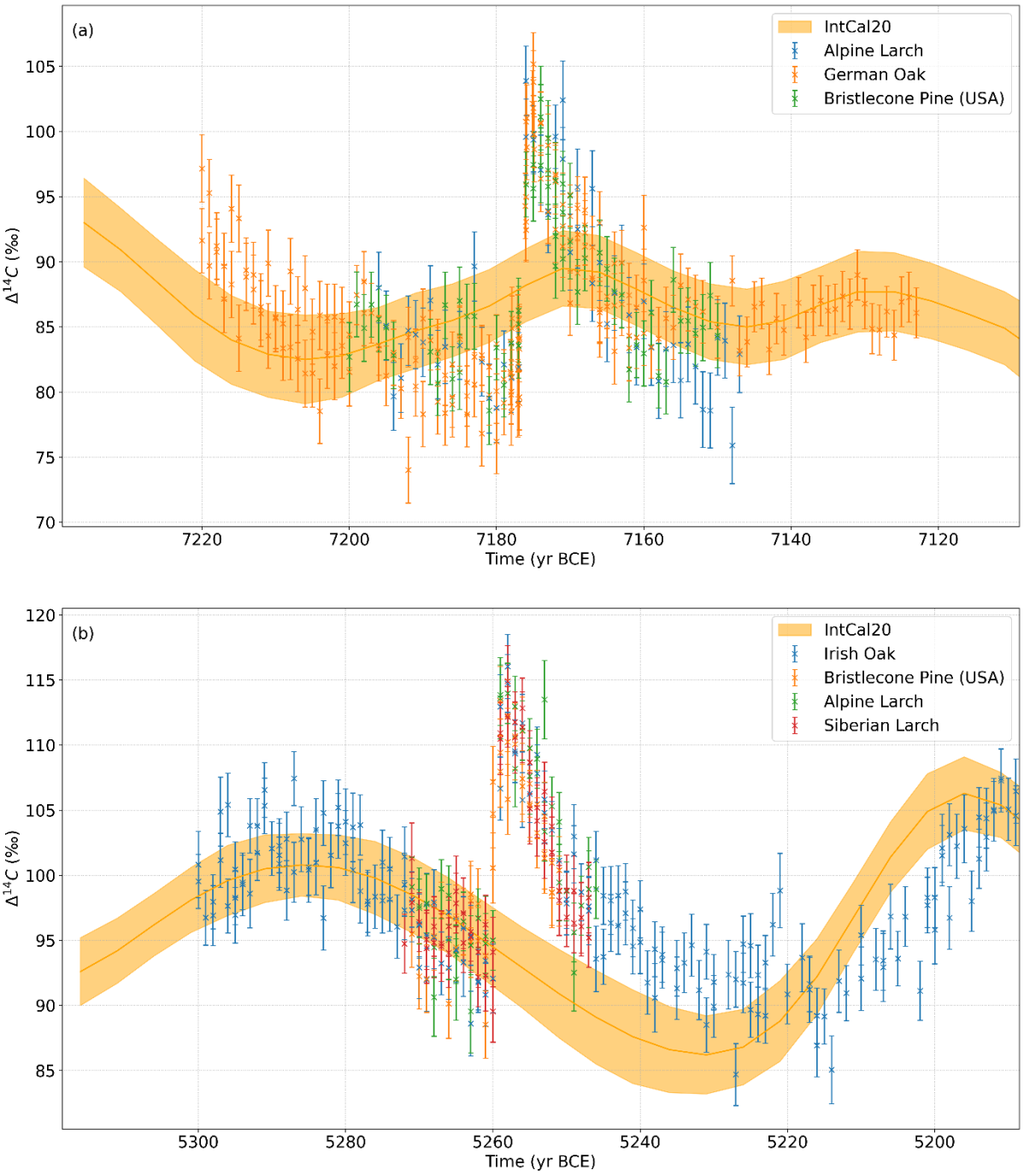
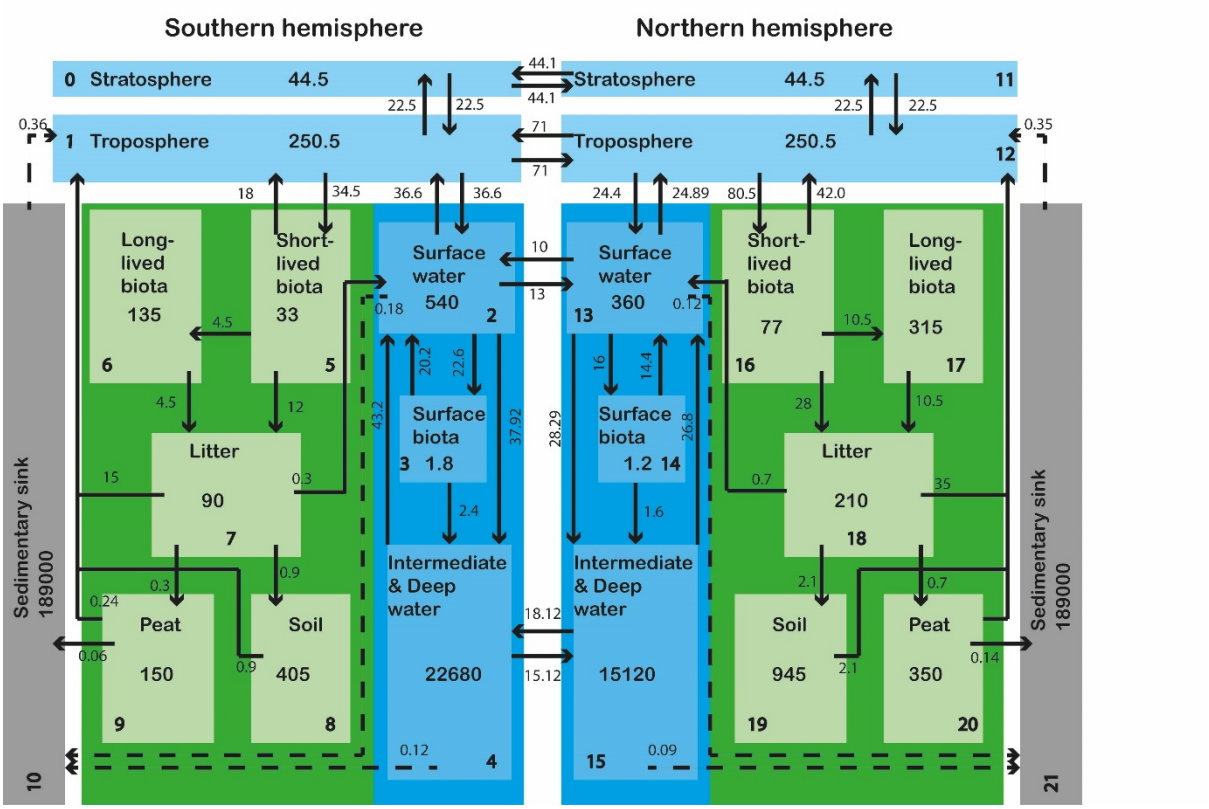
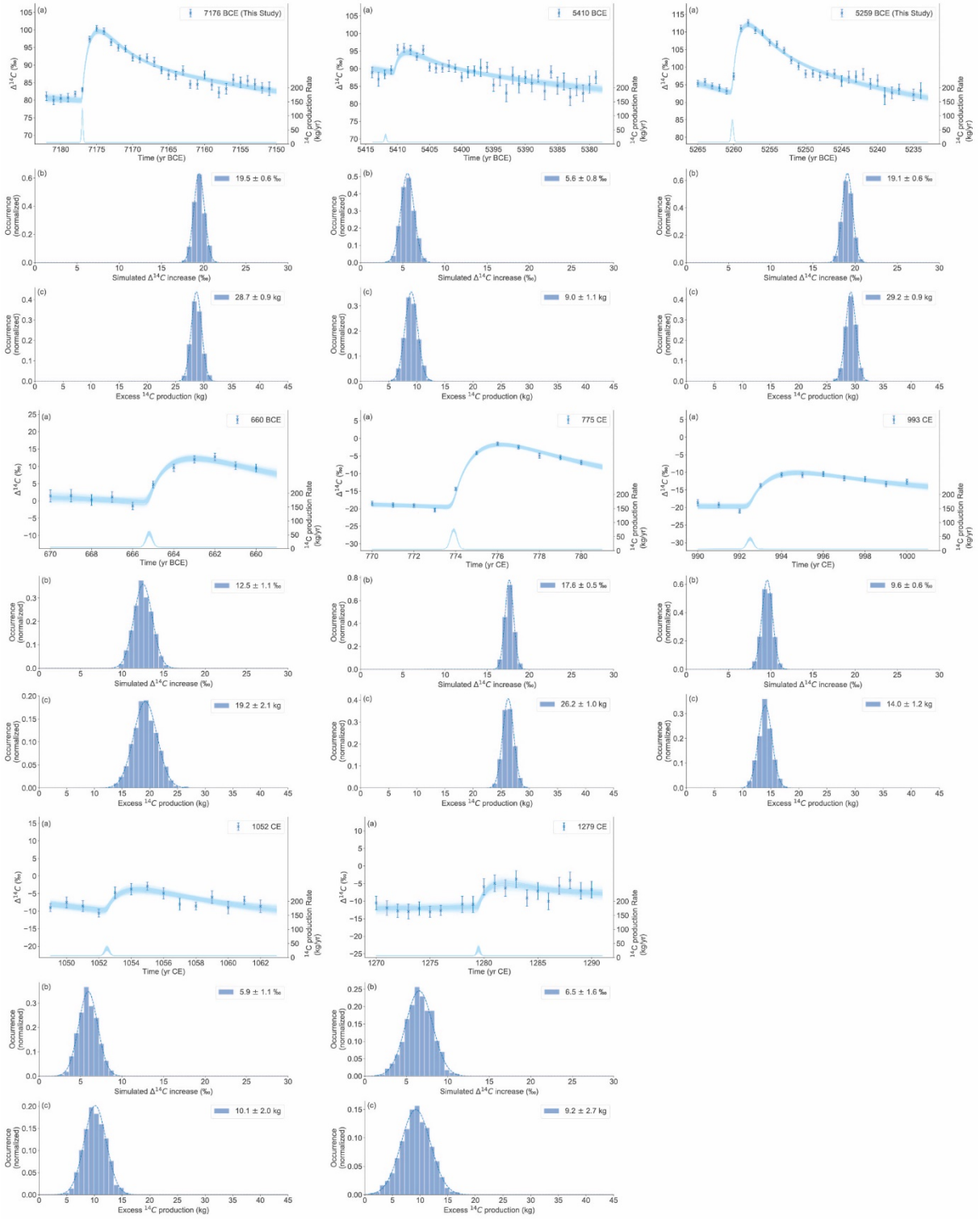
Supplementary figures



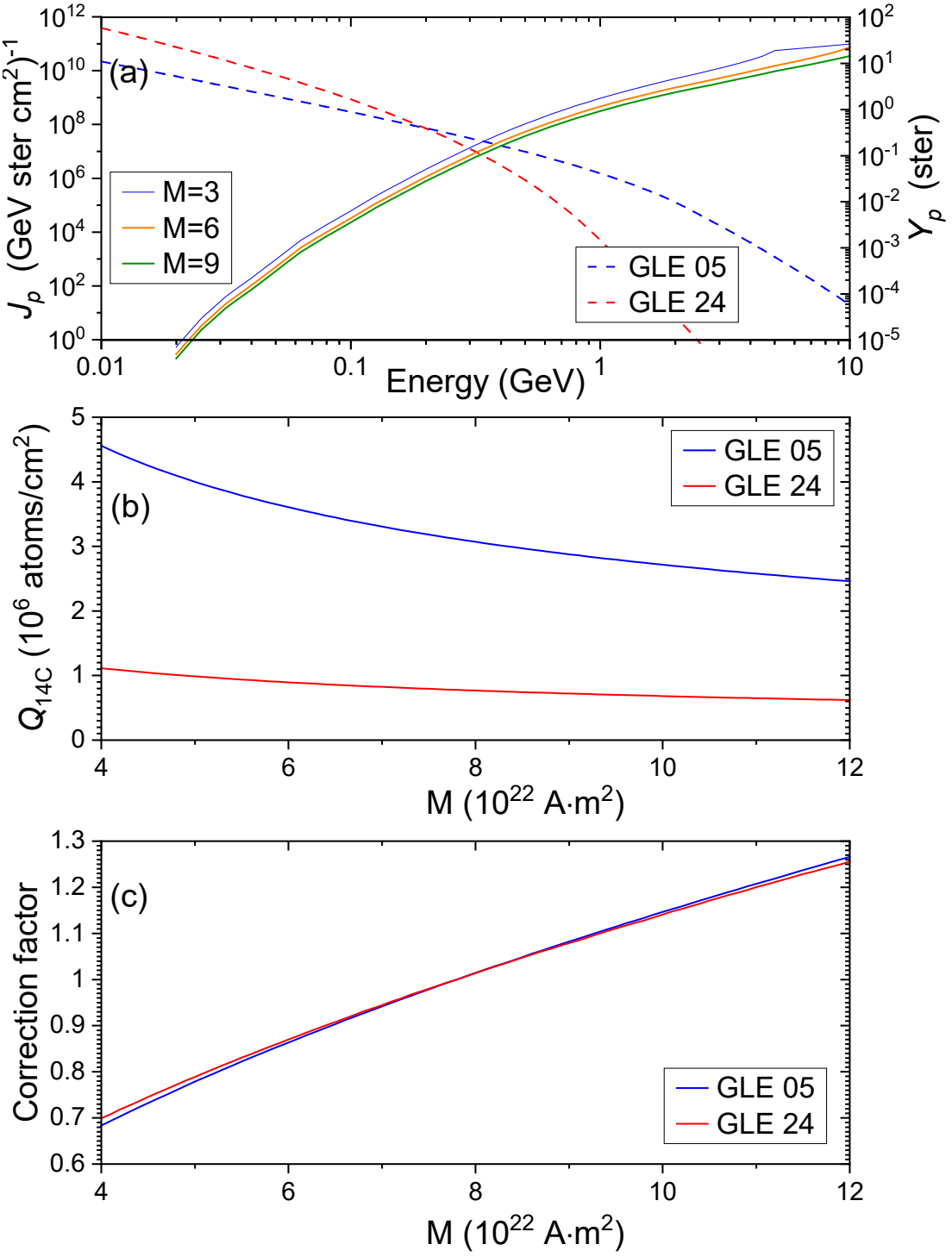
Supplementary Fig. 1 All Measurement results of the 2 new events. *^{14}C measurements with 1-σ errors reported as $\Delta^{14}C$ of the two newly found events (7176 BCE (a), 5259 BCE (b)) in all different trees compared to the IntCal20 calibration curve[1] (orange band). The Irish Oak was independently repeated by two different labs (ETH-Zurich, Bristol).*



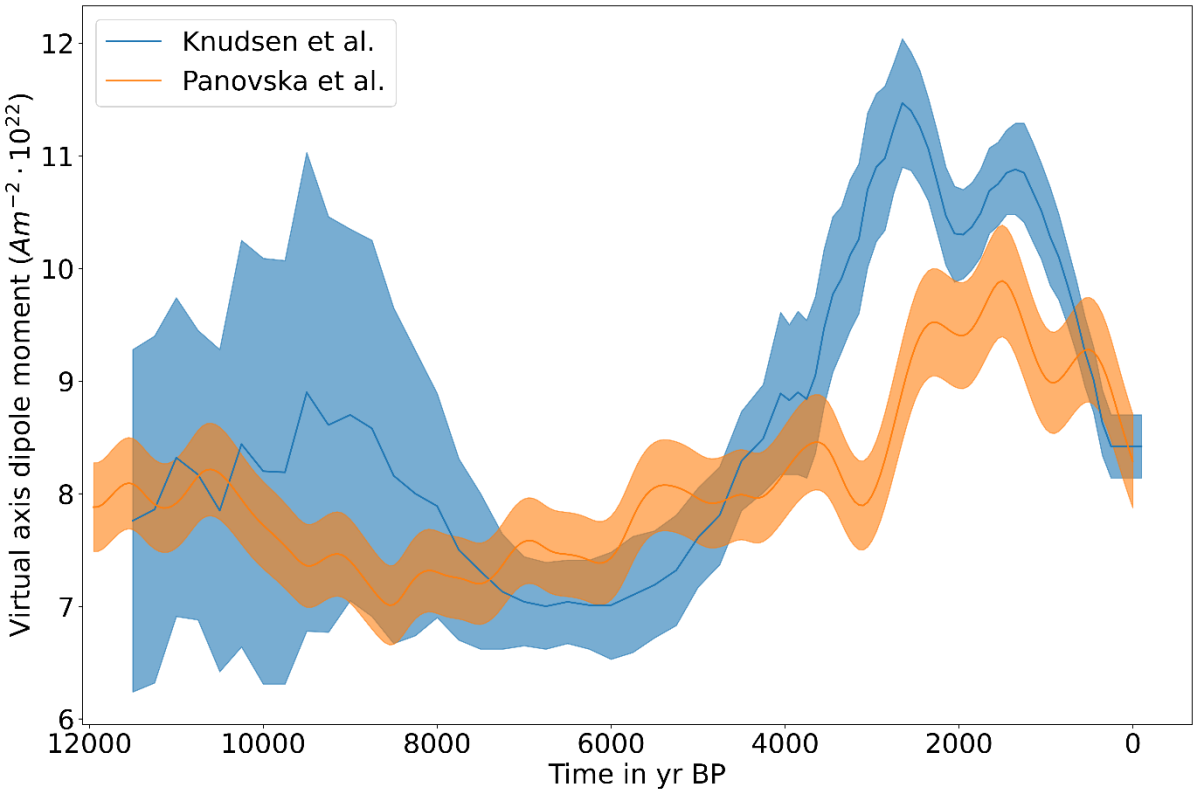
Supplementary Fig. 2 Carbon box model used to reconstruct ^{14}C production. *The carbon fluxes between boxes and their carbon contents are given in Gt/yr and Gt.*



Supplementary Fig. 3 Evaluation of all known ^{14}C events. *(a) Mean data of known ^{14}C events with 1-σ errors and result of 1000 Simulations. The fitted Gaussian shaped production spikes for all simulations are also shown. (b) Distribution of the simulated $\Delta^{14}C$ increases (blue bars) with a Gaussian fit (dashed line). (c) Distribution of excess ^{14}C production (blue bars) with Gaussian fit (dashed line).Data Sources: 5410 BCE: Miyake et al.[2], 660 BCE: Sakurai et al.[3], 775 CE and 993 CE: Büntgen et al.[4], 1052 CE and 1279 CE: Brehm et al.[5].*



Supplementary Fig. 4 *Production of ^{14}C by SEP events for different values of VADM: (a) Global yield functions (Y_p, right-hand-side axis) of ^{14}C by protons for three values of VADM (3x, 6x and 9x10^{22} A m^2 as blue, orange and green solid lines, respectively); as well as spectral omnidirectional fluences (J(E), left-hand-side axis) of SEPs for two bounding cases, the softest and hardest-spectrum known events of 23-Feb-1956 (GLE 05, blue dashed line) and 04-Aug-1972 (GLE 24, red dashed line). (b) Calculated globally-averaged ^{14}C production Q_{14C} during the two SEP events as a function of the geomagnetic field VADM. (c) Correction factor $Q(M_o)/Q(M)$ of the ^{14}C production at the VADM value of M to that for a modern geomagnetic field (M_o=7.8·10^{22} A m^2).*



Supplementary Fig. 5 Comparison of two geomagnetic field reconstructions. *Two geomagnetic field reconstructions by Knudsen et al.[6] (blue) and Panovska et al.[7] (orange) over the last 12000 years including 1-σ uncertainty ranges.*

Supplementay References

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