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Appendix A: Proofs

Proof of Lemma 1. If G remains loyal and does not mount a coup, this threat is realized and G's expected payoff is $p(m, T; \theta)b_i$. If a coup did occur but it failed, G is eliminated entirely so his payoff once the threat is realized remains 0. If a coup succeeded, G obtains the benefit of rule and fights the external threat (R's security resources are assumed lost and unavailable to G). Thus, G's expected payoff from a coup is $p(m, 1; \theta)p(m, T; \theta) - c$. By subgame-perfection, G will remain loyal if

$$p(m, T; \theta)\big[p(m, 1; \theta) - b_i\big] < c,$$

execute a coup if the strict inequality is reversed, and be indifferent otherwise. We can rewrite this as $T < T_i^*(m, \theta)$, where the latter is defined in (1). This establishes the sufficiency part of the claim.

Letting $x \equiv \theta m$, we can observe that

$$\frac{\mathrm{d}\,T_i^*}{\mathrm{d}\,x} = \left(\frac{1}{c}\right)\left[1 - (b_i + c) - \frac{1}{(1 + x)^2}\right],$$

which means that

$$\operatorname{sgn}\left(\frac{\mathrm{d}\,T_i^*}{\mathrm{d}\,x}\right) = \operatorname{sgn}\left(1 - (b_i + c) - \frac{1}{(1 + x)^2}\right).$$

This yields a quadratic, $x^2 + 2x - \frac{b_i + c}{1 - (b_i + c)} > 0$, which is a parabola that opens up. Although the discriminant is $4/(1 - (b_i + c)) > 0$, the smaller root is negative, which means that the inequality is satisfied for all

$$x > \frac{1}{\sqrt{1 - (b_i + c)}} - 1. \tag{4}$$

But now $T_i^*(m, \theta) \geq 0$ implies that

$$x \geq \frac{b_i + c}{1 - (b_i + c)} > \frac{1}{\sqrt{1 - (b_i + c)}} - 1,$$

where the second inequality is readily verified under Assumption 1, and so (4) must be satisfied whenever T_i^* is non-negative. In other words, when T_i^* is non-negative it must be increasing in both θ and m, as claimed.

To prove necessity, we need to show that there is no equilibrium where G executes a coup with positive probability when indifferent. Suppose, to the contrary, that he does execute a coup with positive probability, perhaps even certainty, when indifferent. First, note that if R's expected payoff from a coup is at least as good as the expected payoff from loyalty, then the fact that R strictly prefers not hiring a general to a coup also implies that R would not hire a general in this case. In other words, whenever G gets hired in equilibrium, it must be that R strictly prefers him to remain loyal:

$$p(m, T; \theta) > p(1, \theta m; 1)p(m, T; 1). \tag{5}$$

Second, we show that R can do strictly better by ensuring G's loyalty. Letting $q \in (0, 1]$ denote the probability of a coup, R's expected payoff is $qp(1, \theta m; 1)p(m, T; 1) + (1 - q)p(m, T; \theta) - m$. Since G is indifferent, it must be that $T_i^*(m, \theta) = T > 0$, which further implies that T_i^* is increasing in m. This now means that any $\hat{m} < m$ would result in $T_i^*(\hat{m}, \theta) < T$, ensuring G's loyalty. Consider now some such $\hat{m} < m$ that is arbitrarily close to m, and observe that this means that $p(\hat{m}, T; \theta)$ is arbitrarily close to $p(m, T; \theta)$. By (5), $qp(1, \theta m; 1)p(m, T; 1) + (1-q)p(m, T; \theta) < p(m, T; \theta)$ for any $q \in (0, 1]$, which means that we can always find $\hat{m}$ such that $qp(1, \theta m; 1)p(m, T; 1) + (1 - q)p(m, T; \theta) < p(\hat{m}, T; \theta)$. In other words, R strictly prefers to reduce m by an arbitrarily small amount and ensure G's loyalty. But this contradicts the equilibrium requirement that R's strategy be optimal. Therefore, there can be no equilibrium where G executes a coup with positive probability when indifferent. This establishes the necessity part of the claim. ∎

Proof of Lemma 2. If R's choices avoid a coup, her payoff is $U = p(m, T; \theta) - m$, and the loyalty constraint, $T \geq T_i^*(m, \theta)$, must obtain. Solving for the constraint yields the quadratic $(1 - (b_i + c))\theta^2 m^2 - (b_i + c + cT)\theta m - cT \leq 0$, whose discriminant is $(b_i + c - cT)^2 + 4cT > 0$. Under Assumption 1 the smaller root is negative, so let $S_i^*(T)$ be the larger root defined in (2). Since the coefficient on the squared term is positive, the constraint is satisfied for all $\theta m \leq S_i^*(T)$.

R's payoff is strictly increasing in θ and concave in m:

$$\frac{\mathrm{d}U}{\mathrm{d}\theta} = \frac{mT}{(\theta m + T)^2} > 0 \quad \text{and} \quad \frac{\mathrm{d}U}{\mathrm{d}m} = \frac{\theta T}{(\theta m + T)^2} - 1.$$

Let the solution to the first-order condition on m be defined as

$$\widetilde{m}(\theta) = \max\left(0, \sqrt{\frac{T}{\theta}} - \frac{T}{\theta}\right),$$

so clearly the unconstrained maximum is at $(\widetilde{m}(\overline{\theta}), \overline{\theta})$. Let $S(T) = \overline{\theta}\widetilde{m}(\overline{\theta})$ be the loyalty induced if R were to provide G of maximal competence with the level of resources optimal for dealing with the threat. If this level of disloyalty does not exceed the maximum level that avoids a coup, $S(T) \leq S_i^*(T)$, then the unconstrained maximum is the unique solution to R's maximization problem.

If $S(T) > S_i^*(T)$, then the induced level of disloyalty exceeds the safe maximum, and G would execute a coup if he were provided with such resources. Since this cannot happen in an equilibrium where coups are avoided, the loyalty constraint must bind: $\theta m = S_i^*(T)$. (If it were slack at some θm, then R could strictly increase her payoff by increasing θ until it binds.) This means that R's expected payoff can be written as

$$U = \frac{S_i^*(T)}{S_i^*(T) + T} - \frac{S_i^*(T)}{\theta},$$

which is strictly increasing in θ. Therefore, R will pick $\overline{\theta}$ again except that this time she will handicap G by providing him with fewer resources. ∎

Proof of Lemma 3. When G's resources are not constrained by loyalty considerations, the envelope theorem tells us that

$$\frac{\mathrm{d}\, U(m_i^*(\overline{\theta}), \overline{\theta})}{\mathrm{d}\,\overline{\theta}} = \frac{\partial U(m_i^*(\overline{\theta}), \overline{\theta})}{\partial \overline{\theta}} = \frac{m_i^*(\overline{\theta}) T}{\left(\overline{\theta} m_i^*(\overline{\theta}) + T\right)^2} > 0.$$

Since R's payoff when not hiring a general can be represented by the payoff of hiring a general with competence $\theta = 1$ for whom the constraint is not binding, we conclude that if $\overline{\theta} < 1$, then R strictly prefers not to hire a general than to hire one whose loyalty will not be a problem at the optimal level of resource provision. Since R's payoff is strictly smaller when the loyalty constraint binds, this further implies that R will not want to hire a general at all. This establishes case (i) of the lemma.

If $\overline{\theta} > 1$, then R strictly prefers to hire G provided that his loyalty will not be a problem. We know, however, that for $\overline{\theta}$ sufficiently high, $S(T) > S_i^*(T)$ will obtain, and so R will be forced to reduce the resources in order to ensure G's loyalty. Would she still wish to hire this general? Assume that $S(T) > S_i^*(T)$ so $m_i^*(T) = S_i^*(T)/\overline{\theta}$. Hiring a general yields

$$\frac{S_i^*(T)}{S_i^*(T) + T} - \frac{S_i^*(T)}{\overline{\theta}} > 0,$$

where we can establish the inequality as follows. The inequality holds if, and only if, $\overline{\theta} > S_i^*(T) + T$. But since $S(T) > S_i^*(T)$ here, it follows that $\sqrt{\overline{\theta} T} > S_i^*(T) + T$, which reduces to $\overline{\theta} > \left(S_i^*(T)\right)^2 / T + 2S_i^*(T) + T > S_i^*(T) + T$. Thus, whenever the loyalty constraint binds, R's (constrained) payoff is strictly positive.

Not hiring a general with optimal allocation $m = \sqrt{T} - T$ (provided $T < 1$) yields

$$\frac{\sqrt{T} - T}{\sqrt{T}} - \sqrt{T} + T = 1 + T - 2\sqrt{T} > 0.$$

Since $T \geq 1$ means that not hiring yields a payoff of zero (because the optimal allocation is zero), it follows that in all such cases R strictly prefers to hire a general even if doing so requires R to impose constraints on him. This establishes case (ii) of the lemma.

Suppose then that $T < 1$, so that the payoffs from hiring and not hiring are both positive. We now show that it is possible that R prefers not to hire at all. Note first that

$$\lim_{c \to 0} S_i^*(T) = \frac{b_i}{1 - b_i},$$

and since we require that $S_i^*(T) < S(T)$, the condition that the constraint is binding will be satisfied for any

$$b_i < \frac{S(T)}{1 + S(T)}.$$

This means that as $b_i \to 0$, the constraint must be binding, and since $\lim_{b_i \to 0} b_i/(1 - b_i) = 0$, we obtain

$$\lim_{c \to 0, b_i \to 0} \frac{S_i^*(T)}{S_i^*(T) + T} - \frac{S_i^*(T)}{\overline{\theta}} = 0 < 1 + T - 2\sqrt{T}.$$

In other words, if c and b_i are sufficiently small, then it must be the case that R strictly prefers not to hire. This establishes case (iii) of the lemma. ∎

Proof of Lemma 4. It is clear by inspection of (2) that S_i^* is strictly increasing in b_i. Since $S(T)$ is constant in b_i, it follows that $b^* > 0$ such that $S_i^*(T) = S(T)$ exists and is unique. If $\overline{b} \leq b^*$, then the loyalty constraint is binding, so the military allocation is $m_i^*(T) = S_i^*(T)/\overline{\theta}$, which is increasing in $S_i^*(T)$. Moreover, since this constrained allocation is less than the unconstrained optimum, it follows that R's expected payoff is strictly increasing in m_i^* as well. In other words, in this case R's expected payoff strictly increases in b_i, which implies that she must pick $\overline{b}$. If $\overline{b} > b^*$, then the loyalty constraint is no longer binding, so R's military allocation is at the unconstrained optimum, which itself is independent of b_i. In these cases, R is indifferent among any $b_i \in (b^*, \overline{b}]$, as claimed. ∎

Proof of Lemma 5. Suppose a coup will occur, so R's payoff is

$$U = \left(\frac{m}{1 + \theta m}\right)\left(\frac{q}{m + T_{\mathrm{S}}} + \frac{1 - q}{m + T_{\mathrm{L}}}\right) - m,$$

which is always strictly worse than not hiring a general for any $m > 0$. Since $T_i^*(0, \theta) = 0 < T_{\mathrm{S}}$, the probability of a coup is zero when $m = 0$, which implies that in any subgame where a coup is certain to occur it must be the case that $m > 0$, and so R is strictly better off not hiring a general. In other words, there exists no equilibrium where a coup is certain to occur. ∎

Proof of Lemma 6. NO COUP. Suppose there is an equilibrium in which no coups occur regardless of the size of the threat. We know that this requires m to be such that G remains loyal under T_S. It turns out that $m_i^*(T_\mathrm{S})$ must be the optimal security-preserving allocation under asymmetric information as well. We know that it cannot exceed that level because if it did, G would execute a coup under T_S. It also cannot be less than that level because if it did, R's payoffs under both T_S and T_L (under Assumption 3) would decrease, leading to a decrease in the expected payoff as well. Thus, the best expected payoff that R can obtain where no coup occurs is

$$U_\mathrm{N}(q) = q\left(\frac{\theta m_i^*(T_\mathrm{S})}{\theta m_i^*(T_\mathrm{S}) + T_\mathrm{S}}\right) + (1-q)\left(\frac{\theta m_i^*(T_\mathrm{S})}{\theta m_i^*(T_\mathrm{S}) + T_\mathrm{L}}\right) - m_i^*(T_\mathrm{S}).$$

Since $m_i^*(T_\mathrm{S})$ does not depend on q, U_N is a simple linear function of q. In particular, since $T_\mathrm{S} < T_\mathrm{L}$, it is strictly increasing

$$\frac{\mathrm{d}\,U_\mathrm{N}}{\mathrm{d}\,q} = \frac{\theta m_i^*(T_\mathrm{S})(T_\mathrm{L} - T_\mathrm{S})}{(\theta m_i^*(T_\mathrm{S}) + T_\mathrm{S})(\theta m_i^*(T_\mathrm{S}) + T_\mathrm{L})} > 0.$$

We now show that if $\theta \leq 1$, then R prefers to go it alone when the alternative is hiring a general who would not execute a coup. This follows immediately from the fact that $\theta < 1 \Rightarrow U_\mathrm{A} > U_\mathrm{N}$ for any $m > 0$ and any q. We can write $U_\mathrm{A} > U_\mathrm{N}$ as

$$q\,[p(m, T_\mathrm{S}; 1) - p(m, T_\mathrm{S}; \theta)] + (1-q)\,[p(m, T_\mathrm{L}; 1) - p(m, T_\mathrm{L}; \theta)] > 0,$$

so it is sufficient to show that both bracketed terms are positive. Since $p(m, T; \theta)$ is strictly increasing in θ, they are positive when $\theta < 1$, so the claim holds. Moreover, since $\theta \leq T_\mathrm{S}$ implies that $m_i^*(T_\mathrm{S}) = 0$, we obtain $U_\mathrm{N} = 0 < U_\mathrm{A}$, so R will also prefer to go it alone in this case as well. Thus, the necessary condition for hiring G in such an equilibrium is $\theta > \max(1, T_\mathrm{S})$.

PROBABILISTIC COUP. Suppose there is an equilibrium in which G executes a coup under T_S but remains loyal under T_L. This means that $T_\mathrm{S} < T_i^*(m, \theta) \leq T_\mathrm{L}$. Recalling from Lemma 1 that T_i^* is increasing in both parameters whenever it is positive (as it must be here), we conclude that the optimal allocation must be some $m_\mathrm{C} \in (m_i^*(T_\mathrm{S}), m_i^*(T_\mathrm{L})]$.

When the coup is probabilistic, R's expected payoff is

$$U_\mathrm{C}(q) = q\left[\left(\frac{1}{1+\theta m}\right)\left(\frac{m}{m + T_\mathrm{S}}\right) - m\right] + (1-q)\left(\frac{\theta m}{\theta m + T_\mathrm{L}} - m\right). \quad (6)$$

We now show that $\theta \leq 1 \Rightarrow U_\mathrm{A} > U_\mathrm{C}$, so R will never hire a general that is less competent than herself if she expects the continuation game to involve a probabilistic coup. We can write $U_\mathrm{A} > U_\mathrm{C}$ as

$$q\,[p(m, T_\mathrm{S}; 1) - p(1, \theta m; 1)\,p(m, T_\mathrm{S}; 1)] + (1-q)\,[p(m, T_\mathrm{L}; 1) - p(m, T_\mathrm{L}; \theta)] > 0,$$

so it is sufficient to show that both bracketed terms are positive. The first is positive because $p(1, \theta m; 1) < 1$, and the second is non-negative if $\theta \leq 1$ because $p(m, T; \theta)$ is strictly increasing in θ. Moreover, $\theta \leq T_{\mathrm{S}} < T_{\mathrm{L}}$ implies that $m_i^*(T_{\mathrm{S}}) = m_i^*(T_{\mathrm{L}}) = 0$, so there exists no m_{C} that will induce a probabilistic coup. In other words, if $\theta \leq T_{\mathrm{S}}$, then such an equilibrium does not exist. Thus, the necessary condition for hiring G in such an equilibrium is also $\theta > \max(1, T_{\mathrm{S}})$.

Since R will not hire G with $\theta \leq 1$, for the remainder of this proof we shall assume that $\theta > 1$. The unconstrained FOC for (6) is

$$\begin{aligned}
\frac{\partial U_{\mathrm{C}}}{\partial m} &= \frac{q(T_{\mathrm{S}} - \theta m^2)}{(1+\theta m)^2 (m + T_{\mathrm{S}})^2} + \frac{(1-q)\theta T_{\mathrm{L}}}{(\theta m + T_{\mathrm{L}})^2} - 1 \\
&= q\left[\frac{T_{\mathrm{S}} - \theta m^2}{(1+\theta m)^2 (m + T_{\mathrm{S}})^2} - \frac{\theta T_{\mathrm{L}}}{(\theta m + T_{\mathrm{L}})^2}\right] + \frac{\theta T_{\mathrm{L}}}{(\theta m + T_{\mathrm{L}})^2} - 1 \\
&\equiv q\zeta + \frac{\theta T_{\mathrm{L}}}{(\theta m + T_{\mathrm{L}})^2} - 1 = 0. \qquad (7)
\end{aligned}$$

Since the derivative is strictly decreasing in m, it attains a maximum at $m = 0$, where it is strictly positive if, and only if, $qT_{\mathrm{L}} + (1-q)\theta T_{\mathrm{S}} > 1$. By Assumption 2 and $\theta > 1$, this condition is satisfied, so the fact that $\lim_{m\to\infty} \frac{\partial U}{\partial m} = -1$ implies that there exists a unique $m_{\mathrm{C}}(q) > 0$ for which the FOC is satisfied (i.e., the function is concave). The question now is to ensure that the solution satisfies the constraints.

We begin by showing that $m_{\mathrm{C}}(q)$ must be decreasing. The implicit function theorem tells us that (7) implies that

$$\frac{\mathrm{d}\, m_{\mathrm{C}}}{\mathrm{d}\, q} = -\frac{\partial^2 U_{\mathrm{C}}}{\partial m \partial q} \Bigg/ \frac{\partial^2 U_{\mathrm{C}}}{\partial m \partial m_{\mathrm{C}}}$$

which then tells us that since

$$\frac{\partial^2 U_{\mathrm{C}}}{\partial m \partial m_{\mathrm{C}}} < 0 \Rightarrow \operatorname{sgn}\left(\frac{\mathrm{d}\, m_{\mathrm{C}}}{\mathrm{d}\, q}\right) = \operatorname{sgn}\left(\frac{\partial^2 U_{\mathrm{C}}}{\partial m \partial q}\right) = \operatorname{sgn}(\zeta) = \operatorname{sgn}\left(1 - \frac{\theta T_{\mathrm{L}}}{(\theta m + T_{\mathrm{L}})^2}\right),$$

where the last step also follows from (7) and $q > 0$. This, of course, yields

$$\operatorname{sgn}\left(1 - \frac{\theta T_{\mathrm{L}}}{(\theta m + T_{\mathrm{L}})^2}\right) = -1 \quad \Leftrightarrow \quad m < \sqrt{\frac{T_{\mathrm{L}}}{\theta}} - \frac{T_{\mathrm{L}}}{\theta} \equiv \widetilde{m},$$

where the last expression is the unconstrained optimum for the complete-information case under T_{L}.

We now show that m_{C} can never exceed this value. Consider the payoff in (6). The expression in the square brackets (the expected payoff from a coup with T_{S}) is strictly decreasing in m because

$$\frac{T_{\mathrm{S}} - \theta m^2}{(1+\theta m)^2 (m + T_{\mathrm{S}})^2} - 1 < 0$$

obtains. To see this, observe that it is certainly true for any $T_S - \theta m^2 \leq 0$. When this expression is positive, we can write the inequality as $T_S - \theta m^2 < (1 + \theta m)^2 (m + T_S)^2$, and observe that the left-hand side is strictly decreasing in m while the right-hand side is strictly increasing. Thus, if the inequality holds at $m = 0$, it must hold at $m > 0$ as well. But at $m = 0$ the inequality reduces to $T_S < T_S^2 \Leftrightarrow 1 < T_S$, which holds by Assumption 2. Thus, the first component in the expected payoff is always strictly decreasing in m.

The second component of this payoff is, of course, the complete-information payoff without a coup against T_L, and we know that its unconstrained optimum is $\widetilde{m} = \sqrt{T_L/\theta} - T_L/\theta$. This immediately tells us that $m_C < \widetilde{m}$: if this were not so, one could improve the payoff by decreasing m_C to $\widetilde{m}$ since this will strictly increase both components.

Thus, $m_C(q) < \widetilde{m}$, which in turn means that $\operatorname{sgn}(\zeta) = -1$, and we conclude that $m_C(q)$ is strictly decreasing.

Observe now that at $q = 0$, the payoff in (6) is equivalent to the complete-information case under T_L, which means that $m_C(0) = m_i^*(T_L) > m_i^*(T_S)$, where the inequality follows from Assumption 3, so the constraints are satisfied (the general executes a coup if the threat is T_S but does not if it is T_L). Moreover, since $m_i^*(T_L)$ is the (possibly constrained) optimum against T_L, it follows that

$$U_C(0) = \frac{\theta m_i^*(T_L)}{\theta m_i^*(T_L) + T_L} - m_i^*(T_L) > \frac{\theta m_i^*(T_S)}{\theta m_i^*(T_S) + T_L} - m_i^*(T_S) = U_N(0),$$

which means that at $q = 0$, the ruler must strictly prefer to play the risky strategy by endowing G with enough resources to meet the large external threat. (Of course, at $q = 0$, this risk is zero.)

Consider now what happens as q increases, in which case we have shown that m_C must decrease. There are two cases, depending on whether $m_C(q)$ satisfies the constraints or not.

Case 1: $m_C(q) \geq m_i^*(T_L)$, which implies that the solution must be constrained at $m_i^*(T_L)$ (or else G would execute the coup regardless of the threat size): since the payoff function is concave in m, it must be increasing for all $m < m_C(q)$. Moreover, since $m_C < \widetilde{m}$, it follows that $m_C(q) \geq m_i^*(T_L)$ can only obtain when $m_i^*(T_L)$ is the constrained solution to the complete-information case, which means that $m_i^*(T_L) = S_i^*(T_L)/\theta$. For U_C to be decreasing, it must be the case that

$$\begin{aligned}\frac{\mathrm{d}\, U_C}{\mathrm{d}\, q} &= \frac{\partial U_C}{\partial m_i^*(T_L)} \frac{\mathrm{d}\, m_i^*(T_L)}{\mathrm{d}\, q} + \frac{\partial U_C}{\partial q} = \frac{\partial U_C}{\partial q} \\ &= \frac{m_i^*(T_L)}{(1 + \theta m_i^*(T_L))(m_i^*(T_L) + T_S)} - \frac{\theta m_i^*(T_L)}{\theta m_i^*(T_L) + T_L} < 0,\end{aligned}$$

where the first step follows from the fact that $\frac{\mathrm{d}\, m_i^*(T_L)}{\mathrm{d}\, q} = 0$ at the constrained solution. Letting $m \equiv m_i^*(T_L) > 0$ to simplify notation, we can rewrite the inequality above as

$$\frac{1}{(1+\theta m)(m+T_{\mathrm{S}})} < \frac{\theta}{\theta m + T_{\mathrm{L}}}, \tag{8}$$

Recall that $m_i^*(T_{\mathrm{L}})$ is the constrained solution to the complete information case, which means that $S(T_{\mathrm{L}}) > S_i^*(T_{\mathrm{L}}) > 0$, which in turn implies that $S(T_{\mathrm{L}}) > 0$ must be satisfied, and so $\theta > T_{\mathrm{L}}$ must obtain. But this now implies that

$$\frac{1}{(1+\theta m)(m+T_{\mathrm{S}})} < \frac{1}{(1+mT_{\mathrm{L}})(m+T_{\mathrm{S}})} \quad \text{and} \quad \frac{\theta}{\theta m + T_{\mathrm{L}}} > \frac{T_{\mathrm{L}}}{mT_{\mathrm{L}} + T_{\mathrm{L}}} = \frac{1}{1+m},$$

so it will be sufficient to show that

$$\frac{1}{(1+mT_{\mathrm{L}})(m+T_{\mathrm{S}})} < \frac{1}{1+m} \quad \Leftrightarrow \quad 1+m < (1+mT_{\mathrm{L}})(m+T_{\mathrm{S}}),$$

where the last inequality is easily verified because $mT_{\mathrm{L}} > 0$ and $T_{\mathrm{S}} > 1$ together imply that $(1+mT_{\mathrm{L}})(m+T_{\mathrm{S}}) > m+T_{\mathrm{S}} > m+1$. Thus, U_{C} is strictly decreasing in q for any $m_{\mathrm{C}} \geq m_i^*(T_{\mathrm{L}})$.

Summarizing, start with $q = 0$, where the solution is $m_{\mathrm{C}} = m_i^*(T_{\mathrm{L}})$. If $m_i^*(T_{\mathrm{L}})$ is the constrained solution to the complete-information case, then it is possible that the solution to (7) is actually strictly greater. If this is so, then increasing q will decrease this solution until at some point it will equal $m_i^*(T_{\mathrm{L}})$: in this interval, the optimal allocation is constant at $m_i^*(T_{\mathrm{L}})$, and the payoff is strictly decreasing. If $m_i^*(T_{\mathrm{L}})$ is the unconstrained solution, then the fact that $m_{\mathrm{C}}(q)$ is decreasing means that the second case applies.

Case 2: $m_{\mathrm{C}}(q) \in [m_i^*(T_{\mathrm{S}}), m_i^*(T_{\mathrm{L}}))$. In this region, the constraint that ensures that G remains loyal under T_{L} is no longer binding, and since this means that $\frac{\partial U_{\mathrm{C}}}{\partial m} = 0$ at the optimum, we can apply the envelope theorem to obtain

$$\begin{aligned}\frac{\mathrm{d}\, U_{\mathrm{C}}}{\mathrm{d}\, q} &= \frac{\partial U_{\mathrm{C}}}{\partial m}\frac{\mathrm{d}\, m}{\mathrm{d}\, q} + \frac{\partial U_{\mathrm{C}}}{\partial q} = \frac{\partial U_{\mathrm{C}}}{\partial q} \\ &= \frac{m_{\mathrm{C}}}{(1+\theta m_{\mathrm{C}})(m_{\mathrm{C}}+T_{\mathrm{S}})} - \frac{\theta m_{\mathrm{C}}}{\theta m_{\mathrm{C}} + T_{\mathrm{L}}} < 0,\end{aligned}$$

where we can establish this inequality as follows. If $m_i^*(T_{\mathrm{L}})$ is the constrained solution to the complete-information case, then $\theta > T_{\mathrm{L}}$ must obtain, and the argument following (8) applies. If, on the other hand, $m_i^*(T_{\mathrm{L}})$ is the unconstrained solution to the complete-information case, then we argue as follows. Loosely, since the first component of the payoff in (6) is strictly decreasing in m while the second one is strictly increasing, putting more weight on the first component decreases m_{C} (we showed this already), which in turn decreases U_{C}. We need to show that

$$\frac{m_{\mathrm{C}}}{(1+\theta m_{\mathrm{C}})(m_{\mathrm{C}}+T_{\mathrm{S}})} - m < \frac{\theta m_{\mathrm{C}}}{\theta m_{\mathrm{C}} + T_{\mathrm{L}}} - m.$$

Recall that the left-hand side is strictly decreasing in m and we know that the right-hand side is strictly increasing because m_{C} is smaller than the unconstrained optimum of the complete-information case under T_{L}. But since at $m = 0$ both sides are zero, the inequality must obtain for any $m > 0$ in this region. In other words, U_{C} is strictly decreasing here as well. Note in particular that this also covers the cases where $m_{\mathrm{C}}(q) < m_i^*(T_{\mathrm{S}})$, but this cannot occur because in that case G will not execute a coup at T_{S}, and if the solution to (7) is that small, R's optimal choice is to optimize the "no-coup" scenario.

We conclude that the optimal payoff, $U_{\mathrm{C}}(m_{\mathrm{C}}(q))$, is strictly decreasing in q (it is clearly continuous).

Finally, we show that at $q = 1$, the ruler prefers to play the riskless strategy:

$$U_{\mathrm{C}}(1) = \frac{\widetilde{m}}{(1+\theta\widetilde{m})(\widetilde{m}+T_{\mathrm{S}})} - \widetilde{m} < \frac{\theta m_i^*(T_{\mathrm{S}})}{\theta m_i^*(T_{\mathrm{S}}) + T_{\mathrm{S}}} - m_i^*(T_{\mathrm{S}}) = U_{\mathrm{N}}(1),$$

where the inequality follows from

$$\frac{\widetilde{m}}{(1+\theta\widetilde{m})(\widetilde{m}+T_{\mathrm{S}})} - \widetilde{m} < \frac{\widetilde{m}}{\widetilde{m}+T_{\mathrm{S}}} - \widetilde{m} < \frac{\theta\widetilde{m}}{\theta\widetilde{m}+T_{\mathrm{S}}} - \widetilde{m} \leq \frac{\theta m_i^*(T_{\mathrm{S}})}{\theta m_i^*(T_{\mathrm{S}}) + T_{\mathrm{S}}} - m_i^*(T_{\mathrm{S}}),$$

where the last inequality follows from $m_i^*(T_{\mathrm{S}})$ being the optimizer under complete information.

We have now established that $U_{\mathrm{C}}(0) > U_{\mathrm{N}}(0)$, $U_{\mathrm{C}}(1) < U_{\mathrm{N}}(1)$, that U_{N} is strictly increasing while U_{C} is strictly decreasing. Since both functions are continuous, it follows that there exists precisely one intersection, at some $q^* \in (0, 1)$, such that R strictly prefers the risky strategy for all $q < q^*$, and strictly prefers the riskless one for all $q > q^*$.

Since $\theta > 1$ makes hiring a general strictly preferable to not hiring one as long as the probability of a coup is zero, it follows that with $\theta > 1$ R will always hire a general (if R prefers the risky strategy to the one that ensures that no coup takes place, then she must prefer it to not hiring G as well). Conversely, $\theta < 1$ ensures that R does not hire anyone.

The final claims of the lemma follow immediately: if $m_{\mathrm{C}}(q) < m_i^*(T_{\mathrm{L}})$ when the risky strategy is chosen, the allocation obviously falls short of the optimum to deal with the large threat.[10] Since $m_i^*(T_{\mathrm{S}}) < m_i^*(T_{\mathrm{L}})$, the same is certainly true under the safe strategy. ■

Proof of Lemma 7. We establish this result by showing that both U_{N} and U_{C} are increasing in θ regardless of the value of q.

Consider U_{N} first and start with θ sufficiently small so that $\sqrt{\theta T_{\mathrm{S}}} - T_{\mathrm{S}} \leq S_i^*(T_{\mathrm{S}})$; that is, any θ that makes the complete-information constraint not binding against T_{S}

[10]For example, this happens when $b_i = 0.2$, $c = 0.3$, $\theta = 20$, $T_{\mathrm{S}} = 1$, and $T_{\mathrm{L}} = 7$. In this case $q^* \approx 0.055$, while $m_{\mathrm{C}}(q) < m_i^*(T_{\mathrm{L}})$ for all $q > 0.005$.

so that $m_i^*(T_\mathrm{S}) = \max(0, \sqrt{T_\mathrm{S}/\theta} - T_\mathrm{S}/\theta)$. If $\theta \leq T_\mathrm{S}$, then $m_i^*(T_\mathrm{S}) = 0$, and $U_\mathrm{N} = 0$ for any such θ. (This means that R will rather go it alone than a hire a general even when doing so means no coup will occur.) If, on the other hand, $\theta > T_\mathrm{S}$, then $\theta m_i^*(T_\mathrm{S}) = \sqrt{\theta T_\mathrm{S}} - T_\mathrm{S} > 0$, so we can write

$$U_\mathrm{N} = q\left(1 - \sqrt{\frac{T_\mathrm{S}}{\theta}}\right) + (1-q)\left(\frac{\sqrt{\theta T_\mathrm{S}} - T_\mathrm{S}}{\sqrt{\theta T_\mathrm{S}} - T_\mathrm{S} + T_\mathrm{L}}\right) - \sqrt{\frac{T_\mathrm{S}}{\theta}} + \frac{T_\mathrm{S}}{\theta}.$$

Taking the derivative with respect to θ and setting it greater than zero yields, after some algebra,

$$q + (1-q)\left[\frac{\theta T_\mathrm{L}}{(\sqrt{\theta T_\mathrm{S}} - T_\mathrm{S} + T_\mathrm{L})^2}\right] + 1 > 2\sqrt{\frac{T_\mathrm{S}}{\theta}}.$$

Since $\theta > T_\mathrm{S} \Rightarrow \sqrt{T_\mathrm{S}/\theta} < 1$, this inequality will hold whenever

$$q + (1-q)\left[\frac{\theta T_\mathrm{L}}{(\sqrt{\theta T_\mathrm{S}} - T_\mathrm{S} + T_\mathrm{L})^2}\right] > \sqrt{\frac{T_\mathrm{S}}{\theta}}$$

obtains. But since the left-hand side is a linear combination of 1 and the bracketed term, the fact that $\sqrt{T_\mathrm{S}/\theta} < 1$ further tells us that this inequality will hold whenever

$$\frac{\theta T_\mathrm{L}}{(\sqrt{\theta T_\mathrm{S}} - T_\mathrm{S} + T_\mathrm{L})^2} > \sqrt{\frac{T_\mathrm{S}}{\theta}},$$

obtains, which we can establish as follows. Taking the derivative of the left-hand side with respect to T_L yields

$$\frac{\theta\left(\sqrt{\theta T_\mathrm{S}} - T_\mathrm{S} + T_\mathrm{L}\right)}{\left(\sqrt{\theta T_\mathrm{S}} - T_\mathrm{S} + T_\mathrm{L}\right)^3} > 0,$$

and since this means that it is strictly increasing, it is sufficient to establish the inequality for the smallest value T_L can hold; that is, it is sufficient to establish the inequality for $T_\mathrm{L} = T_\mathrm{S}$. But in this case, the left-hand side reduces to 1, and we already know that $1 > \sqrt{T_\mathrm{S}/\theta}$. Thus, we conclude that U_N is strictly increasing in θ whenever the optimal complete-information allocation is unconstrained and positive.

Consider now θ high enough so that $\sqrt{\theta T_\mathrm{S}} - T_\mathrm{S} > S_i^*(T_\mathrm{S})$; that is, any θ that makes the complete-information constraint binding against T_S so that $m_i^*(T_\mathrm{S}) = S_i^*(T_\mathrm{S})/\theta$. Since $S_i^*(T_\mathrm{S})$ is constant in θ, the inequality will be preserved for any larger θ as well. But now we obtain $\theta m_i^*(T_\mathrm{S}) = S_i^*(T_\mathrm{S})$, so we can write

$$U_\mathrm{N} = \frac{q S_i^*(T_\mathrm{S})}{S_i^*(T_\mathrm{S}) + T_\mathrm{S}} + \frac{(1-q) S_i^*(T_\mathrm{S})}{S_i^*(T_\mathrm{S}) + T_\mathrm{L}} - \frac{S_i^*(T_\mathrm{S})}{\theta}, \tag{9}$$

which is clearly increasing in θ. Thus, once θ is high enough that the complete-information constraint binds, increasing it further will only increase U_{N} as well (since the constraint will continue to bind).

Let us now establish the equivalent claim for U_{C}. We have two cases to consider.

Case 1: $m_{\mathrm{C}} = m_i^*(T_{\mathrm{L}})$, which we recall from the proof of Lemma 6 further means that $m_{\mathrm{C}} = S_i^*(T_{\mathrm{L}})/\theta$. Substituting this into (6) yields

$$U_{\mathrm{C}} = \left(\frac{q}{1 + S_i^*(T_{\mathrm{L}})}\right)\left(\frac{S_i^*(T_{\mathrm{L}})}{S_i^*(T_{\mathrm{L}}) + \theta T_{\mathrm{S}}}\right) + \frac{(1-q)S_i^*(T_{\mathrm{L}})}{S_i^*(T_{\mathrm{L}}) + T_{\mathrm{L}}} - \frac{S_i^*(T_{\mathrm{L}})}{\theta}, \qquad (10)$$

from which we obtain

$$\frac{\mathrm{d}\, U_{\mathrm{C}}}{\mathrm{d}\,\theta} = \frac{S_i^*(T_{\mathrm{L}})}{\theta^2} - \frac{q T_{\mathrm{S}} S_i^*(T_{\mathrm{L}})}{(1 + S_i^*(T_{\mathrm{L}}))(S_i^*(T_{\mathrm{L}}) + \theta T_{\mathrm{S}})^2} > 0,$$

where the inequality can be established with simple algebra. Thus, U_{C} is strictly increasing in θ whenever m_{C} is the constrained solution.

Case 2: m_{C} is the unconstrained optimizer so the FOC is satisfied: $\frac{\partial U_{\mathrm{C}}}{\partial m} = 0$ at the optimum. We can simply apply the envelope theorem to obtain

$$\frac{\mathrm{d}\, U_{\mathrm{C}}}{\mathrm{d}\,\theta} = \frac{\partial U_{\mathrm{C}}}{\partial m}\frac{\mathrm{d}\, m}{\mathrm{d}\,\theta} + \frac{\partial U_{\mathrm{C}}}{\partial \theta} = \frac{\partial U_{\mathrm{C}}}{\partial \theta} = m_{\mathrm{C}}\left[\frac{(1-q)T_{\mathrm{L}}}{(\theta m_{\mathrm{C}} + T_{\mathrm{L}})^2} - \frac{q m_{\mathrm{C}}}{(m_{\mathrm{C}} + T_{\mathrm{S}})(1 + \theta m_{\mathrm{C}})^2}\right],$$

which tells us that U_{C} must be increasing in θ if

$$\frac{(1-q)T_{\mathrm{L}}}{(\theta m_{\mathrm{C}} + T_{\mathrm{L}})^2} > \frac{q m_{\mathrm{C}}}{(m_{\mathrm{C}} + T_{\mathrm{S}})(1 + \theta m_{\mathrm{C}})^2}. \qquad (11)$$

Since (7) is satisfied, we know that

$$\frac{(1-q)T_{\mathrm{L}}}{(\theta m_{\mathrm{C}} + T_{\mathrm{L}})^2} = \left(\frac{1}{\theta}\right)\left[1 - \frac{q(T_{\mathrm{S}} - \theta m_{\mathrm{C}}^2)}{(m_{\mathrm{C}} + T_{\mathrm{S}})^2(1 + \theta m_{\mathrm{C}})^2}\right].$$

We substitute this into (11) and after some algebra reduce that inequality to

$$(m_{\mathrm{C}} + T_{\mathrm{S}})^2(1 + \theta m_{\mathrm{C}}) > q T_{\mathrm{S}},$$

which clearly holds: $(m_{\mathrm{C}} + T_{\mathrm{S}})^2(1 + \theta m_{\mathrm{C}}) > (m_{\mathrm{C}} + T_{\mathrm{S}})^2 > m_{\mathrm{C}} + T_{\mathrm{S}} > T_{\mathrm{S}} > q T_{\mathrm{S}}$. Thus, if m_{C} is the unconstrained optimizer, U_{C} is strictly increasing in θ. ∎

Proof of Lemma 8. We shall establish this result by showing that both U_{N} and U_{C} are either constant in b_i or strictly increasing.

We begin with U_{N}. If $m_i^*(T_{\mathrm{S}})$ is the unconstrained complete-information optimum, then it is independent of b_i, and so U_{N} itself is constant in b_i. If $m_i^*(T_{\mathrm{S}}) =$

$S_i^*(T_{\mathrm{S}})/\overline{\theta}$, on the other hand, then the allocation is strictly increasing in b_i because $S_i^*(T_{\mathrm{S}})$ does. The payoff in this case is given by (9). Since

$$\frac{\mathrm{d}\,U_{\mathrm{N}}}{\mathrm{d}\,b_i} = \frac{\partial U_{\mathrm{N}}}{\partial S_i^*}\frac{\mathrm{d}\,S_i^*}{\mathrm{d}\,b_i} + \frac{\partial U_{\mathrm{N}}}{\partial b_i}$$

but $\frac{\partial U_{\mathrm{N}}}{\partial b_i} = 0$ and $\frac{\mathrm{d}\,S_i^*}{\mathrm{d}\,b_i} > 0$, it follows that

$$\operatorname{sgn}\left(\frac{\mathrm{d}\,U_{\mathrm{N}}}{\mathrm{d}\,b_i}\right) = \operatorname{sgn}\left(\frac{\partial U_{\mathrm{N}}}{\partial S_i^*}\right).$$

Thus, we need to show that

$$\frac{\partial U_{\mathrm{N}}}{\partial S_i^*} = \frac{qT_{\mathrm{S}}}{(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{S}})^2} + \frac{(1-q)T_{\mathrm{L}}}{(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{L}})^2} - \frac{1}{\overline{\theta}} > 0.$$

We are going to split the proof in two cases. First, suppose that $S_i^*(T_{\mathrm{S}}) < \sqrt{T_{\mathrm{S}}T_{\mathrm{L}}}$, which implies that $T_{\mathrm{S}}\left(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{L}}\right)^2 > T_{\mathrm{L}}\left(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{S}}\right)^2$. We can rewrite the condition on the derivative as

$$\overline{\theta} > \frac{\left(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{S}}\right)^2\left(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{L}}\right)^2}{qT_{\mathrm{S}}\left(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{L}}\right)^2 + (1-q)T_{\mathrm{L}}\left(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{S}}\right)^2} \equiv \underline{\theta}.$$

By Assumption 3, $\overline{\theta} > \left(\sqrt{T_{\mathrm{S}}} + \sqrt{T_{\mathrm{L}}}\right)^2$, so it suffices to show that $\left(\sqrt{T_{\mathrm{S}}} + \sqrt{T_{\mathrm{L}}}\right)^2 > \underline{\theta}$. But since $S_i^*(T_{\mathrm{S}}) < \sqrt{T_{\mathrm{S}}T_{\mathrm{L}}}$, it follows that

$$\underline{\theta} < \frac{\left(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{S}}\right)^2\left(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{L}}\right)^2}{T_{\mathrm{L}}\left(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{S}}\right)^2} = \frac{\left(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{L}}\right)^2}{T_{\mathrm{L}}},$$

so we only need to show that

$$\left(\sqrt{T_{\mathrm{S}}} + \sqrt{T_{\mathrm{L}}}\right)^2 > \frac{\left(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{L}}\right)^2}{T_{\mathrm{L}}} \quad \Leftrightarrow \quad S_i^*(T_{\mathrm{S}}) < \sqrt{T_{\mathrm{S}}T_{\mathrm{L}}}.$$

Since the last inequality is true by supposition, the claim holds.

Turning now to the other possibility, suppose that $S_i^*(T_{\mathrm{S}}) > \sqrt{T_{\mathrm{S}}T_{\mathrm{L}}}$, which implies that $T_{\mathrm{S}}\left(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{L}}\right)^2 < T_{\mathrm{L}}\left(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{S}}\right)^2$. Recall that $m_i^*(T_{\mathrm{S}})$ is the binding allocation, which means that $S(T_{\mathrm{S}}) > S_i^*(T_{\mathrm{S}})$, which implies that

$$\overline{\theta} > \frac{\left(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{S}}\right)^2}{T_{\mathrm{S}}}.$$

But this now means that

$$\frac{\partial U_{\mathrm{N}}}{\partial S_i^*} > \frac{(1-q)T_{\mathrm{L}}}{(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{L}})^2} - \frac{(1-q)T_{\mathrm{S}}}{(S_i^*(T_{\mathrm{S}}) + T_{\mathrm{S}})^2},$$

so it suffices to show that

$$\frac{T_{\mathrm{L}}}{(S_i^*(T_{\mathrm{S}})+T_{\mathrm{L}})^2} > \frac{T_{\mathrm{S}}}{(S_i^*(T_{\mathrm{S}})+T_{\mathrm{S}})^2} \quad \Leftrightarrow \quad S_i^*(T_{\mathrm{S}}) > \sqrt{T_{\mathrm{S}} T_{\mathrm{L}}}.$$

Since the last inequality is true by supposition, the claim holds. Thus, U_{N} is non-decreasing in b_i.

Consider now U_{C}. If m_{C} is the unconstrained optimizer, then $\frac{\partial U_{\mathrm{C}}}{\partial m}\Big|_{m_{\mathrm{C}}} = 0$. The envelope theorem then tells us that

$$\frac{\mathrm{d}\, U_{\mathrm{C}}}{\mathrm{d}\, b_i} = \frac{\partial U_{\mathrm{C}}}{\partial m}\frac{\mathrm{d}\, m}{\mathrm{d}\, b_i}\bigg|_{m_{\mathrm{C}}} + \frac{\partial U_{\mathrm{C}}}{\partial b_i} = \frac{\partial U_{\mathrm{C}}}{\partial b_i} = 0,$$

which means that U_{C} is independent of b_i in this case.

If, on the other hand, m_{C} is the constrained optimizer, then $\frac{\partial U_{\mathrm{C}}}{\partial m}\Big|_{m_{\mathrm{C}}} > 0$ and $m_{\mathrm{C}} = m_i^*(T_{\mathrm{L}}) = S_i^*(T_{\mathrm{L}})/\theta$. Since $\frac{\partial U_{\mathrm{C}}}{\partial b_i} = 0$, we obtain

$$\frac{\mathrm{d}\, U_{\mathrm{C}}}{\mathrm{d}\, b_i} = \frac{\partial U_{\mathrm{C}}}{\partial m}\frac{\mathrm{d}\, m}{\mathrm{d}\, b_i}\bigg|_{m_{\mathrm{C}}} > 0,$$

where the inequality follows from $\frac{\mathrm{d}\, m}{\mathrm{d}\, b_i}\Big|_{m_{\mathrm{C}}} = \left(\frac{\mathrm{d}\, S_i^*}{\mathrm{d}\, b_i}\right)/\theta > 0$ and $\frac{\partial U_{\mathrm{C}}}{\partial m}\Big|_{m_{\mathrm{C}}} > 0$. Thus, U_{C} is non-decreasing in b_i as well.

We conclude that the payoffs are strictly increasing whenever $m_i^*(T_{\mathrm{S}}) = S_i^*(T_{\mathrm{S}})/\overline{\theta}$ (in the riskless subgame) or $m_i^*(T_{\mathrm{L}}) = S_i^*(T_{\mathrm{L}})/\overline{\theta}$ (in the risky subgame) are the constrained optima under complete information.

Recall that S_i^* itself is increasing in b_i, that $S(T)$ is independent of b_i, that $S_i^*(T_{\mathrm{S}}) < S_i^*(T_{\mathrm{L}})$, and that $S(T_{\mathrm{S}}) < S(T_{\mathrm{L}})$ under Assumption 3. Consider now very low values of b_i (and possibly c) such that the loyalty constraint binds in both cases: $S_i^*(T) < S(T)$ for $T \in \{T_{\mathrm{S}}, T_{\mathrm{L}}\}$. In other words, consider $b_i < b_1$.[11] The results above indicate that R's payoff from both U_{N} or U_{C} is strictly increasing in b_i, so she must pick the highest such b_i that still ensures that the constraints obtain. If $\overline{b} \leq b_1$, then R must select from the most privileged group regardless of q.

If $\overline{b} \in (b_1, b_2)$, then at least one of the constraints will cease to be binding. The corresponding payoff will now be constant in b_i whereas the other one will continue to increase. If $b_1 = b^*(T_{\mathrm{S}})$, then the constraint that affects U_{N} will no longer bind. R is now indifferent among any $b_i \in [b_1, \overline{b}]$ when the equilibrium outcome is riskless, which we know to be the case for any $q > q^*$. On the other hand, since $b_1 = b^*(T_{\mathrm{S}})$ implies that $b_2 = b^*(T_{\mathrm{L}})$, it follows that $\overline{b} < b^*(T_{\mathrm{L}})$, so

[11] $b^*(T)$ can be concave or strictly decreasing in T, depending on the values of c, which is why we cannot say which constraint will be relaxed first in general.

the constraint is still binding for the risky continuation game. Since U_C is strictly increasing in b_i, R must strictly prefer to pick $\overline{b}$ for any $q \leq q^*$. The situation where $b_1 = b^*(T_L)$ is analogous, *mutatis mutandis*.

Finally, if $\overline{b} \geq b_2$, then the constraints are not binding in either continuation game, so R must be indifferent among any $b_i \in [b_2, \overline{b}]$ regardless of q. ■

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Appendix B: Iraqi Senior Military Leadership from 1987/88

1. Saddam Hussein
    - Rank: Field Marshall
    - Post: Commander in chief of the armed forces (Bengio, 1989, 455; Bengio, 1990, 537).
    - POST WAR: Survived and continued in position.
2. ‘Adnan Khayrallah Talfah
    - Rank: General
    - Post: Defense minister (Bengio, 1989, 455; Bengio, 1990, 537).
    - POST WAR: Died in a helicopter accident in May 1989 while still defense minister. Opponents say that the circumstances of his death are suspicious (Dougherty and Ghareeb, 2013, 367).
3. ‘Abd al-Jabbar Shanshal
    - Rank: General
    - Post: Minister of state for military affairs (Bengio, 1989, 455; Bengio, 1990, 537).
    - Post: Defense minister (Bengio, 1992, 420).
    - POST WAR: Became minister of defense upon death of Khayrallah (1989-1990) (Woods et al., 2011, 41).
4. Nazir ‘Abd al-Karim al-Khazraji
    - Rank: Lt. General
    - Post: Chief of staff (Rai, 2002, 89; Bengio, 1989, 455).
    - POST WAR: Was chief of staff through 1991 (Rai, 2002, 89).
5. Sa’di Tu’ma ‘Abbas al Jaburi
    - Rank: Lt. General
    - Post: Armed Forces assistant chief of staff for training (Bengio, 1989, 455; Bengio, 1990, 537).
    - Post: Commander of the 1st Special Army Corps (“Allah Akbar Forces”).

- POST WAR: Replaced General Shansal as defense minister in December 1990 (Bengio, 1992, 420).

6. Thabit Sultan al-Hajj Ahmad
   - Rank: Lt. General
   - Post: Armed Forces assistant chief of staff for operations (Bengio, 1989, 455; Bengio, 1990, 537).
   - POST WAR: Held his position through at least 1990 (Bengio, 1992, 420).

7. ‘Abd al-Sattar (Ahmad) al-Ma’ini
   - Rank: Lt. General
   - Post: Armed Forces assistant chief of staff for administration and supplies (Bengio, 1989, 455).
   - POST WAR: Held position through at least 1990 (Bengio, 1992, 420).

8. Iyad Fatih Khalifa al-Rawi
   - Rank: Major General
   - Post: Commander of the Presidential Guard Forces (Bengio, 1989, 455; Bengio, 1990, 537).
   - POST WAR: Held his position at least through 1990 (Bengio, 1992, 420). Was the commander of the Fedayeen Saddam leading up to the 2003 invasion (Otterman, 2003).

9. ‘Abd al-Jabbar Muhsin
   - Rank: Major General
   - Post: Head of political guidance in the Defense Ministry (Bengio, 1989, 455; Bengio, 1990, 537).
   - POST WAR: Served as spokesman for Saddam Hussein (Dougherty and Ghareeb, 2013, 431).

10. Husayn Kamil al-Majid
    - Rank: Colonel
    - Post: Supervisor of military industries (Bengio, 1989, 455; Bengio, 1990, 537).

- POST WAR: Minister of industry and military industrialization (1988-1991). Minister of defense (1991). Minister of industry and materials (1993-1995). He was also minister of oil (1990-1991) (Dougherty and Ghareeb, 2013, 358).

11. Sabir ‘Abd al-’Aziz Husayn al-Duri
    - Rank: Major General
    - Post: Head of military intelligence (Bengio, 1990, 537).
    - POST WAR: Still head of military intelligence in 1990 (Bengio, 1992, 420).

12. Sabah Mirza
    - Rank: Lt. General
    - Post: Head of president’s bodyguard unit (Bengio, 1989, 455; Bengio, 1990, 537).
    - POST WAR: Pensioned off in 1990 (Henderson, 1991, 251).

13. Hamid Sha’ban al-Tikriti [Khudayr]
    - Rank: Lt. General
    - Post: Air Force and air defense commander (Bengio, 1989, 455; Bengio, 1990, 537).
    - POST WAR: Continued as commander of the air force into the 1990s. Was suspected of involvement with an attempted coup by the Special Republican Guard in June 1996, but was released (al-Marashi and Salama, 2008, 188).

14. ‘Abd Muhammad ‘Abdallah
    - Rank: Major General
    - Post: Commander of Naval and Coastal Defense Forces (Bengio, 1989, 455).
    - POST WAR: Was minister of agriculture and irrigation in 1991 (Litvak, 1993, 444).

15. Gha’ib Hassun Gha’ib
    - Rank: Brig. General
    - Post: Commander of the Naval and Coastal Defense Forces (Bengio, 1990, 537).

- POST WAR: Held position at least through 1990 (Bengio, 1992, 420).

16. Kamal Jamil ‘Abbud
    - Rank: Lt. General
    - Post: Commander of the I Special Army Corps (“Allah Akbar Forces”) (Bengio, 1989, 455).
    - POST WAR: UNKNOWN.

17. Husayn Rashid [al-Windawi][al-Tikriti]
    - Rank: Lt. General
    - Post: Commander of the I Army Corps (Bengio, 1989, 455).
    - Post: Armed forces chief of staff for operations (Bengio, 1990, 537).
    - POST WAR: Was made military chief of staff in aftermath of Second Persian Gulf War in 1991 (Hiro, 2003, 54).

18. Shawkat Ahmad ‘Ata
    - Rank: Lt. General
    - Post: Commander of the II Army Corps (Bengio, 1989, 455).
    - POST WAR: UNKNOWN.

19. Kamil Sajit ‘Aziz
    - Rank: Major General
    - Post: Commander of the II Army Corps (Bengio, 1990, 537).
    - POST WAR: Continued to serve in the military. Was executed by Hussein in connection with a coup attempt in late 1998 or 1999 after Operation Desert Fox (Hiro, 2003, 167).

20. Diya al-Din Jamal
    - Rank: Maj. General
    - Post: Commander of the III Army Corps (Bengio, 1989, 455).
    - POST WAR: UNKNOWN.

21. Salah ‘Abbud Mahmud
    - Rank: Maj. General
    - Post: Commander of the III Army Corps (Bengio, 1990, 537).
    - POST WAR: Held position at least through 1990 (Bengio, 1992, 420).

22. Muhammad ‘Abd al-Qadir
    - Rank: Major General
    - Post: Commander of the IV Army Corps (Bengio, 1989, 455).
    - POST WAR: Was promoted to assistant army chief of staff at some point before being made governor of Ninwa, where he was serving by at least 2000 (Bengio, 2002, 276).
23. Iyad Khalil Zaki
    - Rank: Major General
    - Post: Commander of the IV Army Corps (Bengio, 1990, 537).
    - POST WAR: Held position at least through 1990 (Bengio, 1992, 420). Was made assistant chief of staff for supply before becoming governor of Muthanna in 2000 (Bengio, 2002, 276).
24. ‘Abd al-’Aziz Ibrahim al-Hadithi
    - Rank: Major General
    - Post: Commander of the V Army Corps (Bengio, 1989, 455).
    - POST WAR: He was killed in action while fighting in northern Iraq during January 1988 (Bengio, 1990, 537).
25. Yunis Muhammad al-Dhirib (aka al-Zareb)
    - Rank: Major General
    - Post: Commander of the V Army Corps (Bengio, 1990, 537).
    - POST WAR: UNKNOWN.
26. Sultan [Qasim] Hashim Ahmad
    - Rank: Major General
    - Post: Commander of the VI Army Corps (Bengio, 1989, 455).
    - Post: Commander of the I Army Corps (Bengio, 1990, 537).
    - POST WAR: Eventually promoted to Lt. General. Armed forces assistant chief of staff for operations. Was the defense minister of Iraq leading up to the 2003 invasion (Burns, 2007).
27. Yaljin ‘Umar ‘Adil
    - Rank: Major General
    - Post: Commander of the VI Army Corps (Bengio, 1990, 537).

- POST WAR: Held position through at least 1990 (Bengio, 1992, 420).

28. Mahir ‘Abd al-Rashid
    - Rank: Lt. General
    - Post: Commander of the VII Corps (Bengio, 1989, 455; Bengio, 1990, 537).
    - POST WAR: He was Qusay Husayn’s father-in-law. Forced into retirement following end of Iran-Iraq War. He was eventually recalled to help suppress Shi’a uprising in 1991, but his role in this is not entirely clear (Woods et al., 2008, 80).

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Appendix C: Additional Implications

Due to space limitations, we use this appendix to outline some additional implications of the theoretical model.

The Sources of Disagreement

The model reveals that the Guardianship Dilemma is triggered by the asymmetric information that militaries hold about the nature of the threats facing the state. One might wonder, however, why the military would not reveal its private information to the rulership. While we abstract away from this question in the model, it is easy to identify circumstances under which military actors will withhold or misrepresent information about the threat environment. A root cause of these asymmetries is the fact that militaries and political elites often want different things. Militaries tend to crave institutional autonomy, and discretion over spending and personnel decisions in particular (Finer, 1988; Huntington, 1957). Alternatively, we show that constraints on the military are crucial for leaders who are trying to manage the Guardianship Dilemma when facing intermediate threats. Because the ruler's beliefs about the threat environment are a key driver of these constraints, military agents have an incentive to misrepresent the true threat environment when they expect that revealing this information would lead to restrictions. Since rulers know that the armed forces possess this incentive, they are likely to be skeptical of the assessments produced by the military, even when the armed forces are providing accurate reports about the threat environment. Rulers can, of course, seek strategic assessments from other sources. In fact, regimes often create independent, redundant security and intelligence services for this purpose. Yet as Egyptian President Gamal Abdel Nasser discovered in the Six-Day War of 1967, poor threat assessment due to contentious civil-military relations is a problem that can be hard to overcome (Brooks, 2008).

While restrictions on the autonomy of military institutions imposed by rulers are an important source of acrimony in civil-military relations, the model reveals that efforts to achieve civil-military harmony may backfire. Maintaining control over the resources that flow to the military and the characteristics of its personnel are essential levers for political elites who seek to avoid defection by the armed forces. However, these efforts compromise the military's institutional autonomy and may otherwise hurt its corporatist interests. In this, the model offers a logical basis for the type of civil-military strife that has troubled many states. Yet paradoxically, the model predicts that *attempts to appease the military can lead to a coup*, since providing any $T_i^*(m, \theta) > T$ will trigger disloyalty by the armed forces. This puts rulers in a tough position, since keeping the armed forces constrained is sometimes necessary to ensure their loyalty, but may also perpetuate the military's incentive to misrepresent the threat environment.

The Military Caste

The model reveals that rulers are better by picking military leaders from among groups that benefit from life under their current regime. However, one may wonder what might happen if rulers lack the kind of politically-salient cleavages that can be leveraged for political purposes. In other words, what if there is no readily-available group that derives privilege from the regime?

Since higher b_i are always better for R when the loyalty constraint is binding, R might want to create a privileged caste from which to draw her generals. That is, if $\overline{b} < b^*$ so that no existing group derives sufficient benefits from the status quo to ensure G's loyalty at the optimal level of resources required to deal with the external threat, R is strictly better off creating such a group, $\overline{b} = b^*$, and appointing a general from it. If benefits are intended to ensure the loyalty of the armed forces, it makes sense to allocate goods directly to these agents, as a ruler could do by creating and privileging a military caste. This logic is consistent with one prominent example of military privilege. President Hosni Mubarak sought to ensure the loyalty of the Egyptian armed forces by providing military personnel with access to better economic opportunities and other benefits than were available in private life, to the point where the military was allowed to engage in for-profit ventures, including the production of Jeep Wranglers (Roston and Rohde, 2011).

Powerless over the Purse?

Since we have posited the existence of the power of the purse, it is important to consider how our argument would change if the government did not possess it; that is, if the military is in direct control of its budget. This sort of affair is only common in military regimes where the government itself is controlled by the armed forces. The problem a junta faces is actually in some sense even more severe because the ruling officers have to worry about being displaced by others while simultaneously being constrained in their ability to reduce the military's budget in order to prevent that outcome. Since the junta cannot starve itself — after all, doing so would negate the reason for grabbing power in the first place — it might be forced to replace potentially disloyal officers with less competent ones. Thus, we would expect to see purges in the military after a coup. Moreover, when these purges are impractical (e.g., because the officers command significant loyalty on their own or because sacking them would jeopardize the legitimacy of the junta in the eyes of the armed forces), we would expect military regimes to succumb to coups with far greater frequency than non-military ones. Indeed, the potentially destabilizing effects created by military control over the budget may be one reason why military regimes tend to be short-lived, and why some militaries are so eager to return to the barracks after a more favorable political regime has been installed (see Magaloni, 2008; Geddes, 2004).

Creating Threats: The Diversionary Incentive

The relationship between large external threats and the security of the regime is counterintuitive, and has significant implications for understanding when rulers are likely to behave aggressively. Although we do not explore the possibility in this model, where the size of the external threat is exogenous, rulers can affect the size of that threat through their actions. In this context, the fact that large threats have a loyalty-inducing effect has another, less salutary implication, since political elites may sometimes provoke opponents in order to defend against a coup.

This dynamic is similar to studies of diversionary war in the sense that states' rulers may pick fights abroad in order to increase the security of their regime at home. However, the behavior implied by our study is different in two key ways. First, in studies of diversionary war, rulers survive by reducing the willingness of the public to replace the regime, either by creating a "rally around the flag" effect or by using conflict as an opportunity to demonstrate their competence (Levy, 1993; Smith, 1996; Tarar, 2006). The circling of the wagons effect supplied by our theory suggests instead that external threats provide regime security by creating a situation in which the armed forces are deterred by a challenge from a third-party, such that these forces will remain loyal even if they might otherwise wish to overthrow the government.

Second, the diversionary war literature focuses on the behavior of regimes vis-a-vis the threat to their rule posed by mass unrest, rather than the military. In fact, scholars have tended largely to ignore the agency of armed forces when connecting domestic instability to the likelihood of conflict. Among the few studies that do explore the diversionary incentive in the context of civil-military relations, however, evidence suggests that rulers are quite responsive to concerns about their armed forces, though the exact mechanism driving this relationship remains unclear.[1] Our model offers a novel explanation for the observed link between civil-military strife and the use of force that does not depend on public opinion, and which is potentially applicable for a range of regime types.

The Permanent Siege

Rulers in coup-prone states may need also enduring threats. The game outlined by the model ends when the external threat is faced. In practice, however, whoever is in charge of the state must continue to rule after the threat is realized. This is potentially important: if the military's loyalty is tied to the presence of a threat, the

[1] See Miller and Elgün (2011); Powell (2014); Belkin (2005); Dassel and Reinhardt (1999).

defeat of this threat could put the regime in grave danger. A general that would be loyal when the state faced a threat would not remain so once the threat from that opponent has disappeared (since $T = 0 < T_i^*(m, \theta)$ whenever $m > 0$ and $\theta > 0$). The regime could resolve this problem by deposing the general or completely depriving the military of resources, but doing so may often be difficult for rulers.

The problem does not exist, however, for rulers who can keep the threat environment elevated. There are two basic strategies for ensuring a permanent external threat. Regimes can cultivate multiple external threats, ensuring that if one enemy is defeated, the state must then deal with others. Alternatively, the rulership can create rivalries with opponents, ensuring that the threats which induce military loyalty persist across time. This suggests that civil-military concerns may limit the aims of belligerents in conflict, in the sense that defeating an opponent completely may reduce the safety of the regime by freeing the military to act against the rulership.

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