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Interfaces with Other Disciplines

Optimizing referral reward programs under impression management considerations

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ABSTRACT

We examine referral reward programs (RRP) that are intended for a service firm to encourage its current customers (inductors) to entice their friends (inductees) to purchase the firm's service. By considering the interplay among the firm, the inductor, and the inductee, we solve a "nested" Stackelberg game so as to determine the optimal RRP in equilibrium. We determine the conditions under which it is optimal for the firm to reward the inductor only, reward the inductee only, or reward both. Also, our results suggest that RRP dominates direct marketing when the firm's current market penetration or the inductor's referral effectiveness is sufficiently high. We then extend our model to incorporate certain key impression management factors: the inductor's intrinsic reward of making a positive impression by being seen as helping a friend, the inductor's concerns about creating a negative impression when making an incentivized referral, and the inductee's impression of the inductor's credibility when an incentive is involved. In the presence of these impression management factors, we show that the firm should reward the inductee more and the inductor less. Under certain conditions, it is optimal for the firm to reward neither the inductor nor the inductee so that the optimal RRP relies purely on unincentivized word of mouth.

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1. Introduction

To gain market share, many service firms have launched various referral reward programs (RRPs) that are intended to encourage their existing customers (inductors) to recommend the firms' service to their friends (inductees). Because RRP's utilize existing customers (who are familiar with the firm's service) as inductors to reach out to the same affinity group, their testimonies tend to be more credible and effective in communicating and tailoring the value proposition than direct marketing (Brown and Reinigen, 1987; Tuk et al., 2009). Consequently, RRP's are generally seen as particularly effective in acquiring new customers in the service industry. At the same time, RRP's can be highly cost effective because the rewards for the inductor and the inductee are contingent on the inductee's successful purchase.

RRP's have become ubiquitous: a recent search on Bing using the keywords "recommend-a-friend program" yielded over 23 million hits. Most of the top hits originated from service firms offering rewards to their existing customers for referring their services to their friends, and, in many instances, their friends also get rewards for signing up as new customers. Examples of various RRP's include: (1) Entertainment services: DirectTV offers \$100 credit to the inductor and the inductee, Time Warner (cable TV) offers the

inductor one month free service and offers the inductee a reduced service fee for the first year, and Netflix (an online video rental/streaming company) offers the inductor \$16 credit and offers the inductee a reduced service fee for the first year, respectively; (2) Wireless services: AT&T wireless, Verizon wireless, and Vonage (a VOIP phone service provider) offer different credits to both the inductor and the inductee; (3) Financial services: Scott Trade (an online stock brokerage firm) offers 3 free trades and Hutchinson Credit Union (a financial service cooperative) offers \$25 rewards to the inductor; however, neither firm offers rewards to the inductee; (4) Fitness center: 24 hours Fitness (a popular fitness center in the US) offers a \$20 coupon to the inductors and offers the inductee a reduced service fee for the first year; and (5) Professional societies: the Institute of Operations Research and Management Science (INFORMS) launched a Member-Get-A-Member program by offering a \$10 Amazon voucher but offers no reward to the inductee.

Despite the popularity of RRP's, little academic research has focused on the optimal design of RRP's or when to use referral rewards. By noting that traditional sales effort models examine ways for a firm to choose a compensation plan to maximize its profit by taking into account the salespersons effort levels under different compensation plans, our base model of RRP's presented in Section 3 is related to the sales effort model in the following sense. First, the inductor in our model is essentially the "salesperson" in the sales effort model. Second, the inductor's reward in our model can be viewed as a specific "compensation plan." (The reader is referred to Basu et al. (1985), Lal and Staelin (1986),

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Rao (1990), and Lal and Srinivasan (1993) for various sales effort models in the marketing literature, and Chen (2000) and Chen (2005) in the operations literature).

Despite the aforementioned similarities between our base model of RRP and the traditional sales effort models, there are two subtle differences that deserve more clarifications. First, in our base model of RRP, the firm chooses the reward (r) for the inductor and the discount incentive (d_1) for the inductee. In traditional sales effort models, the discount incentive for the consumers (inductees in our RRP context) is not considered. One exception is the model presented in Bhardwarj (2001) that deals with a situation in which the firm chooses a compensation plan for the salesperson and the selling price for the customers. Unlike our base model his sales response function is a linear function that is given exogenously (Bhardwarj (2001, p. 146)). Specifically, Bhardwarj (2001) examines a situation in which 2 competing firms sell through sales reps; however, each firm may delegate the pricing decision to its sales reps. Each firm offers a compensation plan that takes the form of $W = \alpha + \beta(q_i)$, where α is the fixed salary and β is the commission on the unit sold for firm $i = 1, 2$. The compensation plan is intended to entice the sales reps to exert effort e_i . For any given retail price p_i and effort level e_i , the demand for firm i 's product is given by: $q_i(p_i, e_i, p_j, e_j) = h - p_i + \theta_p p_j + e_i - \theta_s e_j + \delta$, where θ_p represents price competition intensity, θ_s represents sales reps effort competition intensity, and δ represents demand uncertainty. By examining the non-cooperative game played by these two competing firms, Bhardwarj (2001) determines the Nash equilibrium (Proposition 3 on page 151) that exhibits the following properties: both firms will delegate their pricing decisions to their sales reps in equilibrium when price competition is more intense (i.e., when $\theta_p > \theta_s$); and both firms will not delegate in equilibrium when effort competition is more intense (i.e., when $\theta_s > \theta_p$). Observe that our base model is related to Bhardwarj's model when the firm does not delegate and that the inductor's reward r is related to the sales reps' commission β in Bhardwarj's model. However, we obtain different results because our model is monopolistic while his model is duopolistic; our sales response function (i.e., the demand function) is endogenous while his demand function is exogenous; and our inductor's reward r is a decision variable while the sales reps' commission β is given exogenously. Specifically, Bhardwarj (2001) shows that the commission β has no impact on the selling price in equilibrium (see the no delegation case in Table 1 on page 154), while we show that the inductor's reward r is directly related to the discount incentive d_1 for the inductees in equilibrium (Theorems 1 and 2 in our paper).

Second, in the sales effort model, customers (i.e., inductees in the RRP context) are "passive" participants so that more customers will buy the product if the sales agents (i.e., inductors in the RRP context) exert more efforts. Consequently, the sales response function considered in most sales effort models is usually given *exogenously* so that the purchasing behavior of each customer (or inductee) is not modeled explicitly. However, in order to develop a base model that would enable us to capture the issue of impression management that would affect the inductor's effort and the inductee's purchasing behavior, our base model of RRP captures the purchasing behavior of each inductee in an explicit manner. Specifically, we consider each inductee is an "active" participant and each inductee's purchasing decision depends on the inductee's valuation and the discounted price offered by the firm that are not considered in the sales effort model. Hence, the inductee's response in our model is determined *endogenously* and it is derived from the rational purchasing behavior of each inductee.

In the traditional sales effort models, it is assumed that the sales agent (i.e., the inductor) can increase the number of customers (i.e., inductees) to sign up the firm's service (in expectation) by increasing his efforts. However, as we extend our base model to capture

the issue of impression management, the extended model presented in Section 4 is different from the traditional sales effort models in the following sense. First, each inductee's purchasing decision depends on the incentive for the inductee to sign up the firm's service as well as the inductee's impression about the inductor's referral reward and his effort. Second, due to the potential negative impression about the inductor's intention due to his referral reward and his effort, the inductees may refuse to sign up the firm's service when the inductor's referral reward and his effort create strong negative impression. To our knowledge, the interaction between the incentives given to the agents (inductors in our RRP context) and the consumers (inductees in our RRP context) has not been considered in the sales effort literature. The sales effort model focuses on the dynamics between two parties: the firm and the sales agent (i.e., the inductor in the RRP context); however, because the inductee in the RRP model is an active participant, one needs to examine the inter-plays among three parties: the firm, the inductor, and the inductee.

Our research sheds light on a number of important questions related to RRP. First, by considering the interplay among the firm, the inductor, and the inductee, we solve a "nested" Stackelberg game so as to determine the optimal RRP in equilibrium. We determine the conditions under which it is optimal for the firm to reward the inductor only, reward the inductee only, or reward both. Second, we study under what conditions RRP would dominate direct marketing. Third, we then extend our model by incorporating impression management factors that are important aspects of incentivized referrals but have hitherto not been modeled in the literature. They are the inductor's intrinsic reward of making a positive impression by being seen as helping a friend, the inductor's concerns about creating a negative impression when making an incentivized referral, and the inductee's impression of the inductor's credibility when an incentive is involved. Specifically, we examine how impression management factors affect the reward structure of the optimal RRP, and we show that strong impression management factors can make referral rewards suboptimal. Our paper contributes to the extant literature by developing a comprehensive model to examine a firm's optimal RRP design and by determining how this design is affected by the aforementioned impression management factors.

In this paper, we present a model to capture the inductor's and the inductee's behavior as well as the environmental factors. Specifically, to explicate the mathematical structure of our model, we first present the base model that is intended to analyze the underlying dynamics among the firm, the inductor, and the inductee without considering impression management factors. Our base model uses a "nested" Stackelberg game in which the firm acts as the "super" leader of an "outer" game who decides on the reward mechanism to be given to the inductor and the inductee. For any given reward mechanism, the inductor and the inductee enter an "inner" game in which the inductor is the leader who chooses his referring effort in promoting the service (on behalf of the firm), and the inductee is the follower who makes her purchasing decision. By solving this nested game using backward induction, we show how the regular selling price and the inductor's referral effectiveness affect the structure of the optimal RRP that the firm should adopt in equilibrium. By comparing the firm's profits under the optimal RRP and under direct marketing, we identify conditions under which RRP dominates direct marketing. We then extend our base model of RRP to incorporate impression management factors and show how they significantly change the optimal reward structure of a RRP.

From our analysis we obtain the following results. First, the regular price and the inductor's referral effectiveness affect the reward structure of an optimal RRP. Second, RRP dominates direct marketing when the firm's number of current customers (i.e., its market

penetration) or their referral effectiveness is sufficiently high. Third, when we extend our model to incorporate impression management factors, our analysis reveals that the optimal reward structure shifts from rewarding the inductor only towards rewarding both or rewarding the inductee only. Also, there are cases in which the firm should “reward none” so that the firm should rely on unincentivized word of mouth (WOM) via the inductors’ social network to acquire new customers.

The remainder of this paper is organized as follows. We provide a brief review of the related literature in Section 2. Section 3 presents our base model that is intended to explicate the mathematical structure of the nested Stackelberg game without considering impression management factors. We determine the optimal RRP reward structure and show when a RRP dominates direct marketing. In Section 4, we extend our base model so as to examine the impact of impression management factors on the optimal reward structure. The paper ends with some concluding remarks and future research directions.

2. Literature review

Despite the popularity of RRP in practice, only a few studies have explored the optimal design of a RRP. In the marketing literature, experimental work focused largely on the inductor’s response to referral rewards associated with RRP. [Ryu and Feick \(2007\)](#) examined the impact of tie-strength (the relationship between the inductor and the inductee), brand strength and the RRP’s reward structure (reward only the inductor, only the inductee, or reward both) on the inductor’s likelihood to make referrals. By considering the extent to which the inductor is satisfied with the firms products, [Wirtz and Chew \(2002\)](#) showed that the referral reward is an effective mechanism to increase the inductor’s likelihood to make referrals, especially when the inductor is highly satisfied. In contrast to [Ryu and Feick \(2007\)](#) and [Wirtz and Chew \(2002\)](#) who focused on the inductor’s response, [Tuk et al. \(2009\)](#) examined the inductee’s responses to the RRP reward structure. They showed that the inductor’s reward can reduce the inductee’s likelihood to purchase because the inductor’s reward can be ill-perceived by the inductee and reduces the perceived sincerity of the inductor. In this paper, we capture the key findings of this stream of experimental work as follows: for any given reward structure (r, d) ([Ryu and Feick, 2007](#)), the inductor’s likelihood to make referrals and the inductee’s likelihood to purchase possess the same properties in our model as discovered in [Tuk et al. \(2009\)](#) and [Wirtz and Chew \(2002\)](#). As such, this present study complements the extant stream of experimental work.

There are two recent modeling papers that deal with RRP in the following manner. To our knowledge, [Biyalogorski et al. \(2001\)](#) are the first to analyze a RRP in which the reward is given to the inductor only: either through a referral reward that is contingent upon a successful referral, or through a lower price for the inductor that is intended to delight him so as to improve his willingness to make referrals. They show that, because the referral reward is contingent upon successful referral, the referral reward can be more effective because it avoids the free-rider problem that is present in the lower price strategy. Our model expands the issues examined by [Biyalogorski et al. \(2001\)](#) as follows. In addition to the inductor’s referral reward, we consider an RRP that also offers a reward to the inductee via a price discount. Because there are rewards for the inductor and for the inductee, we develop a nested Stackelberg game that enables us to capture the behavior among three major players (the firm, the inductor, and the inductee).

In addition to [Biyalogorski et al. \(2001\)](#), and [Kornish and Li \(2010\)](#) are the first who incorporated a psychological factor: the inductor will take the expected inductee’s satisfaction level with

a product into consideration when deciding whether to recommend that product. [Kornish and Li](#) consider the situation in which the inductor is more knowledgeable about a product’s performance than his friend, and he knows his friend’s needs and wants, which means that the inductor can then make an informed recommendation to the inductee. They investigate the impact of the inductor’s concern over the inductee’s satisfaction with the product on the optimal inductor’s referral reward and the inductee’s price. Specifically, inductors were modeled to be concerned about potentially negative effects of inductee dissatisfaction on the inductor’s reputation, which led to incentives having to be increased at first to compensate for that risk until it became too high and inductors preferred inductee discounts (and no reward to the inductor) to mitigate that risk. Our paper complements [Kornish and Li’s](#) work as follows. While [Kornish and Li \(2010\)](#) address the inductor’s concern about the inductee’s satisfaction level with the product, we examine impression management factors that directly relate to the reward structure of the RRP: (a) the inductor’s concern of making a negative impression when potentially being seen to be motivated by a reward and by benefiting from a friend’s purchase; (b) the flip side of the inductor’s negative impression management factors is the inductee’s concern about the inductor’s objectivity when an incentivized referral is involved; and (c) the inductor’s intrinsic reward though making a positive impression of being helpful to a friend, especially if the friend can receive an attractive discount as part of the RRP.

In summary, our paper contributes to the existing literature in the following ways: First, we develop a model about RRP that captures the major findings of a stream of experimental work about the inductor’s referral and inductee’s purchasing behaviors. Hence, our work complements the extant stream of experimental work. Second, relative to the existing game theoretical models of RRP, our model deals with additional decision variables (c.f., [Biyalogorski et al., 2001](#)), and the hitherto unexplored impression management factors that are directly related to inductor’s and inductee’s rewards (c.f., [Kornish and Li, 2010](#)). Third, we obtain closed form expressions for the optimal RRP reward structure, which enable us to examine the impact of different impression management and environmental factors. Fourth, besides helping us to formalize our understanding of RRP, our model can also serve as a building block for determining optimal reward structures in more general settings.

3. The base model

Consider an existing service firm (e.g., AT&T or Verizon) who offers a service in a market that is comprised of $m + N$ customers. Currently, the firm has m existing customers and the firm’s market penetration is ϕ , where $\phi = m/(m + N)$. Hence, the potential number of new customers is $N = m(\frac{1}{\phi} - 1)$.¹ Currently, the firm is offering its service at a regular price p to all existing customers. In our model, we assume p is given exogenously for two reasons.² First, this assumption is consistent with actual practice: the regular price p remains the same in the examples mentioned in Section 1. Second, in the context of RRP, a firm should not lower its regular price because it would allow those existing customers who do not serve as

¹ Our model does not deal with a start-up company that offers RRP so that each consumer can decide to be an inductor (i.e., an early adopter who signs up the firm’s service without referral and then refers to his friends) or an inductee (i.e., a late adopter who signs up later and obtains some rewards). To capture the consumers’ strategic behavior over time for a start up firm, one would need to develop a two-period dynamic model which is beyond the scope of this paper. The authors thank one reviewer for this insightful comment.

² Even though p is exogenously given, one can always search for the optimal price p numerically.

inductors to pay a lower price and would cause the firm to lose revenue. Instead of lowering the price to all existing customers by “leaving money on the table”, we focus on the usage of referral reward to inductors and the price discount to inductees. Moreover, we focus on analyzing the equilibrium behavior of the firm, the inductor, and the inductee. To obtain tractable results, we assume each new customer has a service valuation V , where $V \sim U[0, 1]$. To solicit new customers, the firm can use either (1) direct marketing (e.g., advertising and promotions) or (2) a RRP.

3.1. The benchmark model: direct marketing

Under direct marketing, the firm offers a discount d_0 (a decision variable that satisfies $d_0 \in (0, 1]$) so that each new customer can purchase the service at a discounted price pd_0 . The direct marketing program has two cost components: a fixed cost for advertising such as the design for TV and print advertisements K , and a variable cost for mass communication to potential customers k . This direct marketing program will enable the firm to reach αN potential new customers, where $\alpha \in [0, 1]$ is the penetration rate of direct marketing program. Under direct marketing, the interaction between the firm and the new customer can be modeled as a simple Stackelberg game in which the firm acts as the leader who selects the discount rate d_0 , and the new customer acts as the follower who decides whether to purchase (Fig. 1).

Given d_0 , each customer will buy the service if her valuation $V \geq pd_0 + h_0$, where h_0 is the “hurdle rate” which captures the implicit costs associated with the customer’s perceived risk due to unfamiliarity with the service and/or lack of knowledge and skills of using the service (Su, 2009). In this case, the probability of a potential customer to purchase the service is $P\{V \geq pd_0 + h_0\}$. Knowing the customer’s purchase probability, the firm can determine the optimal discount $d_0^* \in (0, 1]$ by solving the following problem:

$$\max_{d_0 \in (0, 1]} [N\alpha \cdot P\{V \geq pd_0 + h_0\} \cdot (pd_0 - c) - K - kN],$$

where c is the unit cost for serving a customer. Notice that $N\alpha$ is the effective number of potential customers generated by the direct marketing program, and kN is the cost associated with mass communication to all potential customers N . By using the fact that $V \sim U[0, 1]$, the firm’s expected profit can be rewritten as: $[N\alpha \cdot (1 - (pd_0 + h_0)) \cdot (pd_0 - c) - K - kN]$.

By considering the first-order condition, we obtain the following result:

Lemma 1. Under direct marketing, the optimal discount $d_0^* = \min \left\{ \frac{1-h_0+c}{2p}, 1 \right\}$, and the firm’s optimal expected profit $\pi_0^*(N, h_0)$ satisfies

$$\pi_0^*(N, h_0) = m \left(\frac{1}{\phi} - 1 \right) \alpha \left(\frac{1 - h_0 + c}{2p} \right)^2 - K - km \left(\frac{1}{\phi} - 1 \right). \quad (1)$$

Lemma 2 suggests that the firm should offer a deeper discount d_0^* if the regular price p or the hurdle rate h_0 is high. By noting that $d_0^* = 1$ when $p \leq \frac{1-h_0+c}{2}$, there is no incentive for the firm to set $p < \frac{1-h_0+c}{2}$ especially when the firm anticipates price discount ex-ante. As such, we focus on the case when $p \geq \frac{1-h_0+c}{2}$ in the remainder of this paper.

3.2. The base model: referral reward program

Note that our model is a one-period model which captures the context in which the firm needs to increase revenue by leveraging existing consumers. Therefore the underlying assumption is the firm has already existing consumers when it makes the decision

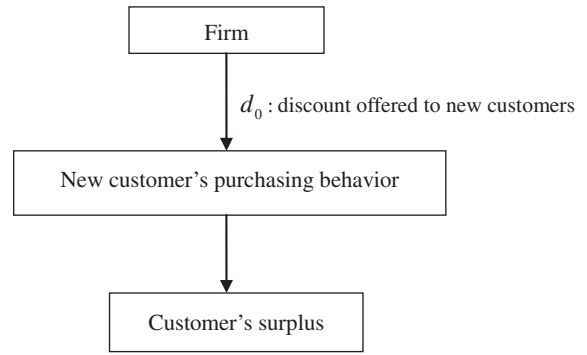


Fig. 1. Game structure for direct marketing.

regarding the structure of the referral reward program. Under RRP, the firm offers a reward structure with two components: (a) a referral reward r (a decision variable that has $r \geq 0$) to be given to each existing customer (an inductor) who refers a friend (an inductee) successfully; and (b) a price discount d_1 (a decision variable that satisfies $d_1 \in (0, 1]$) to be given to each inductee so that the inductee can purchase the service at a discounted price pd_1 . (We shall refer to the term $(1 - d_1)$ as the “discount depth” under RRP throughout this paper.) Observe that the reward structure (d_1, r) can be classified into four basic types: (1) Reward the inductor only; i.e., $r > 0, d_1 = 1$; (2) Reward both; i.e., $r > 0, d_1 \in (0, 1)$; (3) Reward the inductee only; i.e., $r = 0, d_1 \in (0, 1)$; and (4) Reward none; i.e., $r = 0, d_1 = 1$.

Given the reward structure, the interplay among the firm, the inductor and the inductee can be modeled as a “nested” Stackelberg game that integrates an inner game with an outer game (Fig. 2). We assume the inductor’s reward r and inductee’s purchase discount d_1 are common knowledge. In most instances, the reward structure is common knowledge through the firm’s marketing communication or the communication between the inductor and the inductee.

Given any reward (r, d_1) , the inductor and the inductee enter a Stackelberg game in which the inductor acts as the leader who chooses his referral effort e , and the inductee acts as the follower who makes her purchase decision. Throughout this paper, we shall refer to the inductor and the inductee as “he” and “she”, respectively. Anticipating the inductor’s and the inductee’s equilibrium responses in the inner game, the firm acts as the “super” leader who determines its reward structure (r^*, d_1^*) , that maximizes its equilibrium profit in the outer game. Once the equilibrium reward structure (r^*, d_1^*) is determined, one can retrieve other equilibrium outcomes accordingly.

3.2.1. Analysis of the inner game

For any reward structure (r, d_1) and for any referral effort e exerted by the inductor, the inductee will purchase the service if her valuation $V \geq pd_1 + h(e)$, where $h(e)$ is the inductee’s hurdle rate associated with inductor’s effort e (e.g., communication, explanation, demonstration, and persuasion). We assume the inductee’s hurdle rate $h(e)$ is decreasing and convex in the inductor’s effort e . For ease of exposition, we consider the case when $h(e) = h_0 - \theta\sqrt{e}$. Notice that the parameter θ represents the inductor’s referral effectiveness. Managerially speaking, the inductor’s referral effectiveness θ reflects the inductors’ overall satisfaction with the service (Wirtz and Chew, 2002). For instance, when the inductor is highly satisfied with the service, he is more effective in articulating the value of the service to the inductee (e.g., Jones and Sasser, 1995; Rust et al., 1995). Therefore, the inductee will purchase the service with probability $\beta(e) = P\{V \geq pd_1 + (h_0 - \theta\sqrt{e})\}$. Given the inductee’s

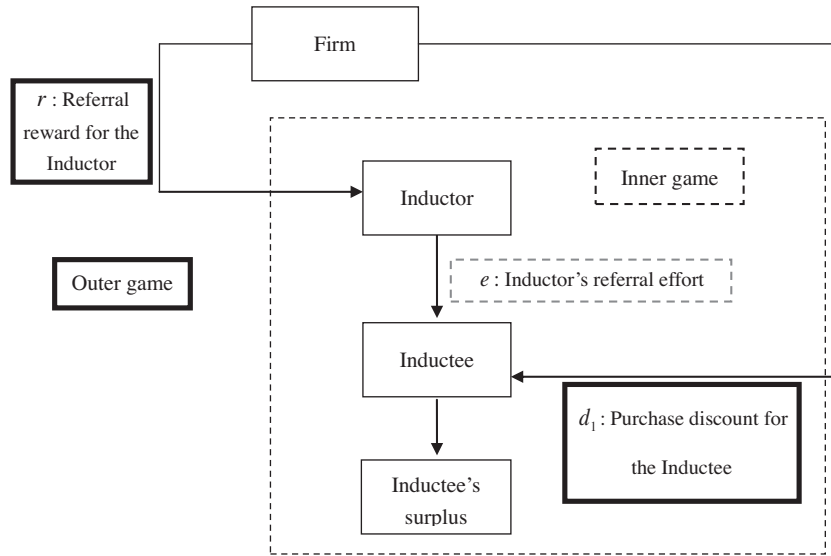


Fig. 2. The inner and outer games of a referral reward program.

purchase probability $\beta(e)$ and the referral reward r , the inductor's expected payoff for his referral effort e is equal to $r\beta(e) - e = r \cdot P\{V \geq pd_1 + (h_0 - \theta\sqrt{e})\} - e$. By using the fact that $V \sim U[0, 1]$, the inductor's problem can be written as:

$$\max_{e \geq 0} [r(1 - pd_1 - h_0 + \theta\sqrt{e}) - e].$$

By considering the first-order condition, the inductor's equilibrium referral rate satisfies:

$$e^* = \left(\frac{r\theta}{2}\right)^2. \tag{2}$$

Observe from (2) that the equilibrium referral effort e^* in the inner game is increasing in the inductor's referral reward r and the inductor's referral effectiveness θ . This result is consistent with the experimental findings reported by Ryu and Feick (2007). Therefore, for any given referral reward r , the inductor will participate the RRP if his net surplus from referring is non-negative, i.e., if $(r\beta(e^*) - e^*) \geq 0$. It follows from (2) that

$$\beta(e^*) = 1 - pd_1 - h_0 + \frac{r\theta^2}{2}. \tag{3}$$

Hence, the inductor's participation constraint can be rewritten as:

$$(r\beta(e^*) - e^*) = r \cdot \left(1 - pd_1 - h_0 + \frac{r\theta^2}{4}\right) \geq 0. \tag{4}$$

Observe from (3) that the inductee's purchase probability $\beta(e^*)$ and from (4) that the inductor's surplus are increasing with the inductor's referral effectiveness θ and the inductee's discount depth $(1 - d_1)$. Hence, these results are consistent with our intuition.

3.2.2. Analysis of the outer game

By anticipating the inductor's and the inductee's response as described in Section 3.2.1, we now analyze the outer game. For tractability, let us assume that each inductor will refer one inductee (when the inductor participates in the RRP).³ Hence, for any given (d_1, r) , the firm's expected profit is equal to

$$\pi_1(d_1, r) = [m \cdot \beta(e^*) \cdot (pd_1 - c) - m \cdot r \cdot \beta(e^*)], \tag{5}$$

³ We make this assumption for tractability. However, the same approach can be used to analyze the case when each inductor refers multiple inductees.

where the first term represents the gross profit obtained from the inductees and the second term represents the referral cost paid to inductors. By consider $\beta(e^*)$ given in (3), we can determine the firm's equilibrium reward structure by solving the following optimization problem:

$$\pi_1^* = \max_{d_1 \in (0,1], r \geq 0} \left[m \cdot \left(1 - pd_1 - h_0 + \frac{r\theta^2}{2}\right) \cdot (pd_1 - c - r) \right], \text{ subject to (4).}$$

By relaxing all constraints and by considering the Langrangean associated with the above optimization problem, we can identify the set of active constraints in the optimal solution. As it turns out, the optimal reward structure depends on the regular price p . For ease of exposition, we shall present our results according to different price levels. First, the following theorem deals with the case when the firm's regular price p is low.

Theorem 1 (Low Regular Price). Suppose $p < \frac{3-3h_0-c}{2}$. Then the firm should never reward both the inductor and the inductee; i.e., either $d_1^* = 1$ or $r^* = 0$. Specifically, the firm's optimal reward structure satisfies

$$(d_1^*, r^*) = \begin{cases} \left(\frac{1-h_0+c}{2p}, 0\right), & \text{if } \frac{\theta^2}{4} < 0.5, \\ \left(1, \frac{\theta^2 p + 2p - c\theta^2 - 2 + 2h_0}{2\theta^2}\right), & \text{otherwise.} \end{cases} \tag{6}$$

Theorem 1 can be interpreted as follows. When the firm's regular price p is low, the firm cannot afford to reward both the inductor and the inductee without spreading its rewards too thin and rendering them ineffective. Hence, the firm should not reward both. To reward either the inductor or the inductee but not both, the firm has to choose the player who offers the highest return for its reward. Consequently, it is intuitive to reward the inductor only when his referral effectiveness is sufficiently high (i.e., when $\frac{\theta^2}{4} \geq 0.5$). Otherwise, the firm should offer the incentive to the inductee only.

Substituting the optimal reward structure given in (6) into the firm's expected profit given in (5), we can determine the firm's optimal expected profit in equilibrium $\pi_1^* = \pi_1(d_1^*, r^*)$ accordingly. By considering the case when $p = 0.7$, $c = 0.2$, $h_0 = 0.4$ so that $p < \frac{3-3h_0-c}{2}$, the top chart in Fig. 3 illustrates the impact of the inductor's referral effectiveness (measured in terms of $\frac{\theta^2}{4}$) on the firm's optimal profit and its optimal reward structure (measured

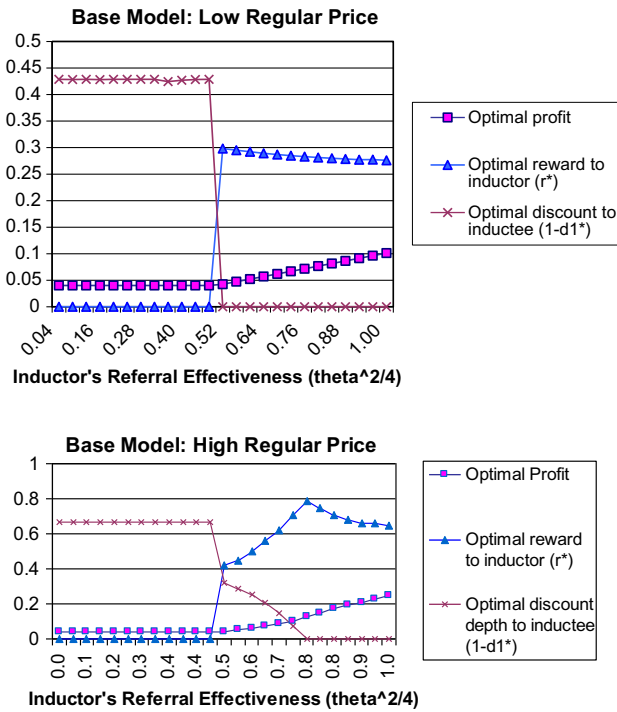


Fig. 3. Optimal RRP under low and high regular price.

in terms of the inductor’s referral reward r^* and the inductee’s discount depth $(1 - d_1^*)$.

By using the same approach as before, we can establish the following result for the case when the firm’s regular price p is high:

Theorem 2 (High Regular Price). Suppose $p \geq \frac{3-3h_0-c}{2}$. Then the firm’s optimal reward structure satisfies

$$(d_1^*, r^*) = \begin{cases} \left(\frac{1-h_0+c}{2p}, 0 \right), & \text{if } \frac{\theta^2}{4} \leq 0.5, \\ \left(\frac{1-h_0}{p} + \frac{\theta^2/4(1-h_0-c)}{2p(1-\theta^2/4)}, \frac{1-h_0-c}{2(1-\theta^2/4)} \right), & \text{if } 0.5 < \frac{\theta^2}{4} \leq \frac{2(p+h_0-1)}{(p-c)+(p+h_0-1)}, \\ \left(1, \frac{4(p+h_0-1)}{\theta^2} \right), & \text{if } \frac{2(p+h_0-1)}{(p-c)+(p+h_0-1)} < \frac{\theta^2}{4} \leq \frac{3(p+h_0-1)}{2(p-c)}, \\ \left(1, \frac{\theta^2 p + 2p - c \theta^2 - 2 + 2h_0}{2\theta^2} \right), & \text{if } \frac{\theta^2}{4} > \frac{3(p+h_0-1)}{2(p-c)}. \end{cases}$$

Contrary to Theorems 1, 2 asserts that it can be optimal for the firm to reward both the inductor and the inductee when p is sufficiently high. For instance, when the inductor’s referral effectiveness $\frac{\theta^2}{4}$ is in the medium range; i.e., when $0.5 < \frac{\theta^2}{4} \leq \frac{2(p+h_0-1)}{(p-c)+(p+h_0-1)}$, the firm should reward both the inductor and the inductee. Also, as the inductor’s referral effectiveness θ increases, the inductee’s hurdle rate $h(e^*) = h_0 - r\theta^2/2$ decreases and becomes more eager to purchase the service. In this case, the firm can discount less (i.e., a higher d_1^*). Finally, when the inductor is very effective in referring, say, when $\frac{2(p+h_0-1)}{(p-c)+(p+h_0-1)} < \frac{\theta^2}{4}$, the third statement reveals that it is optimal for the firm to reward the inductor only (i.e., $d_1^* = 1$). Also, by examining the inductor’s referral reward r^* as stated in Theorem 2, it can be shown that r^* is decreasing in the inductor’s referral effectiveness θ when $\frac{\theta^2}{4} > \frac{3(p+h_0-1)}{2(p-c)}$. This is because, as θ is sufficiently large, the inductee’s hurdle rate $h(e^*) = h_0 - r\theta^2/2$ is sufficiently low. As such, the firm can afford to lower the reward for the inductor r^* without affecting the inductee’s purchase probability significantly.

Substituting the optimal reward structure presented in Theorem 2 into the firm’s expected profit given in (5), we can determine the firm’s optimal expected profit in equilibrium $\pi_1^* = \pi_1(d_1^*, r^*)$

accordingly. By considering the case when $p = 1.2, c = 0.2, h_0 = 0.4$ so that $p \geq \frac{3-3h_0-c}{2}$, the bottom chart in Fig. 3 illustrates the impact of the inductor’s referral effectiveness level (measured in terms of $\theta^2/4$) on the firm’s optimal profit and its optimal reward structure (measured in terms of the inductor’s referral reward r^* and the inductee’s discount depth $(1 - d_1^*)$).

3.3. Comparing direct marketing and referral reward program

Comparing the firm’s optimal profit under direct marketing program $\pi_0(m, h_0)$ given in (1) in Lemma 2 and the firm’s optimal profit under RRP $\pi_1^*(d_1^*, r^*)$ by using the results stated in Theorems 1 and 2, we can establish the following intuitive result formally:

Corollary 1 (Direct Marketing versus RRP). There exists two thresholds τ_1 and τ_2 so that the Referral Reward Program dominates Direct Marketing when the firm’s current market penetration is sufficiently high ($\phi \geq \tau_1$) or when the inductor’s referral effectiveness is sufficiently high ($\theta \geq \tau_2$).

4. Extension: the issue of impression management

In this section, we extend the base model presented in Section 3.2 by incorporating impression management factors associated with referral reward (d_1, r) . First, from the inductee’s perspective, the credibility of the inductor’s recommendation would decrease if the inductor received a referral reward r . Hence, as the inductor’s referral reward r increases, the inductee’s hurdle rate increases because the inductee may have a suspicion about the inductor’s motive. To capture this effect, we modify the inductee’s hurdle rate $h(e)$ defined in Section 3.2.1 by including an additional term Δr so that

$$h(e) = h_0 - \theta\sqrt{e} + \Delta r, \tag{7}$$

where $\Delta > 0$.

Next, when incorporating the inductor’s impression management factors associated with any RRP reward (d_1, r) , the inductor’s effective reward equals to $r + s(r, d_1)$, where r is the inductor’s direct benefit and $s(r, d_1)$ captures the impression management costs and benefits associated with the following factors. First, regardless of the inductor’s referral reward r , the inductor may obtain an intrinsic reward $s_0 \geq 0$ that is associated with helping a friend make a good purchase decision, and make an impression of being helpful and knowledgeable when making a recommendation (e.g., Arndt, 1967; Dichter, 1966; Ryan and Deci, 2000). We shall refer s_0 as the intrinsic reward in the remainder of this paper. Second, when $r > 0$, the inductor may be concerned about being seen as wanting to benefit from his friend’s purchase decision (c.f., Tuk et al., 2009). Hence, there is a negative benefit $-s_1 r$ that is associated with this concern, which we shall refer to as the negative impression management in the remainder of this paper. Third, when $r = 0$ and $d_1 > 0$, the inductor may perceive a reward from managing a “positive impression” $s_2(1 - d_1) \cdot I_{\{r=0\}}$ (with $s_2 > 0$ and $I_{\{r=0\}}$ as the indicator function) that is associated with making the impression of being genuinely interested in helping the inductee to receive a discount without receiving any referral reward himself. In view of these three impression management factors, the inductor’s effective reward under RRP is equal to $r + s(r, d_1)$, where

$$s(r, d_1) = s_0 - s_1 r + s_2(1 - d_1) \cdot I_{\{r=0\}}. \tag{8}$$

Observe from (8) that the inductor’s effective reward $r + s(r, d_1)$ involves a step function $s_2(1 - d_1) \cdot I_{\{r=0\}}$. For this reason, we now first determine the firm’s optimal profit π_2^* for the case when $r = 0$ in

Section 4.1. Then, in Section 4.2, we determine the firm’s optimal profit π_2^* for the case when $r > 0$. Then, by comparing the firm’s optimal profits associated with these two cases, we can obtain the firm’s optimal reward structure that yields the highest profit for the firm numerically in Section 4.3.

4.1. Analysis of the inner game and outer game when the referral reward $r = 0$

When the inductor’s referral reward $r = 0$, we can use the hurdle rate $h(e)$ given in (7) to show that the inductee’s purchase probability $\beta(e)$ satisfies:

$$\beta(e) = P\{V \geq pd_1 + (h_0 - \theta\sqrt{e})\}. \tag{9}$$

By considering the inductor’s effective reward given in (8) for the case when $r = 0$ and by anticipating the inductee’s purchase probability $\beta(e)$ given in (9), one can determine the inductor’s optimal referral effort in equilibrium by solving the following problem:

$$\max_{e \geq 0} [(s_0 + s_2(1 - d_1)) \cdot \beta(e) - e]. \tag{10}$$

By using the fact that the valuation $V \sim U[0, 1]$ and by considering the first-order condition, we obtain the following result:

Lemma 2. When $r = 0$, the inductor’s referral effort in equilibrium satisfies:

$$e^* = \left(\frac{(s_0 + s_2(1 - d_1))\theta}{2} \right)^2. \tag{11}$$

Substituting e^* into (10), the inductor’s participation constraint can be written as:

$$1 - pd_1 - h_0 + \frac{(s_0 + s_2(1 - d_1))\theta^2}{4} \geq 0. \tag{12}$$

Hence, the firm’s problem associated with the outer game can be written as:

$$\begin{aligned} \pi_2^* &= \max_{d_1 \in (0,1)} \left[m \cdot \left(1 - pd_1 - h_0 + \frac{(s_0 + s_2(1 - d_1))\theta^2}{4} \right) \cdot (pd_1 - c) \right] \\ \text{s.t. } & 1 - pd_1 - h_0 + \frac{(s_0 + s_2(1 - d_1))\theta^2}{4} \geq 0. \end{aligned}$$

By considering the Kuhn–Tucker conditions associated with the Lagrangean function, we can establish the following result:

Theorem 3 (No Inductor Referral Reward). When $r = 0$, the firm should offer the inductee a discount d_1^* in equilibrium that satisfies:

$$d_1^* = \min \left\{ 1, \frac{1 - h_0 + (s_0 + s_2)\theta^2/4}{p + s_2\theta^2/4}, \frac{(p + s_2\theta^2/2)c + (1 - h_0 + (s_0 + s_2)\theta^2/2)p}{2p(p + s_2\theta^2/2)} \right\}.$$

By noting that d_1^* is decreasing in the inductor’s referral effectiveness $\theta^2/4$, it is easy to check that $d_1^* = 1$ (i.e., no discount for the inductee) when the inductor’s referral effectiveness $\theta^2/4$ is sufficiently high. This result reveals the possibility of having the firm to “reward none” ($r^* = 0, d_1^* = 1$) in the presence of impression management factors. We examine this possibility in Section 4.3.

4.2. Analysis of the inner game and outer game when the referral reward $r > 0$

When the inductor’s referral reward $r > 0$, we can use the hurdle rate $h(e)$ given in (7) to show that the inductee’s purchase probability $\beta(e)$ satisfies:

$$\beta(e) = P\{V \geq pd_1 + (h_0 - \theta\sqrt{e} + \Delta r)\}. \tag{13}$$

Anticipating the inductee’s purchase probability, one can determine the inductor’s referral effort in equilibrium by solving the following problem:

$$\max_{e \geq 0} [(r + s(r, d_1)) \cdot \beta(e) - e], \tag{14}$$

where (8) yields $s(r, d_1) = s_0 - s_1r$. By using the fact that the valuation $V \sim U[0, 1]$ and by considering the first-order condition, we obtain the following result:

Lemma 3. When $r > 0$, the inductor’s referral effort in equilibrium satisfies:

$$e^* = \left(\frac{(s_0 + (1 - s_1)r)\theta}{2} \right)^2. \tag{15}$$

Because $s_1 \in [0, 1]$, it is easy to check that the inductor’s referral effort e^* is increasing in the inductor’s referral reward r , the inductor’s intrinsic benefit s_0 , and the inductor’s referral effectiveness θ . Also, observe that the inductor’s referral effort e^* given in (15) reduces to the base case as given in (2) when $s_0 = s_1 = 0$.

By using (15) and (14) along with the fact that $s(r, d_1) = s_0 - s_1r$, the inductor’s participation constraint $((r + s(r, d_1))\beta(e^*) - e^* \geq 0)$ can be written as:

$$1 - pd_1 - h_0 + \frac{(s_0 + (1 - s_1)r)\theta^2}{4} - \Delta r \geq 0. \tag{16}$$

Hence, the firm’s problem associated with the outer game can be written as:

$$\begin{aligned} \pi_3^* &= \max_{d_1 \in (0,1)} \left[m \cdot \left(1 - pd_1 - h_0 + \frac{(s_0 + (1 - s_1)r)\theta^2}{4} - \Delta r \right) \cdot (pd_1 - c - r) \right] \\ \text{s.t. } & 1 - pd_1 - h_0 + \frac{(s_0 + (1 - s_1)r)\theta^2}{4} - \Delta r \geq 0. \end{aligned} \tag{17}$$

By considering the Kuhn–Tucker conditions associated with Lagrangean functions, we can determine the optimal reward structure (d_1^*, r^*) that involves the solutions associated with different regions within the feasible set. To simplify our exposition, let us introduce the following quantities. Let: $a = 1 - h_0 + \frac{s_0\theta^2}{2}$, $a' = 1 - h_0 + \frac{s_0\theta^2}{4}$, $k = -1 - \Delta + \frac{(1-s_1)\theta^2}{2}$, and $k' = -1 - \Delta + \frac{(1-s_1)\theta^2}{4}$. Also, we define:

$$\begin{aligned} (d_a^*, r_a^*) &= \left(\frac{a' + (k' + 1)r_a^*}{p}, \min \left\{ \frac{p - a'}{k'}, \max \left\{ -\frac{(k - k')(a' - c) + k'(a - a')}{2(k - k')k'}, \frac{1 - h_0 - c}{1 + \Delta} \right\} \right\} \right) \\ (d_b^*, r_b^*) &= \left(\frac{a + (k + 2)r_b^* + c}{2p}, \min \left\{ \frac{p - a'}{k'}, \frac{1 - h_0 - c}{1 + \Delta} \right\} \right) \\ (d_c^*, r_c^*) &= (1, 0) \\ (d_a^*, r_d^*) &= \left(\frac{a' + (k' + 1)r_d^*}{p}, \frac{1 - h_0 - c}{1 + \Delta} \right) \\ \text{if } \pi_3 \left(d_1 = \frac{a' + (k' + 1)r}{p}, r = \frac{1 - h_0 - c}{1 + \Delta} \right) &> \pi_3 \left(d_1 = \frac{a' + (k' + 1)r}{p}, r = \frac{p - a'}{k'} \right) \\ (d_a^*, r_d^*) &= \left(\frac{a' + (k' + 1)r_d^*}{p}, \frac{p - a'}{k'} \right) \text{ Otherwise} \\ (d_e^*, r_e^*) &= \left(\frac{a + (k + 2)r_e^* + c}{2p}, \min \left\{ \frac{p - a'}{k'}, \frac{1 - h_0 - c}{1 + \Delta} \right\} \right) \\ (d_f^*, r_f^*) &= (1, 0) \end{aligned} \tag{18}$$

By using those quantities defined in (18), we obtain the following results:

Theorem 4 (Reward Inductor ($r > 0$)). Suppose the firm chooses to reward the inductor under RRP so that $r > 0$. Then the firm’s optimal reward structure (d_1^*, r^*) can be described as follows:

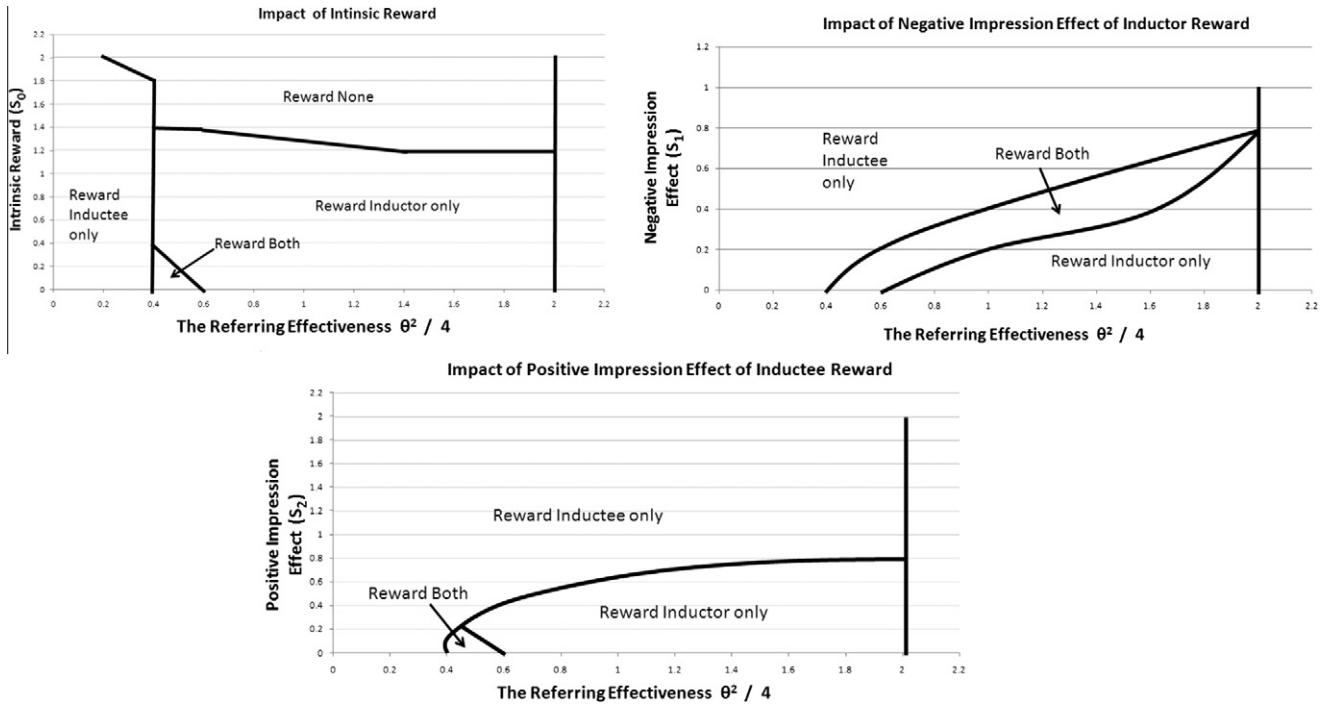


Fig. 4. Numerical analysis of the impacts of impression management factors on the optimal RRP.

1. When $\frac{\theta^2}{4} < \frac{1+\Delta}{2(1-s_1)}$, $(d_1^*, r^*) = (\min\{\frac{a+c}{2p}, \frac{a}{p}, 1\}, 0)$.
2. When $\frac{1+\Delta}{2(1-s_1)} \leq \frac{\theta^2}{4} < \frac{1+\Delta}{1-s_1}$, $(d_1^*, r^*) = \operatorname{argmax}\{\pi_3(d_a^*, r_a^*), \pi_3(d_b^*, r_b^*), \pi_3(d_c^*, r_c^*)\}$.
3. When $\frac{\theta^2}{4} \geq \frac{1+\Delta}{1-s_1}$, $(d_1^*, r^*) = \operatorname{argmax}\{\pi_3(d_d^*, r_d^*), \pi_3(d_e^*, r_e^*), \pi_3(d_f^*, r_f^*)\}$.

When the inductor’s referral effectiveness is low (i.e., when $\frac{\theta^2}{4} < \frac{1+\Delta}{2(1-s_1)}$), Theorem 4 reveals that the firm should never reward the inductor, i.e., $r^* = 0$. This result is consistent with the result obtained in Theorem 1 in the base model. It is easy to check from Theorem 4 that the case associated with $(\frac{\theta^2}{4} < \frac{1+\Delta}{2(1-s_1)})$ reduces to the base model for the case associated with $\frac{\theta^2}{4} < 1/2$ when $\Delta = 0$ and $s_1 = 0$. Also, as indicated in Statement 3, it is possible for the firm to reward none when the inductor’s effectiveness is sufficiently high; say, when $\frac{\theta^2}{4} \geq \frac{1+\Delta}{1-s_1}$. We shall investigate this further numerically in the next section.

By using the results obtained in Theorems 3 and 4 for the case when the inductor’s referral reward $r = 0$ and $r > 0$, respectively, we can determine the firm’s (global) optimal profit by selecting the case that yield the higher profit; i.e., the firm’s optimal profit is equal to $\Pi = \max\{\pi_2^*, \pi_3^*\}$. In addition, one can retrieve the optimal reward structure (d_1^*, r^*) that yields the maximum profit for the firm in the presence of various impression management factors.

4.3. Numerical analysis

We now examine the optimal reward structure (d_1^*, r^*) in the presence of impression management factors. Each of the impression management factors captured by the function $s = s(r, d_1) = s_0 - s_1 r + s_2 (1 - d_1) \cdot I_{(r=0)}$ given in (8) will have impact on the reward structure. Recall from Section 4 that the inductor’s intrinsic reward is captured by the parameter $s_0 \geq 0$; the inductor’s concerns about being seen as taking advantage of his friend are captured by the parameter $0 \leq s_1 \leq 1$ and the inductee’s suspicion is captured by the additional hurdle rate factor $\Delta \geq 0$; and the inductor’s “warm glow,” or the positive impression created for

enabling the inductee to obtain a discount ($0 < d_1 < 1$) while referring without any reward ($r = 0$), is captured by the parameter $s_2 > 0$. To examine how these impression management factors affect the optimal reward structure, we use the same parameter values as in the base case; namely, we set $p = 1.2$, $c = 0.2$, and $h_0 = 0.4$. To isolate the effect of each of the impression management factors, we conduct our numerical examples as follows:

Case 1: The Impact of Intrinsic Reward ($s_0 > 0$).

Using the base model as the benchmark, we examine how the reward structure changes when we vary the parameter s_0 from 0 to 2. To isolate the effects of changing s_0 , we set $s_1 = 0$, $s_2 = 0$, and $\Delta = 0$. Therefore, the optimal reward structure in the presence of intrinsic reward will reduce to the base case when $s_0 = 0$. Recall from Theorem 2 that the optimal reward structure for the base case when the regular price p is sufficiently high can be described as follows: reward the inductee only when the inductor’s referral effectiveness is low; reward both when his effectiveness is at a medium level; and reward the inductor only when his referral effectiveness is high. Observe from the top left chart in Fig. 4 that as the intrinsic reward s_0 increases, the optimal reward structure shifts away from rewarding the inductor and shifts towards “rewarding none” when s_0 exceeds a certain threshold. Therefore, unincincentivized RRP (using word of mouth) can be an effective mechanism for acquiring new customers when the inductors have high intrinsic rewards for referring the firm’s services to their friends.

Case 2: The Impact of Negative Impression Effect ($s_1 > 0$ and $\Delta = \lambda s_1$).

Similarly, using the base model as the benchmark, we examine how the optimal reward structure changes as the negative impression effect s_1 increases from 0 to 1. To isolate the effects of changing s_1 , we set $s_0 = 0$, $s_2 = 0$, and $\Delta = \lambda s_1$ where $\lambda = 0.8$. The top right chart in Fig. 4 depicts the optimal reward structure. It is easy to check that, as the negative impression effect s_1 increases, the

optimal reward structure shifts towards rewarding the inductee only. For instance, as we increase s_1 from 0 to 0.8, the optimal reward structure shifts toward rewarding the inductee and rewarding both. Then, as s_1 exceeds a certain threshold, say, ($s_1 > 0.8$), it is optimal for the firm to reward the inductee only.

Case 3: The Impact of Positive Impression Effect ($s_2 > 0$).

Again, using the base model as the benchmark, we examine how the reward structure changes when we vary the parameter s_2 from 0 to 2. To isolate the effects of changing s_2 , we set $s_0 = 0$, $s_1 = 0$ and $\Delta = 0$. Observe from the bottom chart in Fig. 4 that, as the positive impression effect s_2 increases, the optimal reward structure shifted towards rewarding the inductee only. Specifically, when the positive impression effect s_2 exceeds a certain threshold, it is optimal for the firm to reward the inductee only.

As these cases show, all three types of impression management factors affect the optimal design of RRP and shift optimal reward structures away from the inductor. It becomes optimal to reward none and rely on WOM if strong intrinsic benefits exist and a threshold level of inductor effectiveness is exceeded, and to reward the inductee only if either high negative impression management factors or high positive impression management factors are present. Rewarding the inductor is never optimal in these cases.

5. Concluding remarks

Although referral reward programs (RRPs) have become ubiquitous, their effectiveness has been examined only by a handful of experimental (Ryu and Feick, 2007; Tuk et al., 2009; Wirtz and Chew, 2002) and modeling studies (Biyalogorski et al., 2001; Kornish and Li, 2010). In this paper, we develop a benchmark model that explores the optimal reward structure a firm should offer in equilibrium. We found that the optimal design of a RRP depends on the regular selling price and the referral effectiveness of the inductor. In particular, when the regular selling price is low, the firm cannot afford to offer rewards to the inductor and the inductee without spreading its rewards too thin and rendering them ineffective. Hence, the firm should not reward both. To reward either the inductor or the inductee but not both, the firm has to choose the player who offers the highest return for its reward. Consequently, it is intuitive to reward the inductor only when his referral effectiveness is sufficiently high. Otherwise, the firm should offer the incentive to the inductee only. Comparing the firm's profits under the optimal RRP and under direct marketing, we found that RRP dominates direct marketing when the firm's number of current customers (i.e., its market penetration) or their referral effectiveness is sufficiently high.

We extended our model to include impression management factors that were hitherto unexplored in the modeling literature. Our analysis revealed that the optimal reward structure shifts from rewarding the inductor only toward rewarding both, rewarding the inductee only, and there even exist optimal RRP designs that call for "reward none." When the latter happens, the firm should rely on unincited WOM via inductors social network to acquire new customers.

Our model helped us to formalize our understanding of RRP, but it can also serve as a building block for determining optimal reward structure in more general settings. Specifically, our numerical analysis explored all three impression management factors in isolation for parsimony, but the model can be used to explore optimal RRP designs of any combination of the impression management factors.

Our study offers important managerial implications. The findings of our base model can help firms understand (1) when to

use RRP rather than direct marketing (e.g., when their market penetration is high, or when their customers are highly satisfied or delighted), and (2) how to design an optimal RRP (e.g., reward the inductor, the inductee, both or none). Key variables that determine the optimal design are the inductor's referral effectiveness (increasing effectiveness favors shifting rewards from the inductee to the inductor), and the regular selling price (increasing the regular selling price favors rewarding both the inductor and inductee).

Our extended model provides guidelines for firms whose customers consider impression management factors as important. Conditions with high intrinsic rewards in particular are interesting as they call for reward none once a certain level of inductor referral effectiveness is exceeded. This means that situations, where inductors' objective is to be seen as knowledgeable and/or as helpful, and their referral effectiveness is sufficiently high, do not call for rewarding any of the parties. Such situations may include high involvement services which inductors enjoy talking about and make them appear in a positive light (Arndt, 1967; Dichter, 1966). In contrast, if inductors are concerned about making a potentially negative impression as being motivated by a reward and as benefiting from the inductee's purchase decision, the optimal RRP design shifts with increasing concern from rewarding the inductor, to rewarding both, and finally, to rewarding the inductee only. Impression management concerns may be most important in weak to medium-tie relationships, such as acquaintances and work colleagues, where single actions like a referral can shape impressions more strongly (c.f., Ryu and Feick, 2007; Tuk et al., 2009). This is in contrast to strong tie relationships (e.g., such as immediate family members who may be unlikely to shift their impression because of a referral reward as impressions are quite stable) and very weak ties, where inductors may be much less concerned about the impression they may create (e.g., in an anonymous online context).

Future research could be extended to consider the optimal designing of RRP under competitive markets. Also, it can be interesting to study how a firm can design a menu of RRP considering consumers heterogeneity in impression management concerns. Moreover, as mentioned in Section 3, it would be of interest to develop a 2-period dynamic model for a start-up firm that offers RRP so that each consumer can decide to be an inductor (i.e., an early adopter who signs up the firm's service without referral and then refers to his friends) or an inductee (i.e., a late adopter who signs up later and obtains some rewards).

References

- Arndt, J., 1967. Word-of-mouth advertising and informal communication. In: Cox, D.F. (Ed.), *Risk Taking and Information Handling in Consumer Behavior*. Division of Research, Harvard University, Boston, MA, pp. 188–239.
- Basu, A., Lal, R., Srinivasan, V., Staelin, R., 1985. Salesforce Compensation Plans: An Agency Theoretic Perspective. *Marketing Science* 4 (4), 267–291.
- Bhardwarj, P., 2001. Delegating pricing decisions. *Marketing Science* 20 (2), 143–169.
- Biyalogorski, E., Gerstner, E., Libai, B., 2001. Customer referral management: Optimal reward programs. *Marketing Science* 20 (1), 82–95.
- Brown, J.J., Reinigen, P.H., 1987. Social ties and word-of-mouth referral behavior. *Journal of Consumer Research* 14 (December), 350–362.
- Chen, F.R., 2000. Sales-force incentives and inventory management. *Manufacturing & Service Operations Management* 2 (2), 186–202.
- Chen, F.R., 2005. Salesforce incentives, market information, and production/inventory planning. *Management Science* 51 (1), 60–75.
- Dichter, E., 1966. How word-of-mouth advertising works. *Harvard Business Review* 44 (6), 147–411.
- Jones, T.O., Sasser, W.E., 1995. Why satisfied customers defect. *Harvard Business Review* 73 (6), 88–99.
- Kornish, L.J., Li, Q., 2010. Optimal referral bonuses with asymmetric information: Firm-offered and interpersonal incentives. *Marketing Science* 29 (1), 108–121.
- Lal, R., Srinivasan, V., 1993. Compensation plans for single-and multi-product salesforces. *Management Science* 39 (7), 777–793.
- Lal, R., Staelin, R., 1986. Salesforce-compensation plans in environments with asymmetric information. *Marketing Science* 5 (3), 179–198.

- Rao, R., 1990. Compensating heterogenous salesforces: Some explicit solutions. *Marketing Science* 9 (4), 319–341.
- Rust, R.T., Zahorik, A.J., Keiningham, T.L., 1995. Return on quality (ROQ): Making service quality financially accountable. *Journal of Marketing* 59 (April), 58–70.
- Ryan, R.M., Deci, L., 2000. Intrinsic and extrinsic motivations: Classic definitions and new directions. *Contemporary Educational Psychology* 25, 54–67.
- Ryu, G., Feick, L., 2007. A penny for your thoughts: Referral reward programs and referral likelihood. *Journal of Marketing* 71 (1), 84–94.
- Su, X., 2009. A model of consumer inertia with applications to dynamic pricing. *Production & Operations Management*.
- Tuk, M.A., Verlegh, P.W.J., Smidts, A., Wigboldus, D.H.J., 2009. Sales and sincerity: The role of relational framing in word-of-mouth marketing. *Journal of Consumer Psychology* 19, 38–47.
- Wirtz, J., Chew, P., 2002. The effects of incentives, deal proneness, satisfaction and tie strength on word-of-mouth behavior. *International Journal of Service Industry Management* 13 (2), 141–162.