) 782–785

''Monophoto' Faces'. *Monotype Newsletter* (1971)

''Monotype' 4-line composition solves mathematical setting headaches', *Printing News* (25 September 1958)

James Moran
'Stanley Morison, 1889–1967'. *The Monotype Recorder.* 43 (3) (1968)

James Moran
Stanley Morison: His typographical achievement. London: Lund Humphries (1971)

Stanley Morison
'The Times New Roman: The designing and cutting of the new type; how matrices are made'. *The Monotype Recorder.* 31 (247) (1932) 12–15

Arthur Phillips
'Mathematical typography', *Print in Britain.* 4 (12) (1957)

'Pixel-ated by digitisation? A layman's guide', *The Monotype Recorder*. new series, no. 3 (1981)

David Saunders
'Two decades of change, 1965–1986', *The Monotype Recorder.* new series (10) (1997)

Ellen Swanson
Mathematics into type. Providence, RI: The American Mathematical Society (1986)

Lawrence W Wallis
'Monotype time check', *The Monotype Recorder.* new series (10) (1997) 46–55

Karel Wick
Rules for type-setting mathematics. Prague: Publishing House of the Czeckoslovak Academy of Sciences (1965)

Hermann Zapf
Alphabet stories: A chronicle of technical developments. Bad Homburg, Germany: Merganthaler Edition/Linotype GmbH (2007)

Hermann Zapf
'My collaboration with Don Knuth and my font design work'. *TUGboat.* 22 (1–2) (2001) 26–30

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