BOOK REVIEWS

Arnold's problems, Vladimir I. Arnold (Editor), with a preface by V. Philippov, A. Yakivchik, and M. Peters, Springer-Verlag, Berlin; PHASIS, Moscow, 2004, xvi+639 pp., EUR 99.95, ISBN 3-540-20614-0

1. The genre

The literature of mathematics comprises millions of works, published ones as well as ones deposited in electronic archives. The number of papers and books included in the Mathematical Reviews database since 1940 (the first year of operation of MR) is now more than 1.9 million, and more than 75 thousand items are added to the database each year [28]. The overwhelming majority of works in this ocean contain new mathematical theorems and their proofs. In addition, many works also formulate unsolved problems, often in the form of precise conjectures. How essential is it for the development of mathematical science to draw the readers' attention unceasingly to open problems? Maybe it would suffice to publish only new results? The first-rank mathematicians of the present time give a definitive answer to this question.

In his preface to the first Russian edition [20] of the book under review,[1] V. I. Arnold reminisced: "I. G. Petrovskiĭ, who was one of my teachers in Mathematics, taught me that the most important thing that a student should learn from his supervisor is that some *question is still open*. Further choice of the problem from the set of unsolved ones is made by the student himself. To select a problem for him is the same as to choose a bride for one's son."[2] In his preface to the present edition, Arnold writes: "For a working mathematician, it is much more important to know what questions are not answered so far and failed to be solved by the methods already available, than all lists of numbers already multiplied, and than the erudition in the ocean of literature that has been created by previous generations of researchers over twenty thousand years."

In accordance with this, such a specific sort of mathematical literature as *lists of unsolved problems* has been of great importance in mathematics. Of all the works by Arnold, about ten papers are just lists of open questions in one branch of mathematics or another, accompanied by a discussion (see, e.g., [5], [7], [8], [9], [10], [11], [12], and [13]).[3] The total number of such lists published by various authors in

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2000 *Mathematics Subject Classification.* Primary 00A07, 01A72.

[1] The English translation of this preface is included in the present edition.

[2] Concerning this maxim of Petrovskiĭ, see also [11] and [13].

[3] Of course, all the problems in these papers have been included in the book under review.

different years and pertaining to different mathematical areas cannot be estimated even roughly, even taking into account the current power of MathSciNet (not to mention compiling the bibliography of all the lists, although that would be very useful). However, there have also been lists of open problems whose scope is, in a certain sense, the whole of mathematics and even some adjoining fields of natural science.

Of all such lists, the most famous one is Hilbert's 23 problems. As is rather well known [35], [45], these problems were compiled by D. Hilbert in 1900, and ten of them (namely, problems 1, 2, 6, 7, 8, 13, 16, 19, 21, and 22) were described in Hilbert's lecture delivered before the Second International Congress of Mathematicians in Paris on August 8, 1900 (see the English translation [31] of Hilbert's paper with the full list of 23 problems; this translation was reprinted in, e.g., [25, pp. 1–34], [30], [32], and [52]). Contrary to widespread opinion, Hilbert himself did not think that the problems he had selected were indeed the most significant ones for mathematics in the coming twentieth century. He wrote: "Permit me in the following, tentatively as it were, to mention particular definite problems, drawn from various branches of mathematics, from the discussion of which an advancement of science may be expected" and "The problems mentioned are merely samples of problems, yet they will suffice to show how rich, how manifold and how extensive the mathematical science of to-day is" [25, pp. 1–34], [30], [31], [32], [52]. Nevertheless, Hilbert's 23 problems deeply influenced the development of mathematics in the last century, ideologically as well as scientifically proper. Getting ahead of the story, note that Arnold solved Hilbert's 13-th problem (about the representability of continuous functions in three variables as superpositions of continuous functions in two variables) and obtained profound results concerning the 16-th problem (about the topology of real algebraic curves and surfaces). On the other hand, Arnold said in one of his interviews: "I very much doubt the importance of such fashionable mathematicians-axiomatists as Hilbert and Bourbaki" [38] (see also [18]).

In May 1974, a special international symposium on the mathematical developments arising from Hilbert's Problems was held at Northern Illinois University in De Kalb, Illinois. The proceedings of this symposium, edited by F. E. Browder, were published in 1976 [25]. The book [25] contained also an extensive new list of unsolved problems divided into 27 groups. This list, prepared by 26 authors in all, constituted the article "Problems of present day mathematics" [25, pp. 35–79]. It is worthwhile to note that 12 problems in this list were due to Arnold (jointly with G. Shimura in one of the cases).[4] Another book where the progress achieved by the end of the 1960s in solving Hilbert's Problems was analyzed is the collection [1]. The current state of the research related to Hilbert's Problems is expounded in the recent books [30] and [52]; see also the articles [29], [35].

In the 1990s Arnold, on behalf of the International Mathematical Union, wrote to a number of prominent mathematicians of the world with a suggestion that they describe some important unsolved problems that twentieth-century mathematics bequeathed to mathematics of the next century. I am aware of only one *explicit* response to this suggestion. In June 1997, at The Fields Institute in Toronto, there took place a conference to celebrate Arnold's sixtieth birthday (the 'Arnoldfest'). At this conference, S. Smale delivered the lecture 'Great problems' on June 19 [23, p. xiv], where he listed 18 problems. The content of Smale's talk was published in

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[4]Of course, all these 12 problems have also been included in the book under review.

[48] and (in an enlarged version with three more problems of lesser significance) in [49].

One should emphasize that, as Arnold wrote, "most researchers carefully avoid any thinking on the old classical problems," because these problems are in many cases extremely difficult and "it is much easier to obtain a new result in an unexplored domain" [12]. That is why the Clay Mathematics Institute (CMI) decided to financially stimulate attempts at solving such classical problems, and on May 24, 2000, the CMI announced the 'Millennium Prize Problems' project [26], [33]. This is a list of seven problems that the CMI regards as "important classic questions that have resisted solution over the years." The first person to solve each problem will be awarded a prize of $1 million by the CMI.

It is well known that the famous prize of 100 thousand marks for proving Fermat's Last Theorem, arranged by P. Wolfskehl in January 1905 (the conditions for this prize were published in 1908) [22], resulted in a horrible avalanche of absurd 'proofs' as well as in the widespread idea that Fermat's Last Theorem is the central question of all mathematics. The latter opinion seems to have been very popular among laymen until now, even after the Wolfskehl Prize was conferred on A. Wiles in 1997. Some admirers of Fermat's Last Theorem confuse Pierre de Fermat with Enrico Fermi (I have encountered a case like that myself). Since none of the Millennium Prize Problems can be formulated as elementarily as Fermat's Last Theorem is, one may hope that those problems will never cause such excitement.

Interestingly, the intersection of Hilbert's problem list of 1900, Smale's list of 1997, and the CMI list of 2000 is not empty: the Riemann Hypothesis is included in all three lists.

The book under review belongs, to some extent, to the same genre of problem lists, especially well-liked at the turn of each century. However, in Hilbert's paper [25, pp. 1–34], [30], [31], [32], [52]; in the article [25, pp. 35–79]; in Smale's lecture [48], [49]; and in works by many other authors,[5] there are listed a small number of more or less fundamental problems open as of the writing of the paper in question. On the other hand, the book under review comprises *plenty of problems* of various degrees of importance which have been posed at Arnold's seminars and in his works *during almost half a century*—from 1956 to 2003. This fact, together with the presence of detailed accounts of the results achieved by now in solving many problems, determines the unique nature of the book.

2. The author of the problems

Of course, there is no necessity at all to introduce Vladimir Igorevich Arnold to the *Bulletin*'s readers. One of the leading mathematicians of our day, he has made a fundamental contribution to such quite different mathematical and mechanical sciences as dynamical system theory, the theory of singularities of differentiable mappings, function theory, symplectic and contact topology, algebraic topology, real algebraic geometry, hydrodynamics and magnetohydrodynamics, the theory of hyperbolic partial differential equations, number theory, etc. The list of Arnold's

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[5]Let me point out ten more problem lists included in the book [19], just as examples: [24], [27], [34], [36], [41], [42], [43], [46], [50], and [53].

main results until 1997 compiled by him was published in the book [14, pp. xliii–xlviii]. Among Arnold's mathematical achievements, there are solutions of individual hard problems (e.g., that of Hilbert's 13-th problem) as well as the creation of new mathematical theories and the foundation of new directions of research.

One of the peculiarities of Arnold's mathematical style is that he often finds unexpected links between quite different questions. Arnold himself writes about this as follows: "Mysterious connections between areas of mathematics that seem to be completely diverse at first glance remain an enigma to me until now. Discovering such connections is one of the greatest delights mathematics can give, and I have been lucky to experience this delight several times in various branches of mathematics" (this is a quotation from Arnold's papers [15] and [21]; see also his lectures [16], [17] and interview [37]). For instance, Arnold discovered links between turbulence problems and the geometry of infinite-dimensional Lie groups (1966), between braid theory and singularity theory/algebraic geometry (1968), between Hilbert's 16-th problem and four-dimensional topology (1971), and between the theory of critical points of functions and the theory of Coxeter groups (1972).

Many notions, statements, and theories in mathematics are named after Arnold, and despite the so-called 'Arnold Principle' about the pioneers (formulated by M. Berry),[6] he is the true inventor (or one of the inventors) of those notions, statements, and theories. It suffices to mention the Kolmogorov–Arnold–Moser (KAM) theory, Arnold diffusion, the Liouville–Arnold theorem, Arnold tongues in perturbation theory, Arnold–Beltrami–Childress (ABC) flows, the Arnold method in hydrodynamics, the Arnold conjectures in symplectic topology, the Arnold classification of singularities, the Hilbert–Arnold problem (the infinitesimal Hilbert 16-th problem), the Maslov–Arnold characteristic class(es), the Arnold–Jordan normal form of matrix families, the Arnold complexity of dynamical systems, etc.

Arnold has created a huge mathematical school. For over 35 years, there has been his famous seminar on the theory of singularities of differentiable mappings at the Department for Mechanics and Mathematics of the Moscow State University. In 1993, the 'Parisian branch' of this seminar started (initially the Parisian branch met at the École Normale Supérieure; now it meets at the Institut de Mathématiques de Jussieu). During the last decade Arnold spent, as a rule, the spring semester (the first half of each year) in Paris and the autumn semester (the second half) in Moscow. Every semester, at the first meeting of the seminar (after the vacation), Arnold posed and discussed several (usually from 10 to 15) open problems. These problems collected under one cover, together with the problems published by Arnold in numerous papers and books, constitute the first part ('The Problems', pp. 1–179) of the book under review.

3. The book

Arnold explained in his preface to the first Russian edition [20]: "Mainly, I did not write my problems down, especially in the sixties; therefore most of them are probably lost. Some problems are included in my papers and books. Sometimes I reconstructed my problems to the seminar from conversations with my colleagues and friends. I hope that below the authors are quoted in most of such situations."

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[6]This principle reads: "If a notion bears a personal name, then this name is not the name of the discoverer;" see [16].

As was already mentioned, the problems in the first part of the book under review span a period of 48 years, from 1956 to 2003. They are arranged in chronological order. In a number of cases, some problems from different years are similar or almost identical to each other. There are no problems dated from the six early years 1957, 1960, 1961, 1962, 1964, and 1967—most of the problems posed by Arnold in the 1960s seem to have been lost indeed. The number of problems per year varies from 1 (in 1956 and 1959) to 53 (in 1994). The total number of problems is equal to 772, so that the average number of problems per year is $772/42 = 18.4$.

The problems pertain to quite diverse branches of mathematics (not only to singularity theory) and are remarkably heterogeneous in their nature. They differ very much in their scope and difficulty. Some problems are fundamental for the area under consideration; others are devoted to minor details. A number of problems are short questions to be answered 'yes' or 'no' (e.g., problem *1975-14* reads: "Is the corank a topological invariant?"). However, for almost all the problems, such an answer in one word would not be appropriate, even if the formulation of the problem is very short (e.g., problem *1980-12* is: "Complexify the homology theory"). Yet the formulations of some problems (especially of those of the 1990s and 2000s) take several pages, include a detailed discussion with various examples and a bibliography, and outline an extensive program of research. In his preface to the first edition [20], Arnold wrote: "Poincaré used to say that precise formulation, as a question admitting a '*yes* or *no*' answer, is possible only for problems of little interest. Questions that are really interesting would not be settled this way: they yield *gradual* forward motion and *permanent* development." In the preface to the present edition, Arnold emphasizes again: "Problems of binary type admitting a 'yes–no' answer (like the Fermat problem) are of little value here. One should rather speak of wide-scope programs of explorations of new mathematical (and not only mathematical) continents, where reaching new peaks reveals new perspectives, and where a preconceived formulation of problems would substantially restrict the field of investigations that have been caused by these perspectives."

The second part of the book under review ('Comments', pp. 181–636) is a collection of comments to the problems. The main goal of these comments is to describe what progress has been achieved by now in the solution of one problem or another and, on a broader scale, in the research that has arisen from the problem in question. The comments are written by 59 persons (including Arnold himself); they are mostly Arnold's former students and/or participants in his seminar. At the end of the book, there is an author index for comments (featuring the problem numbers). Some problems are given several comments by different authors. As is explained in the editorial, "Each comment is opened by a notation indicating its nature: the letter $\mathcal{H}$ means that the comment is *historic*, and $\mathcal{R}$ means that the comment is devoted to the *results* of the research on the problem" (in fact, the book uses much larger and 'nicer' characters $\mathcal{H}$ and $\mathcal{R}$ than those displayed here). The 'historical comments' are, as a rule, editorial and basically say that the problem in question is in such-and-such a paper by Arnold. The 'comments on results' are as heterogeneous as the problems themselves. These comments range from a short piece of information[7] to a detailed survey with an extensive bibliography. All the comments

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[7]The comment by V. A. Vassiliev to problem *1972-32* and that by S. M. Gusein-Zade to problem *1985-17* consist of five words each: "Nothing is known to me" and "It is not known yet," respectively.

(as well as formulations of all the problems) have been checked by Arnold. About 40% of the problems are accompanied by short 'historical' comments only or no comments at all. In almost all the cases, this means only that nobody has undertaken the task of preparing a suitable comment rather than the absence of any progress in the problem's solution. In the first edition [20] of the book, the number of authors of comments was equal to 29 in all, and about 55% of the problems were left without comments or with short 'historical' comments only.

Although the mathematical research originating from Arnold's problems is far from being recorded *in full measure* by the comments in the second part of the book, even those comments show what an enormous role these problems have played in the development of many diverse areas of mathematics since the 1960s. In the preface to the present edition of the book, Arnold writes about his problems: "The observed half-life of the problem (of its more or less complete solution) is about seven years on average. Thus, many problems are still open, and even those that are mainly solved keep stimulating new research appearing every year in journals of various countries of the World.

"The invariable peculiarity of these problems was that Mathematics was considered there not as a game with deductive reasonings and symbols, but as a part of natural science (especially of Physics), that is, as an experimental science (which is distinguished among other experimental sciences primarily by the low costs of its experiments)."

Many problems collected in the first part of the book under review have led to the creation of vast new mathematical theories and keep attracting the attention of a great number of actively working mathematicians. I will confine myself to three examples here:

1) Studies of the genericity of Arnold diffusion (the evolution of the action variables in nearly integrable Hamiltonian systems)—problems *1963-1* (this is a problem in the paper [2]), *1966-3* (in [5]), *1994-33* (in [12]). Arnold constructed the famous first example of diffusion [3] (hence the name).

2) The Nekhoroshev theory on exponential smallness of the rate of Arnold diffusion—problem *1966-2* (in [5]).

3) Research related to the Arnold conjectures about the number of fixed points of symplectomorphisms (symplectic diffeomorphisms) homologous to the identity—problems *1965-1*, *1965-2*, *1965-3* (in [4]), *1966-4*, *1966-5* (in [5]), *1972-17*, *1972-33* (in [6]), *1976-39* (in [8] and [25, p. 66]). Those conjectures started symplectic topology. Arnold proved the conjectures for exact symplectomorphisms of a torus that are not too far from the identity [4].

Of course, the book under review does not ideally fit in with the purpose Arnold mentioned in his prefaces to both editions: letting the reader know a lot of interesting unsolved problems. Many of the problems gathered in the book have already been solved, many problems are given without comments, and the book does not indicate whether the latter problems are still open or not. Many comments are becoming out of date fast: for instance, the recent new fundamental results by D. V. Treschëv [51], M.-R. Herman, P. Lochak, J.-P. Marco, and D. Sauzin (see, e.g., [39], [40], [47]), and L. Niederman [44] on Arnold diffusion and the Nekhoroshev theory have been left outside the scope of the corresponding comments. However, this book will undoubtedly help at least some readers master the high art of raising problems, at least to some extent. G. Cantor asserted that "to ask the right question is harder than to answer it" (these words were used as the epigraph to

the editorial to the first edition [20]). Reading the book under review, especially its first part 'The Problems', is a gripping pastime. A. S. Pushkin noticed in his tale 'The black of Peter the Great' that "following thoughts of a great person is the most interesting science." The book enables one to plunge into a fascinating kaleidoscope of ideas and results which constitute, taken together, a rather sizeable part of mathematics of the second half of the last century. And, last but not least, the design of the book is really beautiful. To summarize, the book under review is a wonderful gift PHASIS and Springer-Verlag have presented to the mathematical community. PHASIS completely prepared the book (a great part here was taken by A. N. Yakivchik); it was printed and is being distributed by Springer-Verlag.

I would like to finish with M. L. Gromov's words used as the epigraph to the second part, 'Comments', of the book: "You are never sure whether or not a problem is good unless you actually solve it."

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