Keplerian Elements for Approximate Positions of the Major Planets

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Lower accuracy formulae for planetary positions have a number of important applications when one doesn't need the full accuracy of an integrated ephemeris. They are often used in observation scheduling, telescope pointing, and prediction of certain phenomena as well as in the planning and design of spacecraft missions.

Approximate positions of the nine major planets may be found by using Keplerian formulae with their associated elements and rates. Such elements are not intended to represent any sort of mean; they are simply the result of being adjusted for a best fit. As such, it must be noted that the elements are not valid outside the given time-interval over which they were fit.

The elements are given below in Table 1 or in Tables 2a and 2b, depending upon the time-interval over which they were fit and within which they are to be used.

Formulae for using them are given here.

Formulae for using the Keplerian elements

Keplerian elements given in the tables below are

- $a_o, \dot{a}$: semi-major axis [au, au/century]
- $e_o, \dot{e}$: eccentricity [, /century]
- $I_o, \dot{I}$: inclination [degrees, degrees/century]
- $L_o, \dot{L}$: mean longitude [degrees, degrees/century]
- $\varpi_o, \dot{\varpi}$: longitude of perihelion [degrees, degrees/century] ($\varpi = \omega + \Omega$)
- $\Omega_o, \dot{\Omega}$: longitude of the ascending node [degrees, degrees/century]

In order to obtain the coordinates of one of the planets at a given Julian Ephemeris Date, $\mathrm{T_{eph}}$,

1. Compute the value of each of that planet's six elements: $a = a_o + \dot{a}\mathrm{T}$, etc., where T, the number of centuries past J2000.0, is $\mathrm{T}=(\mathrm{T_{eph}}\text{-}2451545.0)/36525$.

2. Compute the argument of perihelion, ω, and the mean anomaly, M :

$$\omega = \varpi - \Omega \;\; ; \;\; M = L \; - \; \varpi \; + \; b\mathrm{T}^2 \; + \; c\cos(f\mathrm{T}) \; + \; s\sin(f\mathrm{T}) \qquad (8-30)$$

   where the last three terms must be added to M for Jupiter through Pluto when using the formulae for 3000 BC to 3000 AD.

3. Modulus the mean anomaly so that $-180^o \leq M \leq +180^o$ and then obtain the eccentric anomaly, E, from the solution of Kepler's equation (see below):

$$M \;=\; E - e^* \sin E, \qquad (8-31)$$

   where $e^* \;=\; 180/\pi \; e \;=\; 57.29578 \; e$.

4. Compute the planet's heliocentric coordinates in its orbital plane, $\mathbf{r}'$, with the x'-axis aligned from the focus to the perihelion:

$$x' = a(\cos E - e) \quad ; \quad y' = a\sqrt{1-e^2}\ \sin E \quad ; \quad z' = 0. \qquad (8-32)$$

5. Compute the coordinates, $\mathbf{r}_{ecl}$, in the J2000 ecliptic plane, with the x-axis aligned toward the equinox:

$$\mathbf{r}_{ecl} = \mathcal{M}\mathbf{r}' \equiv \mathcal{R}_z(-\Omega)\mathcal{R}_x(-I)\mathcal{R}_z(-\omega)\mathbf{r}' \qquad (8-33)$$

so that

$$\begin{array}{lclclcl} x_{ecl} & = & (\cos\omega\cos\Omega - \sin\omega\sin\Omega\cos I) & x' & + (-\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos I) & y' \\ y_{ecl} & = & (\cos\omega\sin\Omega + \sin\omega\cos\Omega\cos I) & x' & + (-\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos I) & y' \\ z_{ecl} & = & (\sin\omega\sin I) & x' & + (\cos\omega\sin I) & y' \end{array} \qquad (8-34)$$

6. If desired, obtain the equatorial coordinates in the "ICRF", or "J2000 frame", $\mathbf{r_{eq}}$:

$$\begin{array}{lcllll} x_{eq} & = & x_{ecl} & & & \\ y_{eq} & = & & + \cos\varepsilon\ \ y_{ecl} & - \sin\varepsilon\ \ z_{ecl} \\ z_{eq} & = & & + \sin\varepsilon\ \ y_{ecl} & + \cos\varepsilon\ \ z_{ecl} \end{array} \qquad (8-35)$$

where the obliquity at J2000 is $\varepsilon = 23^\circ\!.43928$.

Solution of Kepler's Equation, $M = E - e^* \sin E$

Given the mean anomaly, M, and the eccentricity, e^*, both in degrees, start with

$$E_0 = M + e^* \sin M \qquad (8\text{-}36)$$

and iterate the following three equations, with $n = 0, 1, 2, ...$, until $|\Delta E| \leq tol$ (noting that e^* is in degrees; e is in radians):

$$\Delta M = M - (E_n - e^* \sin E_n) \quad ; \quad \Delta E = \Delta M/(1 - e\cos E_n) \quad ; \quad E_{n+1} = E_n + \Delta E. \qquad (8\text{-}37)$$

For the approximate formulae in this present context, $tol = 10^{-6} degrees$ is sufficient.

Table 1

Keplerian elements and their rates, with respect to the mean ecliptic and equinox of J2000, valid for the time-interval 1800 AD - 2050 AD.

	a	e	I	L	ϖ	Ω
	$[au, au/cty]$	$[\ ,\ /cty]$	$[deg, deg/cty]$	$[deg, deg/cty]$	$[deg, deg/cty]$	$[deg, deg/cty]$
Mercury	0.38709927	0.20563593	7.00497902	252.25032350	77.45779628	48.33076593
	0.00000037	0.00001906	-0.00594749	149472.67411175	0.16047689	-0.12534081
Venus	0.72333566	0.00677672	3.39467605	181.97909950	131.60246718	76.67984255
	0.00000390	-0.00004107	-0.00078890	58517.81538729	0.00268329	-0.27769418
EM Bary	1.00000261	0.01671123	-0.00001531	100.46457166	102.93768193	0.0
	0.00000562	-0.00004392	-0.01294668	35999.37244981	0.32327364	0.0
Mars	1.52371034	0.09339410	1.84969142	-4.55343205	-23.94362959	49.55953891
	0.00001847	0.00007882	-0.00813131	19140.30268499	0.44441088	-0.29257343
Jupiter	5.20288700	0.04838624	1.30439695	34.39644051	14.72847983	100.47390909
	-0.00011607	-0.00013253	-0.00183714	3034.74612775	0.21252668	0.20469106

Saturn	9.53667594	0.05386179	2.48599187	49.95424423	92.59887831	113.66242448
	-0.00125060	-0.00050991	0.00193609	1222.49362201	-0.41897216	-0.28867794
Uranus	19.18916464	0.04725744	0.77263783	313.23810451	170.95427630	74.01692503
	-0.00196176	-0.00004397	-0.00242939	428.48202785	0.40805281	0.04240589
Neptune	30.06992276	0.00859048	1.77004347	-55.12002969	44.96476227	131.78422574
	0.00026291	0.00005105	0.00035372	218.45945325	-0.32241464	-0.00508664
Pluto	39.48211675	0.24882730	17.14001206	238.92903833	224.06891629	110.30393684
	-0.00031596	0.00005170	0.00004818	145.20780515	-0.04062942	-0.01183482

Table 2a

Keplerian elements and their rates, with respect to the mean ecliptic and equinox of J2000, valid for the time-interval 3000 BC – 3000 AD. **NOTE :** the computation of M for Jupiter through Pluto ***must*** be augmented by the additional terms described above and given in Table 2b.

	a	e	I	L	ϖ	Ω
	$[au, au/cty]$	$[\ ,\ /cty]$	$[deg, deg/cty]$	$[deg, deg/cty]$	$[deg, deg/cty]$	$[deg, deg/cty]$
Mercury	0.38709843	0.20563661	7.00559432	252.25166724	77.45771895	48.33961819
	0.00000000	0.00002123	-0.00590158	149472.67486623	0.15940013	-0.12214182
Venus	0.72332102	0.00676399	3.39777545	181.97970850	131.76755713	76.67261496
	-0.00000026	-0.00005107	0.00043494	58517.81560260	0.05679648	-0.27274174
EM Bary	1.00000018	0.01673163	-0.00054346	100.46691572	102.93005885	-5.11260389
	-0.00000003	-0.00003661	-0.01337178	35999.37306329	0.31795260	-0.24123856
Mars	1.52371243	0.09336511	1.85181869	-4.56813164	-23.91744784	49.71320984
	0.00000097	0.00009149	-0.00724757	19140.29934243	0.45223625	-0.26852431
Jupiter	5.20248019	0.04853590	1.29861416	34.33479152	14.27495244	100.29282654
	-0.00002864	0.00018026	-0.00322699	3034.90371757	0.18199196	0.13024619
Saturn	9.54149883	0.05550825	2.49424102	50.07571329	92.86136063	113.63998702
	-0.00003065	-0.00032044	0.00451969	1222.11494724	0.54179478	-0.25015002
Uranus	19.18797948	0.04685740	0.77298127	314.20276625	172.43404441	73.96250215
	-0.00020455	-0.00001550	-0.00180155	428.49512595	0.09266985	0.05739699
Neptune	30.06952752	0.00895439	1.77005520	304.22289287	46.68158724	131.78635853
	0.00006447	0.00000818	0.00022400	218.46515314	0.01009938	-0.00606302
Pluto	39.48686035	0.24885238	17.14104260	238.96535011	224.09702598	110.30167986
	0.00449751	0.00006016	0.00000501	145.18042903	-0.00968827	-0.00809981

Table 2b

Additional terms which must be added to the computation of M for Jupiter through Pluto, 3000 BC to 3000 AD, as described above.

	b	c	s	f
Jupiter	-0.00012452	0.06064060	-0.35635438	38.35125000
Saturn	0.00025899	-0.13434469	0.87320147	38.35125000
Uranus	0.00058331	-0.97731848	0.17689245	7.67025000
Neptune	-0.00041348	0.68346318	-0.10162547	7.67025000
Pluto	-0.01262724			