Subject: SOME ASPECTS OF TREE THEORY (U) Project 702.0

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Abstract

A set of axioms for various types of trees (in particular, for the labeled ordered trees used in current linguistics) is presented, and definitions for some notions relevant to linguistic work are given. Several alternative abstract representations of trees are discussed.

Foreword

During the summer of 1963 a group of linguists, logicians, and mathematicians joined the Information Sciences Subdepartment of D-15, System Sciences, to work on a number of well-defined tasks for Project 702, Language Processing Techniques. This paper, the result of one of these special studies, is in the area of mathematical formalizations. In our development of a program to establish natural language as an operational language for command and control, logico-mathematical formalism is basic to: (1) defining the complex aspects of linguistic structure in generative grammars; (2) developing translation algorithms that relate structural descriptions of sentences to representations of data; and (3) solving mathematical problems of translatability between formal language of differing complexity.

In current linguistic work the notion of a <u>tree</u> is an important one (Chomsky, 1955; Fraser, 1963; Postal, 1963; Meyers and Wang, 1963). The object of this paper is to characterize formally the particular variety of tree --labeled, ordered, finite, directed, singly-rooted, connected graph without circuits-- useful in describing the constituent structure of sentences. Our interest in the algebraic-set theoretic formulation presented stems from dissatisfaction with other existing abstract representations of trees, many of which are examined briefly in the final section of the paper. A basis for a new formalization of operations on trees, operations such as those central to Chomsky's (1955, 1957, 1962) transformational theory of grammar, is presented first.

The authors are indebted to E. C. Haines and J. R. Ross for their suggestions and advice in formulating the axioms presented in this paper.

SOME ASPECTS OF TREE THEORY

I. Axioms and Definitions - First Order Calculus

A. Unordered Tree

We begin with the notion of an (unordered) tree;[1] in a model for this theory, a universe $\mathcal{U}$ of objects called nodes and one binary relation D on $\mathcal{U}$ must be specified. D(X,Y) is read, "X dominates Y."

A1. $(X)(Y)(Z)\ (D(X,Y)\ .\ D(Y,Z) \longrightarrow D(X,Z))$

D is transitive.

A2. $(X)(Y)\ (D(X,Y) \longrightarrow \sim D(Y,X))$

D is asymmetric.

D1. Root (X) for $(Y)\ (Y \neq X \longrightarrow D(X,Y))$

X is a root if it dominates every node.

A3. $(\exists! X)$ Root(X)

There is exactly one root.

D2. $\bar{D}(X,Y)$ for $D(X,Y)\ .\ \sim(\exists Z)\ (D(X,Z)\ .\ D(Z,Y))$

X covers Y if X dominates Y and no Z intervenes.

A4. $(X)(\sim \text{Root}(X) \longrightarrow (\exists! Y)\ \bar{D}(Y,X))$

Every node except the root is uniquely covered.

B. Ordered Tree

A tree is ordered[2] by the specification of an additional binary relation B on $\mathcal{U}$. B(X,Y) is read, "X precedes Y."

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[1] Note that our concept of tree differs from the usual notion in graph theory (Ore, 1962, Chapter 4) in that our trees are directed, and, moreover, directed in a particular way.

[2] Our ordered trees, since they are actually doubly-ordered, may be considered as double lattices if three closure points (besides the root) are added and if the conditions on our relation B are changed somewhat. For a presentation of lattice theory, see Ore (1962), pp. 175-182.

A5. $(X)(Y)(Z)\ (B(X,Y)\ .\ B(Y,Z) \longrightarrow B(X,Z))$

B is transitive.

A6. $(X)(Y)\ (B(X,Y) \longrightarrow \sim B(Y,X))$

B is asymmetric.

A7. $(X)(Y)\ (B(X,Y) \longrightarrow (\exists Z)(\bar{D}(Z,X)\ .\ (\bar{D}(Z,Y)))$

If X precedes Y, both are covered by the same node.

D3. Init (X,Y) *for* $\bar{D}(Y,X) \cdot \sim(\exists Z)\ B(Z,X)$

X is an *initial* of Y if Y covers X and nothing precedes X.

D4. $\bar{B}(X,Y)$ *for* $B(X,Y) \cdot \sim(\exists Z)\ (B(X,Z) \cdot B(Z,Y))$

X is *the predecessor* of Y if X precedes Y and no Z intervenes.

D5. Fin (X,Y) *for* $\bar{D}(Y,X) \cdot \sim(\exists Z)\ B(X,Z)$

X is a *final* of Y if Y covers X and nothing succeeds X.[3]

A8. $(X)\ (\sim(\exists Y)\ \text{Fin}\ (X,Y) \longrightarrow (\exists! Z)\ \bar{B}(X,Z))$

If X is not the final of some Y, then X has a unique successor.

D6. Term (X) *for* $\sim(\exists Y)\ D(X,Y)$

X is *terminal* if it dominates nothing.

A9. $(X)\ (\sim \text{Term}\ (X) \longrightarrow (\exists! Y)\ \text{Init}\ (Y,X))$

Every node that is not terminal has a unique initial.

C. Labeled Tree

A tree is labeled by the addition of a predicate A and a binary relation N. A(X) is read, "X is a *label*," and N(X,Y) is read, "X names Y." $\mathfrak{U}$ now consists of objects called *labels* as well as objects called *nodes* (where the nodes are simply those objects which are not labels).

A10. $(X)(Y)\ (D(X,Y) \longrightarrow \sim A(X)\ .\ \sim A(Y))$

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[3] We used *succeed* as the converse of *precede* in the informal discussion. Similarly, *successor* is used with the obvious meaning.

If X dominates Y, both X and Y are nodes. From D2, A7, and A10 it follows that if X precedes Y, both X and Y are nodes.

A11. $(X)\ (\sim A(X) \longrightarrow (\exists! Y)\ (A(Y)\ .\ N(Y,X)))$

For every node X there is a unique label Y which names X.

A12. $(Y)\ (A(Y) \longrightarrow (\exists X)\ (\sim A(X)\ .\ N(Y,X)))$

Every label names some node.

A13. $(X)(Y)\ (N(X,Y) \longrightarrow A(X)\ .\ \sim A(Y))$

If X names Y, X is a label and Y is a node.

For labeled trees, adjustments are necessary in the statements of three earlier axioms:

A4'. $(X)\ (\sim A(X)\ .\ \sim \mathrm{Root}\ (X) \longrightarrow (\exists! Y)\ \overline{D}(Y,X))$

A8'. $(X)\ (\sim A(X)\ .\ \sim (\exists Y)\ \mathrm{Fin}\ (X,Y) \longrightarrow (\exists! Z)\ \overline{B}(X,Z))$

A9'. $(X)\ (\sim A(X)\ .\ \sim \mathrm{Term}\ (X) \longrightarrow (\exists! Y)\ \mathrm{Init}\ (Y,X))$

II. Axioms and Definitions - Second Order Calculus

In the above axioms we have used, as the underlying logic, a first-order calculus with equality. By weakening the restriction on the underlying logic, we can take $\overline{D}$ and $\overline{B}$, rather than D and B, as primitive and can use the inductive definitions.[4]

D2'. $\left\{ \begin{array}{l} \overline{D}(X,Y) \longrightarrow D(X,Y) \\ D(X,Y)\ .\ D(Y,Z) \longrightarrow D(X,Z) \end{array} \right\}$

D4'. $\left\{ \begin{array}{l} \overline{B}(X,Y) \longrightarrow B(X,Y) \\ B(X,Y)\ .\ B(Y,Z) \longrightarrow B(X,Z) \end{array} \right\}$

[4] Here, and in later inductive definitions, we have omitted the clause requiring that the relation being defined is the smallest relation satisfying the defining conditions. It is the writing out of this extra clause which requires second-order logical notation.

A1, A2, A5, and A6 must then be replaced by axioms of intransitivity and asymmetry for $\bar{D}$ and $\bar{B}$.

A1'. $\sim(\exists X)(\exists Y)(\exists Z)\ (\bar{D}(X,Y)\ .\ \bar{D}(Y,Z)\ .\ \bar{D}(X,Z))$

A2'. $(X)(Y)\ (\bar{D}(X,Y) \longrightarrow \sim\bar{D}(Y,X))$

A5' and A6' are similarly replaced.

A. Labeled Ordered Tree

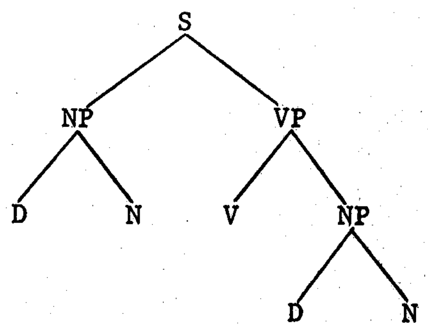
For an example of a labeled ordered tree, let $\mathcal{U} = \{0,1,\ldots,8,$ 'S','NP','VP','D','N','V'$\}$, where the nodes are 0,1,...,8, the labels are 'S','NP','VP','D','N', and 'V', and the relations are given by the following ordered pairs:

$\bar{D}$: (0,1), (0,4), (1,2), (1,3), (4,5), (4,6), (6,7), (7,8)

$\bar{B}$: (1,4), (2,3), (5,6), (7,8)

N: ('S',0), ('NP',1), ('D',2), ('N',3), ('VP',4), ('V',5), ('NP',6), ('D',7), ('N',8)

In the customary graphic representation of trees, symbols used as labels represent nodes, $\bar{D}$ is represented by lines drawn downward from the covering node to the covered node, and $\bar{P}$ is represented by the placement of nodes directly to the right of their immediate predecessors; the labeled ordered tree given above is ordinarily drawn as



B. Unordered and Ordered Sets of Trees

Unordered sets of unordered trees and ordered sets of ordered trees are easily represented as unordered and ordered trees, respectively, by the addition of a new root covering the roots of the given trees (with the requisite additions to $\bar{D}$ and $\bar{B}$.)

To deal with ordered sets of unordered trees or with unordered sets of ordered trees, however, we would have to begin with the notion of a forest (set of trees) by amending D1 and A3 to

D1'. Root (X) for $\sim(\exists Y)\ D(Y,X)$

X is a root if no node dominates X.

A3'. $(X)\ (\sim \text{Root}(X) \longrightarrow (\exists Y)\ (\text{Root}(Y)\ .\ D(Y,X)))$, or

A3". $(X)\ (\sim A(X)\ .\ \sim \text{Root}(X) \longrightarrow (\exists Y)(\text{Root}(Y)\ .\ D(Y,X)))$

For every node X not a root, there is a root which dominates X.

A forest is ordered by a new binary relation T which chain-orders its roots. A forest is a tree if A3 is satisfied. Since this paper is primarily concerned with ordered sets of ordered trees, we will not provide axioms involving T.

C. Bar and Cap Relations

In the presentation above, we have employed the notation of a bar above a binary relation symbol to stand for the intransitive relation corresponding to the given transitive one. $\bar{\mathcal{R}}(X,Y)$ replaces $\mathcal{R}(X,Y)$. $\sim(\exists Z)\ (\mathcal{R}(X,Z)\ .\ \mathcal{R}(Z,Y))$ for any transitive relation $\mathcal{R}$; or, conversely, given $\bar{\mathcal{R}}$ intransitive we define the ancestral $\mathcal{R}$:

$$\left\{\begin{array}{l} \bar{\mathcal{R}}(X,Y) \longrightarrow \mathcal{R}(X,Y) \\ \mathcal{R}(X,Y)\ .\ \mathcal{R}(Y,Z) \longrightarrow \mathcal{R}(X,Z) \end{array}\right\}$$

In the remainder of the paper, we will provide definition-numbers and readings for additional "bar-relations" without explicitly writing out the definitions. For example,

D7. $L(X,Y)$ <u>for</u> $B(X,Y) \vee (\exists U)(\exists V)(D(U,X) \,.\, D(V,Y) \,.\, B(U,V))$

X is <u>to the left</u> of Y if X precedes Y or if a dominator of X precedes a dominator of Y.

D8. $\overline{L}(X,Y)$: X is <u>immediately to the left of</u> Y.

D9. $Q(X,Y)$ <u>for</u> $D(X,Y) \vee L(X,Y)$

X <u>Polishly precedes</u> Y if X dominates Y or is to the left of Y.

D10. $\overline{Q}(X,Y)$: X is the Polish predecessor of Y.

When they exist, covers (A4), predecessors and successors (A7, A8), and Polish predecessors and successors are unique.

Finally, to each relation (or predicate) $\mathcal{R}$ on the set of nodes, we associate a cap relation $\hat{\mathcal{R}}$ obtained from $\mathcal{R}$ by replacing all nodes by their unique labels.

III. <u>Definitions for the Relativized Case</u>

D11. $D_Z(X,Y)$ <u>for</u> $D(Z,X) \,.\, D(Z,Y) \,.\, D(X,Y)$

D12. $B_Z(X,Y)$ <u>for</u> $B(Z,X) \,.\, B(Z,Y) \,.\, B(X,Y)$

The definitions of $\overline{D}_Z$, $\overline{B}_Z$, L_Z, $\overline{L}_Z$, Q_Z, and $\overline{Q}_Z$ are, as before with relativized notions, replacing absolute ones.

D13. $Lext_Z(X)$ <u>for</u> $D(Z,X) \,.\, \sim(\exists Y)\, L_Z(Y,X)$

D14. $Rext_Z(X)$ <u>for</u> $D(Z,X) \,.\, \sim(\exists Y)\, L_Z(X,Y)$

A node X dominated by Z is <u>left-extreme</u> (<u>right-extreme</u>) relative to Z if no node dominated by Z is to the left (right) of X.

D15. $Isa_Z \left[X_1,\ldots,X_n\right]$ <u>for</u> $Lext_Z(X_1) \,.\, Rext_Z(X_n)$.

(i) $(1 < i \leq n \longrightarrow \overline{L}_Z\ (X_{i-1}, X_i))$

If $Isa_Z [X_1,\ldots,X_n]$, we say that $[X_1,\ldots,X_n]$ <u>is a</u> Z (or Z <u>consists of</u> $[X_1,\ldots,X_n]$).

D16a. $Pran_1^Z [X_1,\ldots,X_n]$ <u>for</u>

$$Isa_Z [X_1,\ldots,X_n] . (\exists Y)(Root(Y) . \bar{D}(Y,Z))$$

D16b. $Pran_2 [X_1,\ldots,X_n]$ <u>for</u> $Isa_Z [X_1,\ldots,X_n] . Root(Z)$

If $Pran [X_1,\ldots,X_n]$, we say that $[X_1,\ldots,X_n]$ is a <u>proper analysis</u>,[5] of type 1 or type 2.

For examples, consider the tree in Section II. The nodes 1 and 2 are left-extreme relative to 0, and the nodes 4, 6, and 8 are right-extreme relative to 0. Node 1 consists of [2,3]; node 4 consists of [5,6] and of [5,7,8]; node 6 consists of [7,8]. The following are proper analyses of type 2: [1,4], [2,3,4], [1,5,6], [1,5,7,8], [2,3,5,6], and [2,3,5,7,8].

D17a. $Teran_1^Z [X_1,\ldots,X_n]$ <u>for</u> $Pran_1^Z [X_1,\ldots,X_n]$.

(i) $(1 \leq i \leq n \longrightarrow Term(X_i))$

D17b. $Teran_2 [X_1,\ldots,X_n]$ <u>for</u> $Pran_2 [X_1,\ldots,X_n]$.

(i) $(1 \leq i \leq n \longrightarrow Term(X_i))$

$[X_1,\ldots,X_n]$ is a <u>terminal analysis</u> (of type 1 or 2) if it is a proper analysis (of the correct type) and if each node is terminal.

In the example above, [2,3,5,7,8] is the only terminal analysis of type 2. Analyses of the first type are used for sets of trees; those of the second type, for single trees.

IV. Finiteness Conditions

In a great many applications of trees, the required finiteness condition is simply

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[5]The term "proper analysis" was introduced by Chomsky (1955), p. 379.

C1. $\mathcal{U}$ is finite.

Should we wish to speak of denumerable sets of finite trees, possible conditions are

C2. The set of all nodes covered by the root is denumerable and is well-ordered by B.

C3. For all X covered by the root, the set of all nodes dominated by X is finite.

For denumerable sets of infinite trees, possible conditions are C2 and the following:

C4. Each node covers a finite number of nodes.

C5. If X dominates Y, there is a path from X to Y (where $X_1, \ldots, X_n$ is a <u>path</u> if for every i, $1 < i \leq n$, X_{i-1} covers X_i).

V. <u>Other Representations of Finite-ordered Trees</u>

We now relate our discussion of trees to several representations suggested elsewhere. We do this by providing definitions, within the framework of the earlier part of this paper, for some additional terms in this section.

A. The Polish Representation

If the labels of a tree are symbols, the tree can be represented as the string of labels in their Polish order (beginning with the root), where each label $\mathcal{l}$ bears as its subscript the number of nodes immediately dominated by the node named by $\mathcal{l}$. The Polish representation of our example is

$$S_2 NP_2 D_0 N_0 VP_2 V_0 NP_2 D_0 N_0$$

B. The Labeled Bracketing Representation

D18. $\text{Fam}\left[Y;\ X_1, X_2, \ldots, X_n\right]$ for

$\text{Init}\ (X_1,\ Y)\ .\ \text{Fin}\ (X_n,\ Y)\ .\ (i)\ (1 < i \leq n \rightarrow \overline{P}(X_{i-1},\ X_i))$

$\left[X_1, \ldots, X_n\right]$ is the family of Y if X_1 is the initial of Y, X_n is the final of Y, and the X_i's are in the order of immediate succession.

To obtain a labeled bracketing for a given tree, we associate the following string to each node with a non-empty family.

$$\left[_Y\ X_1\ X_2\ \ldots\ X_n\right]$$

where $\widehat{\text{Fam}}\left[Y;\ X_1, \ldots, X_n\right]$. (Note that the variables now range over labels and not nodes.) Beginning with the string S_0 associated with the root, we substitute for each label X in S_0 the string associated with X. This process is repeated with each resultant string, until no more substitution is possible. The final resultant is the labeled bracketing associated with the tree. The labeled bracketing for our example is

$$\left[_S \left[_{NP}\ D\ N\right] \left[_{VP}\ V \left[_{NP}\ D\ N\right]\right]\right]$$

Algorithms for converting the Polish representation into the labeled bracketing, and vice versa, are well known (Rosenbloom, 1950, Chapter IV, Section 1; Oettinger, 1961).

C. The "Rotated Polish" Representation

For this representation, we define the level of a node as its distance from the root. The nodes are ordered first by increasing level, then by a new kind of precedence on a given level.

$$\text{D19.} \left\{ \begin{array}{l} \text{Root}\ (X) \rightarrow \text{Le}(X) = 0 \\ \overline{D}(X,Y)\ .\ \text{Le}(X) = n \rightarrow \text{Le}(Y) = n+1 \end{array} \right\}$$

The level of a root is 0; if the level of a node X is n, the level of any node covered by X is n+1.

D20. $K(X,Y)$ for $(Le(X) < Le(Y))$ v

$(Le(X) = Le(Y) \; . \; L(X,Y))$

A node X is <u>prior to</u> a node Y if the level of X is less than the level of Y, or if the levels are equal and X is to the left of Y.

D21. $\bar{K}(X,Y)$: X is <u>immediately prior to</u> Y.

The "rotated Polish" representation is the string of labels in their order (beginning with the root) where each label bears as its subscript the number of immediately dominated nodes. For our example, this is

$$S_2NP_2VP_2D_0N_0V_0NP_2D_0N_0$$

D. Representations Involving Complete Paths and Levels

The definition of <u>path</u> (C4 of Section III) can be formalized as

D22. Path $\left[X_1,\ldots,X_n\right]$ <u>for</u> (i) $(1 < i \leq n \rightarrow \bar{D}(X_{i-1}, X_i))$

Similarly, we define

D23. Level $\left[X_1,\ldots,X_n\right]$ <u>for</u> (i) $(1 < i \leq n \rightarrow$

$Le(X_{i-1}) = Le(X_i) \; . \; \bar{K}(X_{i-1}, X_i))$

and then <u>complete path</u> and <u>complete level</u>:

D24. CPath $\left[X_1,\ldots,X_n\right]$ <u>for</u> Root (X_1) . Term (X_n) . Path $\left[X_1,\ldots,X_n\right]$

D25. CLevel $\left[X_1,\ldots,X_n\right]$ <u>for</u> Lext (X_1) . Rext (X_n) . Level $\left[X_1,\ldots,X_n\right]$

A tree is uniquely determined by the specification of CPath, CLevel, and N; or of CPath, the sequence S_Q of all nodes in their Polish order, and N; or of CLevel, the sequence S_K of all nodes in the order of their priority, and N.

D26. $S_Q\left[X_1,\ldots,X_n\right]$ <u>for</u> Root (X_1) . Term (X_n) . Rext (X_n).

(i) $(1 < i \leq n \rightarrow \bar{Q}(X_{i-1}, X_i))$

D27. $S_K\left[X_1,\ldots,X_n\right]$ for Root (X_1) . Term (X_n) . Rext (X_n).

(i) $(1 < i \le n \rightarrow \bar{K}(X_{i-1}, X_i))$

D28. $M\left[X_1,\ldots,X_n\right]$ for $(n = 1$. Root $(X_1))$ v $\text{Pran}_2\left[X_1,\ldots,X_n\right]$

The specification of M and N determines a unique tree if there is no node which covers only one node (i.e., if M does not contain two sequences differing in only one position); in this case, it is impossible to determine which of the two nodes is the covering node and which is the covered node. Either the further specification of CPath, CLevel, S_Q, S_K, or D, or a suitable ordering of M uniquely determines a tree.

Unique representation of trees with the cap relations is more complicated. The following two trees are indistinguishable even if $\widehat{\text{CPath}}$, $\widehat{\text{CLevel}}$, and $\hat{M}$ are all specified:

We have indicated in (A) and (C) above how representation by $\hat{S}_Q$ and $\hat{S}_K$ can be made unique by the addition of subscripts.

In our example, $\widehat{\text{CPath}}$ is true of the sequences [S, NP, D], [S, NP, N], [S, VP, V], [S, VP, NP, D], [S, VP, NP, N]; $\widehat{\text{CLevel}}$ is true of [S], [NP, VP], [D, N, V, VP], [D, N].

E. Representation by Quadruples[6]

A tree can also be uniquely represented by giving all the quadruples (X, Y, Z, W), where X is a node, Y the cover of X (or X itself if X is a root), Z the immediate successor of X (or X itself if

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[6] For some applications of this and the following notations, see Iverson (1962).

X has no immediate successor), and W the label that names X. That is,

D29. $F(X,Y,Z,W)$ for $(\overline{D}(Y,X) \vee (\text{Root}(X) \,.\, Y = X)) \,.\, (\overline{B}(X,Z) \vee (\sim(\exists V)\, \overline{B}(X,V) \,.\, Z = X)) \,.\, N(W,X)$

F. Representation by Integer-Sequences

The nodes of a tree can be coded as sequences of positive integers in the following way: To the root X is assigned the sequence (1); to the initial Y of X is assigned the sequence (1, 1) and to the immediate successor of Y the sequence (1,2) and so on through the other nodes covered by X; to the initial of Y is assigned the sequence (1, 1, 1), etc. To formalize this process we first define two operations on finite sequences of positive integers; the lower-case Latin variables in the definitions range over positive integers, the Greek variables over sequences.

D30. $[X_1, X_2, \ldots, X_m] ^\frown [Y_1, Y_2, \ldots, Y_n] = [X_1, X_2, \ldots, X_m, Y_1, Y_2, \ldots, Y_n]$

D31. $[X_1, X_2, \ldots, X_m]' = [X_1, X_2, \ldots, X_{m-1}, X_m + 1]$

D32. $\text{Root}(X) \rightarrow \text{Seq}(X) = [1]$

$\text{Seq}(X) = \alpha \,.\, \text{Init}(Y,X) \rightarrow \text{Seq}(Y) = \alpha ^\frown [1]$

$\text{Seq}(X) = \alpha \,.\, \overline{B}(X,Y) \rightarrow \text{Seq}(Y) = \alpha'$

In our example, the values of Seq for the terminal nodes are

Seq (2) = [1, 1, 1]

Seq (3) = [1, 1, 2]

Seq (5) = [1, 2, 1]

Seq (7) = [1, 2, 2, 1]

Seq (8) = [1, 2, 2, 2]

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Attachments: References
Distribution List

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